Literature Review: Reducing Soft Costs of Rooftop Solar Installations Attributed to Structural Considerations

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Abstract

Typical engineering methods utilized to calculate stresses on a roof structure involve simplifying assumptions that render a complex non-linear structure a simple and basic determinate beam. That is, instead of considering the composite action of the entire roof structure, the engineer evaluates only a single beam that is deemed conservatively to represent an affected rafter or top chord of a truss. This simplification based on assumptions of a complex problem is where significant conservatism can be introduced. Empirical data will be developed to evaluate this issue.

Simple wood beams will be tested to failure. More complex and complete sections of roof structures that include composite action will also be tested to failure. The results can then be compared.

An initial step in this process involves a literature review of any work that has been performed on roof structure composite action. The following section summarizes the literature review that was completed.
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1. Introduction

Many jurisdictions across the United States require assurance that the structural integrity of a residential roof structure will not be compromised due to the installation of a solar system on it. This often requires an analysis by a qualified structural engineer to determine the applied loading on the roof is less than the allowable loads or stresses. There is believed to be conservatism in both the determination of applied loads that meet applicable regulations (typically summarized in ASCE 7-10) and the allowable stresses of the roof structure. Furthermore, there is believed to be significant conservatism in the applied engineering methodology utilized in calculating these values.

The International Building Code (IBC) and International Residential Code (IRC) both default to ASCE 7-10 for structural code guidance. Based on ASCE7-10, the actual weight of the solar installation (often less than 5 psf) can be a small percentage of the overall load to be considered. Wind loads, live loads, snow loads, drift loads are loads to be considered as well as other loads such as seismic in certain circumstances. Furthermore, they are to be considered in combination as defined by ASCE 7-10. The following are load combinations utilized by the allowable stress design methodology (ASCE 7-10):

1. $D + F$
2. $D + H + F + L + T$
3. $D + H + F + (L_r \text{ or } S \text{ or } R)$
4. $D + H + F + 0.75(L + T) + 0.75(L_r \text{ or } S \text{ or } R)$
5. $D + H + F + (W \text{ or } 0.7E)$
6. $D + H + F + 0.75(W \text{ or } 0.7E) + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$
7. $0.6D + W + H$
8. $0.6D + 0.7E + H$

where:

$D$ = dead load
$E$ = earthquake load
$F$ = load due to fluids with well-defined pressures and maximum heights
$H$ = load due to lateral earth pressure, ground water pressure, or pressure of bulk materials (generally zero for roof applications)
$L$ = live load
$L_r$ = roof live load
$R$ = rain load
$S$ = snow load
\[ T = \text{self-straining force (generally zero for roof applications)} \]
\[ W = \text{wind load} \]

2. Literature Review

Previous researchers have studied the interaction between wood joists/trusses and structural sheathing used for roof and floor assemblies. System effects such as composite action, load sharing, and stiffness variability, among others have been studied and typically recognized to improve the overall performance of the assemblies. This literature review concentrated on composite action.

Full Composite Action (FCA) is defined as the combined strength of the entire structural system. For a roof system, it cannot be achieved between the sheathing and the joist because of the non-rigidity of their connection (typically nailed) and the presence of gaps in the sheathing. Thus only “Partial Composite Action” (PCA) can be achieved.

![Figure 1. Sheathing Nailed to Rafters.](image)

The shear force, \( F \), generated at the interface between the sheathing and the joist is determined by the load-displacement curve (shown in Fig. 2) of the connectors and the magnitude of the differential lateral displacement (interlayer slip), \( \delta \), between these two members of the composite section. If the forces being carried by the connectors are below their yield strength (linear elastic region in Fig. 2), then the behavior of the interface will be linear and “Linear Partial Composite Action” (LPCA) will be developed; if these forces in the connectors are higher than the yield
strength (nonlinear region), then the behavior of the interface will be nonlinear and “Nonlinear Partial Composite Action” (NPCA) will be developed.

![Graph of load-displacement curve for a nail](image)

**Figure 2. Typical load-displacement curve a nail.**

### 2.1 Composite Action and System Effects

Kuenzi and Wilkinson [Kuenzi and Wilkinson 1971], developed deflection and maximum stress equations for two specific loading conditions: four point bending, as shown in Fig 3, and distributed load for simply-supported beams with continuous sheathing and LPCA.

![Diagram of geometric parameters of composite beam](image)

**Figure 3. Definition of geometric parameters of the composite beam.**

According to Kuenzi and Wilkinson [Kuenzi and Wilkinson 1971], the midspan deflection for four point bending, \( \Delta \), is given by

\[
\Delta = \frac{k(3-4k^2)P L^3}{48(EI)_R} \left\{ 1 + \frac{6}{(3-4k^2)[(EI)_u - 1]} \left[ \frac{2}{aL} \right]^2 \left( 1 - \frac{\sinh \alpha k L}{a k L \cosh \frac{\alpha L}{2}} \right) \right\},
\]  \hspace{1cm} (1)

Where

- \( P \) is the applied load,
- \( L \) is the span length,
\( k \) defines load position (shown in Fig. 3),

\((EI)_R\) is the stiffness of the T-beam assuming full composite action,

\((EI)_u = \text{stiffness of all beam components as if unconnected},\)

\(\alpha^2 = \frac{h^2S}{(EI)_R - (EI)_u} \left[ \frac{(EI)_R}{(EI)_u} \right], \quad (2)\)

\(h\) is the distance between the centroid of the joist and centroid of the sheathing, and

\(S\) is the shear force, \(F\), per unit nail spacing, \(s\), per unit slip, \(\delta\), between principal members given by

\[ S = \frac{F/f}{s} . \quad (3)\]

Kuenzi and Wilkinson [Kuenzi and Wilkinson 1971] also derived equations for maximum tensile and compressive stresses of the composite section for the same two loading conditions. However, these equations are not applicable when calculating the bending strength of the composite T-beam because of the assumption of continuous sheathing and LPCA, and because some parameters in the equations were not clearly defined.

A method to estimate the linear interlayer stiffness value was also developed by Wilkinson, where he took into account the properties of the nails like diameter, bending stiffness, and length (note that the yield strength of the nails was not considered), and the properties of the different materials in the assembly [Wilkinson 1972, 1974]. Unreasonably, this interlayer stiffness was assumed to remain linear elastic even when the equations were utilized to calculate the bending strength of the composite beams based on experimental failure loads.

McCUTCHEON modified the deflection equations of Kuenzi and Wilkinson [Kuenzi and Wilkinson 1971] and developed a method for predicting the stiffness of wood-joist floor systems with partial composite action, with a layer of sheathing on only one side of the joist [McCUTCHEON 1977] and on both sides of the joist [McCUTCHEON 1986].

Rearranging terms in Eq. (1)

\[ \Delta = \frac{k(3-4k^2)bt^3}{48(EI)_R} \left\{ 1 + \left[ \frac{6}{(3-4k^2)} \left( \frac{2}{aL} \right)^2 \left( 1 - \frac{\sinh akL}{akL \cosh \frac{akL}{2}} \right) \right] \frac{(EI)_R}{(EI)_u} - 1 \right\} , \quad (4)\]

and replacing the term in the first square brackets by the empirical formula:

\[ f_\Delta = \frac{10}{(L_f a)^2 + 10} , \quad (5)\]

where \(L_f\) is the distance between gaps in the sheathing, and \(a\) is defined by Eq. (2).
now Eq. (4) becomes:

$$\Delta = \frac{k(3-4k^2)PL^3}{48(EI)_R} \left[ 1 + f_\Delta \left( \frac{(EI)_R}{(EI)_u} - 1 \right) \right].$$  \hfill (6)

McCutcheon [McCutcheon 1977, 1986] also defined the effective stiffness of the partial composite section (to be utilized in the elementary beam deflection formulas) for T-beams as

$$EI = (EI)_u + \frac{(EA_f)(EA_w)}{(EA_f)+(EA_w)} h^2,$$  \hfill (7)

$$\overline{EA_f} = \frac{EA_f}{1+10.425L_f},$$  \hfill (8)

where

$EA_f$ is the axial stiffness of the flange,

$EA_w$ is the axial stiffness of the web, and

$\overline{EA_f}$ is the reduced axial stiffness of the flange.

These T-beam and I-beam models continued to assume LPCA, but now they also considered the effects of discontinuities in the sheathing along the span of the joists due to gaps between the panels, and estimated the interlayer slip of the connection when calculating this effective stiffness.

McCutcheon’s analytical method [McCutcheon 1977] was utilized in further research by R.W. Wolfe [Wolfe 1990], with defined values of interlayer stiffness based on some APA recommendations (only for glued-and-nailed and glued tongue & groove connections) mainly to calculate the increase in stiffness of the composite section, although the method was also utilized to calculate an increase in strength. Unreasonably, this increase in strength was calculated as the ratio of an effective section modulus (from McCutcheon’s T-beam model [McCutcheon 1977]) to the section modulus of a bare joist. Results were presented for glued-and-nailed joists only, while only-nailed joists calculations were not performed because they were said to produce no increase in strength due to inconsistencies in experimental results of some assemblies. Nevertheless, a glued-and-nailed connection is not commonly utilized in roof assemblies and can be highly inefficient and expensive as a possible retrofit to account for extra loads of solar panels. Thus, Wolfe’s methods were not adopted.

Wolfe [Wolfe, 1991] also tested three roof assemblies with two different truss configurations (Fink and Scissors). He concluded that at failure loads, the nailed connections between the sheathing and the top-chord of the trusses were stressed beyond the elastic limit, and he said that NPCA has little effect on reducing the tensile stresses of the top-chord due to bending moments and thus, composite action should be ignored. This conclusion is premature and will be refuted during our testing program.
Other system effects of roof assemblies like “load sharing” or “two-way action” have also been investigated over the years. Many researchers [Thompson, Vanderbilt, Goodman 1977; Foschi 1982; Wheat, Vanderbilt, and Goodman, 1983; Wolfe 1990; Wolfe, and LaBissoniere 1991; Wolfe, and McCarthy 1989; McCutcheon 1984; Liu, and Bulleit 1995; Cramer, Drozdek, and Wolfe 2000; Cramer, and Wolfe 1989; Rosowsky, Yu 2004; and Yu 2003] have developed a variety of analytic and numeric methods to try to account for these effects when calculating the strength and stiffness of the assembly. Also, some other researchers [Limkatanyoo 2004; Gupta, Miller, and Dung 2004] have focused on analyzing the roof as a whole and complex system, taking into consideration other effects. Limkatanyoo [Limkatanyoo 2004] included system effects in three-dimensional roof assemblies like the “reduced applied load effect”, the “truss-to-truss support effect” and the “stiff-truss effect”. He also performed a parametric study with the LPCA T-beam model developed by McCutcheon but he only found a 3% to 5% increase in the stiffness of the truss, so he ignored this composite action effect.

Several computer programs have also been developed to take into account these system effects for floors and framing walls: linear analysis programs like FEAFLO (Finite Element Analysis of Floors) by Thompson [Thompson 1977], FAP (Floors Analysis Program) by Foschi [Foschi 1982], FINWALL, by Polensek [Polensek 1976], and non-linear analysis programs like NONFLO (Nonlinear Floors) by Wheat [Wheat 1983], and BSAF (Beam-Spring Analog for Floors) by Liu and Bulleit [Liu and Bulleit 1995]. Although the engineering community recognizes the existence of these additional system effects, they are typically ignored.

As a result of system effects, the repetitive member factor (C_r) was established by the National Design Standards (NDS) [NDS 2005] and permits a 15% increase in the allowable joist bending stress value for the ASD method and a factor of 1.15 for the nominal strength value for the LRFD method, when the roof of floor assemblies meet certain requirements [NDS Section 4.3.9] It is important to note that this 15% increase in the allowable bending stresses ignores effects like gluing the joists to the sheathing or a closer spacing between joists, which would produce a higher strength for the assembly.
<table>
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<tr>
<th>Application</th>
<th>Recommended C_r Value</th>
<th>References</th>
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<tbody>
<tr>
<td>Two adjacent members sharing load</td>
<td>1.1 to 1.2</td>
<td>AF&amp;PA, 1996b</td>
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<tr>
<td></td>
<td></td>
<td>HUD, 1999</td>
</tr>
<tr>
<td>Three adjacent members sharing load</td>
<td>1.2 to 1.3</td>
<td>ASAE, 1997</td>
</tr>
<tr>
<td>Four or more adjacent members sharing load</td>
<td>1.15</td>
<td>NDS</td>
</tr>
<tr>
<td>Wall framing (studs) of three or more members spaced not more than 24 inches on center with minimum 3/8 inch-thick wood structural panel sheathing on one side and 1/2 inch-thick gypsum board on the other side</td>
<td>1.5 – 2x4 or smaller</td>
<td>AF&amp;PA, 1996b</td>
</tr>
<tr>
<td></td>
<td>1.35 – 2x6</td>
<td>SBCCI, 1999</td>
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<tr>
<td></td>
<td>1.25 – 2x8</td>
<td>Polensek, 1975</td>
</tr>
<tr>
<td></td>
<td>1.2 – 2x10</td>
<td></td>
</tr>
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Table 1. Recommended repetitive member factors for dimension lumber used in framing systems [Residential Structural Design Guide: 2000 edition].

Also, increases of up to 50% in the bending strength of wall framing studs have been established due to system effects and composite action as shown in Table 1, based on previous research by Polensek [Polensek 1976] and Douglas and Line [Douglas and Line 1996]. However, these factors have not been recognized yet by the NDS, and the increases where composite action is taken into account are only for wall studs with two layers of sheathing with specific characteristics and not for roof joists or trusses.

Rosowsky and Yu [D.V. Rosowsky and G. Yu, 2004] developed another approach for wall framing, although it may be suitable also to any system composed of repetitive members. They proposed a “partial system factor approach for repetitive members”, which considers a different factor for each of the system effects they studied (LPCA, system size, post-yield behavior, and load sharing), instead of having just one single factor like the 1.15 given by the NDS to account for all of them together.

The proposed factor by Rosowsky and Yu for partial composite action was calculated as the ratio of the maximum bending stress of the bare joist to the maximum bending stress of the composite section:

\[ K_{PCA} = \frac{\sigma_{bare}}{\sigma_{PCA}} , \]  

(9)
where $\sigma_{PCA}$ is derived as follows.

The total resisting moment, $M_{Tot}$, of the composite section shown in Fig. 4 is given by

$$M_{Tot} = M_j + M_s + Qh,$$  \hspace{1cm} (10)

Assuming each layer is bent to the same radius of curvature

$$M_j = \frac{E_{I_j}}{E_I}, \hspace{0.5cm} \text{and}$$  \hspace{1cm} (11)

$$M_s = \frac{E_{I_s}}{E_I},$$  \hspace{1cm} (12)

where

$E_{I_j}$ is the bending stiffness of the joist,

$E_{I_s}$ is the bending stiffness of the sheathing, and

$E_I$ is the effective bending stiffness of the LPCA (using McCutcheon’s method).

Substituting Eqs. (11) and (12) into eq. (10) and solving for $Q$

$$Q = \frac{M_{Tot}}{h} \left(1 - \frac{E_{I_s} + E_{I_j}}{E_I} \right).$$  \hspace{1cm} (13)

The maximum bending stress of the partial composite section, $\sigma_{PCA}$, is given by:

$$\sigma_{PCA} = \frac{Q}{A_j} + \frac{M_j}{S_j},$$  \hspace{1cm} (14)

where

$A_j$ is the cross sectional area of the bare joist, and

$S_j$ is the section modulus of the bare joist.

Figure 4. Rosowsky and Yu T-beam model [Rosowsky and Yu 2004]
The maximum bending stress at the bottom of the bare joist, $\sigma_{bare}$ is given by:

$$\sigma_{bare} = \frac{M_{Tot}}{S_{bare}},$$ (15)

Eq. (16) is obtained by substituting Eqs. (11), (12), (13), (14), and (15) into Eq. (9):

$$K_{PCA} = \frac{6(EI)h}{6(EI)h + h_j[EI - (EI_j + EI_s)]},$$ (16)

where $h_j$ is the depth of the bare joist. As explained further below, this method cannot be correct because $EI$ does not account for NPCA.

On the other hand, an approach to achieve FCA was utilized by Rancourt [Rancourt 2008] using wood I-joists/oriented strand board (OSB) roof panel assemblies. He utilized OSB sheathing on the top and bottom of the I-joist with glued joints, obtaining up to a 124% increase in strength and a 115% increase in stiffness compared to the bare I-joist. This was achieved using continuous OSB sheathing (up to 16 feet long), developing FCA of the panel. However, such an assumption cannot be made for typically-sheathed roof framing, in which the sheathing is typically nailed to the framing as explained earlier.

### 3. Conclusions

It is important to note that in all of the previous models where McCutcheon’s approach [McCutcheon 1977] was utilized to calculate an increase in strength in the T-beam with LPCA, there are certain effects that are not considered in his analysis:

1. Nonlinear behavior of the nails. This behavior strongly affects the magnitude of the axial force being carried by the sheathing, as the axial force in the sheathing remains constant after the nails have yielded, and hence, the maximum tensile stress at the bottom of the joist will also be affected.

2. Effect of gaps in sheathing on strength. Considering the assumptions Rosowsky and Yu made, the bottom tensile stresses are constant throughout the length of the joist and might be calculated as in Eq. (14) if the nails have not reached their nonlinear range (LPCA). Recall that Eq. (14) is based on McCutcheon’s approach, which only considers the effect of the gaps when calculating the effective bending stiffness of the composite section. If the nails have started yielding, then a new formulation is needed to take into account this behavior (NPCA). However, NPCA is developed only at sections away from a gap location. In sections close to a gap, the tensile stresses will increase, reaching a maximum value close to that of a bare joist at the location of the gap (as will be shown in this
research), meaning that there is no overall increase in strength in the partial composite section if no other parameters are considered.

The described research focused on calculating the stiffness of the composite section with LPCA, considering two-way action based on the stiffness variability of the members, analyzing load redistribution of a system once the first member fails, and analyzing the roof assembly as a complex 3-D system, but none mentioned the effects on the bending strength of an assembly with gaps in sheathing considering NPCA. It is hypothesized that a quantified increase in the nominal bending strength of traditionally-sheathed composite roof systems can be achieved even when considering a discontinuous flange.

4. References

13. Laboratory.
15. Madison, WI. U.S. Department of Agriculture, Forest Service, Forest Products
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