Compressive Sensing for Nuclear Security

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Abstract

Special nuclear material (SNM) detection has applications in nuclear material control, treaty verification, and national security. The neutron and gamma-ray radiation signature of SNMs can be indirectly observed in scintillator materials, which fluoresce when exposed to this radiation. A photomultiplier tube (PMT) coupled to the scintillator material is often used to convert this weak fluorescence to an electrical output signal. The fluorescence produced by a neutron interaction event differs from that of a gamma-ray interaction event, leading to a slightly different pulse in the PMT output signal. The ability to distinguish between these pulse types, i.e., pulse shape discrimination (PSD), has enabled applications such as neutron spectroscopy, neutron scatter cameras, and dual-mode neutron/gamma-ray imagers. In this research, we explore the use of compressive sensing to guide the development of novel mixed-signal hardware for PMT output signal acquisition. Effectively, we explore smart digitizers that extract sufficient information for PSD while requiring a considerably lower sample rate than conventional digitizers. Given that we determine the feasibility of realizing these designs in custom low-power analog integrated circuits, this research enables the incorporation of SNM detection into wireless sensor networks.
ACKNOWLEDGMENTS

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CONTENTS

1. Introduction .......................................................................................................................... 9
   1.1. PSD using conventional sampling techniques ............................................................. 9
   1.2. PSD using non-traditional sampling techniques ........................................................ 10
      1.2.1. Hybrid analog/digital acquisition system ............................................................ 10
      1.2.2. Compressive sensing based system ................................................................. 11

2. Sparse Representation of PMT Signals ............................................................................. 13
   2.1. Discrete Haar Wavelet study ..................................................................................... 13
   2.2. Discrete Haar Wavelet compression study ............................................................. 14
      2.2.1. Experiment methodology ................................................................................. 14
      2.2.2. Qualitative Results ......................................................................................... 15
      2.2.3. Quantitative Results ....................................................................................... 15

3. Compressive-Sensing study for PMT signals ................................................................. 19
   3.1. Generalized setup for compressive-sensing study .................................................... 19
   3.2. Potential hardware realization .................................................................................. 20
   3.3. Experimental study results ..................................................................................... 20

4. Pulse-height Spectrum Information Extraction realization on Mixed-signal Platform ..... 23

5. Conclusions ....................................................................................................................... 25

6. References ........................................................................................................................ 27

Distribution ............................................................................................................................. 29

FIGURES

Figure 1: Example output pulses produced by a PMT for both neutron (blue) and gamma-ray interaction (red) with a coupled scintillation material. ................................................................. 9
Figure 2: Potential hybrid analog/digital acquisition system for acquiring information about PMT output pulses ........................................................................................................ 11
Figure 3: Discrete Haar Wavelet representation study results ............................................. 13
Figure 4: Haar Wavelet Model for PMT output signals ....................................................... 14
Figure 5: Processing flow for the model-based Haar Wavelet compression study ............... 15
Figure 6: PSD plot for original data set, no compression .................................................... 16
Figure 7: PSD plot for data set under 8/64 model-based compression ................................... 16
Figure 8: Haar Wavelet model-based compression results for pulse energy, pulse location, and extracted PSD value .......................................................................................... 17
Figure 9: Compressive-sensing study processing flow ........................................................ 19
Figure 10: Mapping of generalized compressive-sensing framework to hardware architecture .. 20
Figure 11: PSD plot for compressive-sensed PMT signal, 16/64 compression ................. 20
Figure 12: PSD plot for compressive-sensed PMT signal, 8/64 compression ......................... 21
Figure 13: Compressive-sensing results for pulse energy, pulse location, and extracted PSD value ...................................................................................................................... 22
Figure 14: Shaper stage of a gamma-ray spectrometer ....................................................... 23
Figure 15: Peak detector output when fed by the output of the gamma-ray spectrometer shaper circuit.
# NOMENCLATURE

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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<tbody>
<tr>
<td>dB</td>
<td>decibel</td>
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<td>SNM</td>
<td>Special Nuclear Material</td>
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<tr>
<td>PMT</td>
<td>Photomultiplier tube</td>
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<td>PSD</td>
<td>Pulse shape discrimination</td>
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<td>ADC</td>
<td>Analog to digital converter</td>
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<td>CFD</td>
<td>Constant fraction discriminator</td>
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<td>FPGA</td>
<td>Field programmable gate array</td>
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<td>PGA</td>
<td>Pulse gradient analysis</td>
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<tr>
<td>CS</td>
<td>Compressive sensing</td>
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<tr>
<td>DHWT</td>
<td>Discrete Haar Wavelet Transform</td>
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<td>IDHWT</td>
<td>Inverse Discrete Haar Wavelet Transform</td>
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Special nuclear material (SNM) detection has applications in nuclear material control, treaty verification, and national security. The neutron and gamma-ray radiation signature of SNMs can be indirectly observed in scintillator materials, which fluoresce when exposed to this radiation. A photomultiplier tube (PMT) coupled to the scintillator material is often used to convert this weak fluorescence to an electrical output signal. As illustrated in Figure 1, the fluorescence produced by a neutron interaction event differs from that of a gamma-ray interaction event, leading to a slightly different pulse in the PMT output signal. The ability to distinguish between these pulse types, i.e., pulse shape discrimination (PSD), has enabled applications such as neutron spectroscopy, neutron scatter cameras, and dual-mode neutron/gamma-ray imagers.

Realizations of these applications are based on conventional digitization of PMT output signals followed by PSD and statistical processing. Given that currently these pulses are sampled at 200MHz–8GHz, an enormous amount of data must be processed in real-time or stored for off-line processing. Clearly, this approach is not feasible in a resource-constrained, remote-deployment setting, such as a wireless sensor network. Therefore, reduced-data sampling and processing techniques specifically for PSD must be developed.

1.1. PSD using conventional sampling techniques

Before the advent of fast analog-to-digital converters (ADCs), PSD was exclusively implemented with analog circuits. Although these analog circuits continue to be used [1], digitizers based on fast ADCs have lead to the approach of utilizing the uniformly sampled PMT output signal for the PSD processing. Often the sampling is initiated by a level-sensitive
threshold circuit or constant fraction discriminator (CFD) circuit. This digital approach is more 
flexible than the analog circuit approach because PSD algorithms can be implemented in 
reconfigurable field-programmable-gate-arrays (FPGAs) or on general-purpose computing 
platforms. As a result of this flexibility, a variety of PSD algorithms have been recently 
developed, including the pulse gradient analysis (PGA), frequency gradient analysis (FGA), and 
wavelet-based methods [2].

Consider an example 200 MHz system that is based on an ADS5527, which is a 12-bit 200 MHz 
ADC that is available from Texas Instruments. Assume that the pulse duration of the PMT 
output signals is 100 ns – 400 ns. It then follows that 32 signal samples should be sufficient to 
capture the duration of the pulse. Using data that is readily available on the Texas Instruments 
website, we see that the ADC component alone contributes 1230 mW of power consumption and 
$56.25 to the overall cost of the acquisition system. Given that many radiation detection systems 
utilize a large number of channels, we are motivated to reduce this cost.

1.2. PSD using non-traditional sampling techniques

Additionally, the drawback of the digital approach, however, is the large amount of data 
generated during digitization. For example, the state-of-the-art neutron scatter camera in [3] has 
32 digitizer channels (One channel per PMT). To capture the 1ns - 2ns rise time and 100ns to 
400ns duration of the pulses in the PMT output signal, a minimum sample rate of 200 MHz is 
used. Assuming that 10 bits are used to represent each sample and that pulses are received 20% 
of the time on average, the resulting average data rate is 12.8 Gbps. Given that such high data 
rates and corresponding processing rates are not amenable for remotely-deployable systems, we 
are motivated to consider alternative acquisition methods. Specifically, we seek an acquisition 
solution that is somewhere between a dedicated analog approach and full digitization approach. 
Such a solution would have the relative simplicity of an analog circuit yet maintain the versatility 
of the digital approach.

1.2.1. Hybrid analog/digital acquisition system

Figure 2 illustrates a potential hybrid analog/digital analog acquisition system. The potential 
system utilizes a trigger circuit similar to a system based on conventional digitization 
approaches. Once triggered, however, an analog circuit extracts information from the input 
pulses throughout the duration of the pulses. At the end of a predetermined time interval, the 
output of the information extraction circuit is sampled by ADCs. Assuming output pulses from 
the PMT arrive at a minimum interval of 333 ns, the required ADC sampling rate need only be 3 
MHz.

For such a system based on the ADS7883, which is a 12-bit 3 MHz ADC that is available from 
Texas Instruments, the cost is easily quantified. Again using the data that is readily available 
from the Texas Instruments website, we see that the power cost is 13.5 mW per ADC, such that 
that total power cost is 54 mW. The monetary cost is $1.85 per ADC, such that the total cost is 
$7.40. Lastly, given that samples from the ADCs are only acquired once per pulse (each pulse is 
represented by 4 samples), the data storage cost is only 4 samples per pulse. Comparing this 
potential hybrid system to the conventional sampling system from Section 1.1, we see that the
cost savings are readily apparent. Specifically, the example hybrid system has the potential to reduce power consumption by 95%, monetary cost by 86%, and data storage cost by 87%.

Figure 2: Potential hybrid analog/digital acquisition system for acquiring information about PMT output pulses

1.2.2. Compressive sensing based system

Compressive sensing (CS) [5-10] has emerged as a mathematical method for efficiently representing signals. There has been considerable work in this area at Sandia National Laboratories for radar applications, but compressive sensing has never been applied to PSD anywhere. The compressive-sensing framework is a suitable starting point for this research. The basic idea of this framework is to find a sparse representation of a signal that captures sufficient information for a particular application. The compressive sensing framework then provides an efficient acquisition scheme to extract this sparse representation. Specifically for radiation detection applications, this information includes PSD, signal energy, and time-of-flight (ToF) information, which is necessary for the system in [3].

Two observations about PMT output signals suggest that this framework is a viable approach. These observations are that (1) empirical models of the component neutron and gamma-ray pulses exist [4] and (2) a sparse representation of PMT output signals in the frequency domain (Below 13 MHz) appears to be sufficient for PSD [2]. Therefore, our approach is to utilize this empirical model in the compressive-sensing framework such that the PMT output signal can be acquired at sampling rates significantly lower than the generally accepted minimum sample rate of 200 MHz.
2. SPARSE REPRESENTATION OF PMT SIGNALS

Before the compressive sensing framework can be applied to PMT output pulses, a suitable sparse representation of these signals must be found. To conduct this study, we can utilize the MISTI data set from a previous Sandia National Laboratories radiation detection project. This data set contains a combination of neutron and gamma-ray PMT pulses that result from the mixed field produced by a $^{241}$Am-Be source. These pulses are continuously sampled at 200 MHz, and the gain of the PMT is adjusted such that saturation does not occur. These pulses generally have a pulse duration of 160 ns, such that the pulses can be adequately captured with 64 ADC samples.

2.1. Discrete Haar Wavelet study

In this study, we consider representing these pulses by transforming the 64 samples using the Discrete Haar Wavelet Transform (DHWT) [11]. The DHWT can be formulated to transform any size column vector to the Discrete Haar Wavelet Basis. For our study, we generate a 64 x 64 sized orthogonal matrix $H$ that transforms a 64 x 1 column vector $x$ to another 64 x 1 column vector $s$. This resulting column vector can then be transformed back to the original vector using $H^T s$.

To form a representative column vector $x$ for a pulse from the MISTI data set, we extract 64 samples at a consistent sample location for each pulse. The samples that occur before the prompt part of the pulse can then be used to form the baseline estimate, which effectively characterizes the common mode output of the PMT. This baseline estimate can then be subtracted from each sample in the extracted 64-sample pulse segment to form the representative $x$ vector for the study. Each representative $x$ in the data set is then transformed to the Discrete Haar Wavelet Basis using the $H$ matrix. By tracking the average magnitude of each element of the $s$ vector over the data set, we can observe how efficiently the Discrete Haar Wavelet Basis represents the PMT output signals. The results of this study are shown in Figure 3. This figure demonstrates that on average, the magnitude of the majority of Discrete Haar Wavelet coefficients is close to zero.

![Figure 3: Discrete Haar Wavelet representation study results](image)
2.2. Discrete Haar Wavelet compression study

In this subsequent study, we examine how efficiently the Discrete Haar Wavelet Basis captures relevant information required for radiation detection applications. Specifically, we examine how well this representation captures the pulse energy, pulse location (for applications requiring time-of-flight information), and PSD information.

2.2.1. Experiment methodology

For a given \( x \) vector, we extract the relevant information using established techniques. To extract pulse energy information, we sum the elements of the \( x \) vector. To extract the pulse location, we first compute the first difference of the \( x \) vector. This produces an output signal that has a zero crossing around the peak of the pulse. The exact location of the zero crossing can then be extracted by using interpolation between the signal sample before and after the zero crossing. Lastly, the PSD information can be found by forming the ratio of the pulse prompt energy to the tail energy present in the \( x \) elements. Here the prompt part of the pulse is defined as the time between 10 ns before the pulse peak and 16 ns after the pulse peak. The tail part of the pulse is defined as the time between 10 ns before the pulse peak and 128 ns after the pulse peak.

Using these techniques, the three forms of pulse information can be extracted from each \( x \) vector formed from each pulse in the data set. This information then forms the baseline for comparison as a greater degree of Haar Wavelet compression is applied to an \( x \) vector. To apply greater degrees of Haar Wavelet compression, we require a model of the pulse in the Haar Wavelet domain. This model can be generated by sorting the Haar Wavelet coefficient indexes by greatest to smallest average magnitude. For a given number of largest-magnitude Haar Wavelet coefficients, we can then use this sorted list to determine which coefficients are included in a subset that only contains a particular number of the largest-magnitude coefficients. The result of the exercise is illustrated in Figure 4.

![Figure 4: Haar Wavelet Model for PMT output signals](image-url)
This Haar Wavelet model can then be used in the processing flow shown in Figure 5. The $x$ vector is first transformed into the Discrete Haar Wavelet Basis using the $H$ matrix, forming the $s$ vector. The $N$ elements of the $s$ vector are then preserved while the remaining are forced to zero (based on the model in Figure 4), forming the $s'$ vector. This vector is then transformed back to the original time domain using the Inverse Discrete Haar Wavelet Transform (IDHWT). The pulse height, pulse energy, and PSD information are then extracted from the vector $x'$.

![Figure 5: Processing flow for the model-based Haar Wavelet compression study](image)

### 2.2.2. Qualitative Results

PSD plots are often used to qualitatively examine the performance of radiation detection systems. This two-dimensional histogram encodes the occurrence frequency of amplitude (usually simply pulse energy) and PSD value across a large data set. This histogram also generally conveys a visual distinction between (amplitude, PSD) pairs generated from neutron pulses and those generated from gamma-ray pulses. Therefore, qualitatively the operation of a radiation detection system can be evaluated by examining these plots.

The PSD plot for the original data set, i.e., no Haar Wavelet compression enabled, is shown in Figure 6. The PSD plot for a compression ratio of 8/64, i.e., 8 coefficients are kept based on the generated model, is shown in Figure 7. Comparing these two figures, we see that the distinction between the neutron and gamma-ray pulses is generally preserved even under considerable compression.

### 2.2.3. Quantitative Results

Given these encouraging results, it is reasonable to rerun the experiment using 12/64 compressions and then gradually reduce the number of non-zeroed-out coefficients. In this study, however, we examine how the pulse energy, pulse location, and PSD values degrade relative to the original value extracted when no compression is applied.

The results of the comparison are shown in Figure 8. As demonstrated in this figure, the pulse energy, pulse location (shown as fraction of a 200 MHz sample period), and extracted PSD value are relatively well-preserved under model-based Haar Wavelet compression.
Figure 6: PSD plot for original data set, no compression

Figure 7: PSD plot for data set under 8/64 model-based compression
Figure 8: Haar Wavelet model-based compression results for pulse energy, pulse location, and extracted PSD value.
3. COMPRESSIVE-SENSING STUDY FOR PMT SIGNALS

Given the encouraging results in the previous sections, it appears reasonable to use the Discrete Haar Wavelet Basis for the sparse representation required for compressive sensing techniques. In this section, we first formulate a generalized compressive-sensing study. We then describe how the framework in this study can be mapped to the potential hybrid analog/digital system described in Section 1.2.1. Lastly, we use this framework to guide an experimental study based on the data set utilized in the previous section.

3.1. Generalized setup for compressive-sensing study

A generalized setup for a compressive-sensing study is shown in Figure 9. Again, we start with the \( x \) vector. Rather than multiply this vector by the \( H \) matrix directly, we multiply the \( x \) vector by an \( N \times 64 \) sized matrix \( B \), where \( N \) is related to the compression ratio that we would like to achieve. This \( B \) matrix satisfies the restricted isometric property [11], such that well-established compressive-sensing techniques are possible. A frequently used \( B \) matrix involves forming the elements of the matrix by using independently identically distributed samples of a Bernoulli random variable [10].

The resulting \( N \times 1 \) vector \( y \) then contains the compressive samples of the \( x \) vector. Various optimization techniques can then be utilized to estimate the original \( x \) vector from the sensed vector \( y \). These optimization techniques attempt to find the sparsest solution given knowledge of the sensing matrix, which is \( BH^T \) in this study, and the compressive samples contained in the \( y \) vector. The output of this optimization operation, which is also referred to as the reconstruction step, is an estimate of the original \( s \) vector, denoted as \( s' \). This \( s' \) vector is the estimate of the original \( x \) vector in the Discrete Haar Wavelet Basis. Therefore, an estimate of the original \( x \) vector can be obtained using \( H^Ts' \).

Figure 9: Compressive-sensing study processing flow
3.2. Potential hardware realization

The mapping of the compressive-sensing processing flow to the potential hardware system described in Section 1.2.1 is shown in Figure 10. The information extraction circuit must implement the $Bx$ multiplication step. This can be realized by switched-capacitor circuits. The outputs of the ADCs form the $y$ vector. Lastly, the digitized results can then be stored and used for signal reconstruction in an FPGA.

![Figure 10: Mapping of generalized compressive-sensing framework to hardware architecture](image)

3.3. Experimental study results

For the experimental study, we utilize the processing flow described in Figure 9 and the model-based compressive-sensing technique that is described in [12] for the signal reconstruction step. The qualitative results of this study are shown in Figure 11 and Figure 12. As demonstrated in these plots, the extracted PSD information is almost completely degraded even when 16 compressive samples are used for the signal reconstruction.

The quantitative results of the experiment are shown in Figure 13. The results indicated in the percent PSD error plot are consistent with the qualitative results. Even when 16 compressive samples are used for the signal reconstruction step, the percent PSD error is approximately 33%. The results for the pulse energy and pulse location are considerably more encouraging, however. As shown in the fraction peak time error plot, both gamma-ray and neutron pulses can be relatively well-localized when only 8 compressive samples are used for the signal reconstruction step. As shown in the fraction amplitude error plot, the extracted pulse energy value is also relatively well-preserved when only 8 compressive samples are used in the signal reconstruction step.

Taken together, these results suggest that compressive-sensing can be used to extract pulse energy and pulse location information. Additional exploration is required, however, such that sufficient PSD information can be extracted from compressively-sensed PMT signals.
Figure 11: PSD plot for compressive-sensed PMT signal, 16/64 compression

Figure 12: PSD plot for compressive-sensed PMT signal, 8/64 compression
Figure 13: Compressive-sensing results for pulse energy, pulse location, and extracted PSD value
Pulse-height spectrum measurements are often required in chain-of-custody applications. These systems generally need to have low power consumption because they are often powered by a battery and generally difficult to access readily once deployed. Toward the goal of reduced power consumption and reduced monetary cost, an existing circuit for a gamma-ray spectrometer was considered for realization on a reconfigurable mixed-signal system-on-chip platform. Specifically, the circuit shown Figure 14 was realized on a Cypress Semiconductor PSoC 5. This low-power chip contains reconfigurable analog and digital blocks and represents the state-of-the-art in reconfigurable mixed-signal technology.

The pulse-shaped circuit shown in Figure 14 was implemented on the PSoC 5. The first stage of this circuit consists of a charge amplifier that would be AC-coupled to a suitable photodiode. The second stage of the circuit consists of a shaper circuit that was implemented as a band-pass filter.

Figure 14: Shaper stage of a gamma-ray spectrometer

The shaper output of this circuit was then coupled to an available peak detector module on the PSoC 5. A simulated photodiode current pulse was then input to the circuit. The results of this experiment are shown in Figure 15.
Figure 15: Peak detector output when fed by the output of the gamma-ray spectrometer shaper circuit
5. CONCLUSIONS

We proposed the use of compressive sensing to guide the development of novel mixed-signal hardware for PMT output signal acquisition. Effectively, we explored smart digitizers that extract sufficient information for PSD while requiring a considerably lower sample rate than conventional digitizers. Given that the feasibility of realizing these designs in custom low-power analog integrated circuits was considered, this research enables the incorporation of SNM detection into wireless sensor networks.

We first demonstrated that a hybrid analog/digital approach has the potential to reduce the cost of a radiation detection system in the areas of monetary cost, power consumption, and data storage requirements. Such a system would be initially triggered by a level-sensitive threshold circuit or a suitable CFD circuit. Once triggered, the system would extraction information in the analog circuit domain throughout the duration of the PMT output pulse. Only after the duration of the pulse passed would the resulting output of the analog circuit be digitized, rather than continuous digitization of the pulse signal. This fundamental difference is the key driver for the overall cost reduction of the potential system.

The compressive-sensing framework appeared to be a suitable technique for realizing this hybrid system. As a first step toward utilizing this framework, we explored sparse signal representations of PMT output signals. Specifically, we discovered an efficient representation of these signals in the Discrete Haar Wavelet domain. We demonstrated that when a PMT output signal segment is transformed to the Discrete Haar Wavelet domain, the majority of the resulting wavelet coefficients have a magnitude close to zero. We then further demonstrated that only a small number of Discrete Haar Wavelet coefficients was required to reconstruct a PMT output signal that contained sufficient pulse energy, pulse location, and PSD information.

Encouraged by these results, we then developed and implemented a compressive-sensing framework based on representing PMT output signals in the Discrete Haar Wavelet domain. We found that while the compressive-sensing framework could be easily mapped to our originally proposed hybrid analog/digital acquisition system, the compressive samples did not contain sufficient PSD information. We did demonstrate, however, that a low number of compressive samples is sufficient to extract sufficient pulse energy and pulse location information.

Alternative approaches could be taken in future work. In this study, we focused on reconstructing the pulse first and then extracting information from the reconstructed pulse. Given that information could be lost during the reconstruction step, an alternative method could involve sensing the required PSD information directly. Specifically, a classifier could be initially developed based on a handful of extracted features. The compressive sensing matrix could then be generated such that only those relevant features are estimated during the reconstruction step.
6. REFERENCES


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