# SANDIA REPORT

SAND2015-
Unlimited Release
Printed January 2016

# Approximate Model for Turbulent Stagnation Point Flow

Lawrence J. DeChant

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Issued by Sandia National Laboratories, operated for the United States Department of Energy by Sandia Corporation.



Printed in the United States of America. This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from
U.S. Department of Energy
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831

Telephone: (865) 576-8401
Facsimile: (865) 576-5728
E-Mail: reports@adonis.osti.gov
Online ordering: http://www.osti.gov/bridge

Available to the public from
U.S. Department of Commerce
National Technical Information Service
5285 Port Royal Rd.
Springfield, VA 22161

Telephone: (800) 553-6847
Facsimile: (703) 605-6900
E-Mail: orders@ntis.fedworld.gov
Online order: http://www.ntis.gov/help/ordermethods.asp?loc=7-4-0#online

SAND2016-0226
Printed January 2016 2015

# Approximate Model for Turbulent Stagnation Point Flow

Lawrence J. DeChant
Aerosciences Department
Sandia National Laboratories
P.O. Box 5800
Albuquerque, New Mexico 87185-0825

## ABSTRACT

Here we derive an approximate turbulent self-similar model for a class of favorable pressure gradient wedge-like flows, focusing on the stagnation point limit. While the self-similar model provides a useful gross flow field estimate this approach must be combined with a near wall model is to determine skin friction and by Reynolds analogy the heat transfer coefficient. The combined approach is developed in detail for the stagnation point flow problem where turbulent skin friction and Nusselt number results are obtained. Comparison to the classical Van Driest (1958) result suggests overall reasonable agreement. Though the model is only valid near the stagnation region of cylinders and spheres it nonetheless provides a reasonable model for overall cylinder and sphere heat transfer. The enhancement effect of free stream turbulence upon the laminar flow is used to derive a similar expression which is valid for turbulent flow. Examination of free stream enhanced laminar flow suggests that the rather than enhancement of a laminar flow behavior free stream disturbance results in early transition to turbulent stagnation point behavior. Excellent agreement is shown between enhanced laminar flow and turbulent flow behavior for high levels, e.g. 5% of free stream turbulence. Finally the blunt body turbulent stagnation results are shown to provide realistic heat transfer results for turbulent jet impingement problems.

# ACKNOWLEDGMENTS

The author would like to thank Justin Smith, Srinivasan Arunijatesan and Jonathan Murray; Aeroscience Department; Sandia national Laboratories for their technical insight and support. Special thanks to Dean Dobranich; Thermal Science and Engineering Department and Adam Hetzler, V&V, UQ and Credibility Processes Department; Sandia National Laboratories for their technical and programmatic support.

# CONTENTS



# FIGURES



# NOMENCLATURE

## Symbols

| | |
|---|---|
| a | Dimensionless model constant |
| B | Stagnation point inviscid flow constant; U=Bx |
| c | Locally defined constant |
| D | Cylinder/sphere diameter |
| $C_f$ | Skin friction $C_f = \frac{2\tau_w}{\rho U^2}$ |
| $C_{f\_0}$ | Undisturbed free-stream skin friction |
| const | Constant |
| f | Dependent similarity variable |
| I | Turbulence intensity (absolute value) |
| K | Clauser turbulent viscosity constant |
| L | Streamwise length scale |
| $L_{lam}$ | Streamwise location extent of laminar stagnation point |
| m | Similarity/Faulkner-Skan model coefficient |
| M | Free-stream Mach number |
| Nu | Nusselt Number |
| Pr | Prandtl number |
| Re | Reynolds number |
| $Re_x$ | Streamwise flat plate Reynolds number |
| $Re_{x\_t}$ | Transition Reynolds number |
| $Re_\delta$ | Boundary layer thickness R |
| $Re_\theta$ | Momentum thickness Reynolds number |
| St | Stanton number |
| t | time |
| u | Streamwise turbulent mean flow |
| U | Free stream turbulent mean flow velocity |
| u' | Root Mean Square (RMS) streamwise velocity fluctuation amplitude |
| W | Local turbulent velocity scale |
| x | Streamwise spatial coordinate |
| $x^*$ | $x/\delta$ |

| | |
|---|---|
| y | Cross-stream spatial coordinate |
| $y^*$ | y/δ |

**Greek**

| | |
|---|---|
| α | Turbulence power law constant |
| ξ | y/δ |
| δ | Boundary layer thickness |
| $\delta_0$ | Dimensionless constant for boundary layer thickness approximation $\delta=\delta_0 x$ |
| $\delta^*$ | Displacement thickness |
| $\delta^+$ | Boundary layer thickness inner law length scale $\delta^+ = \frac{\delta v^*}{\nu_w}$ |
| η | Local scaled similarity |
| κ | Von Karman constant κ=0.41 |
| $\widetilde{\kappa}$ | Pulsatile flow modified Von Karman constant |
| ν | Kinematic viscosity |
| ω | Frequency |
| $\omega_0$ | Dimensionless frequency $\omega_0 = \frac{\omega\delta}{U}$ |
| Φ | Power Spectral Density, i.e. spectra |
| ρ | Density |
| τ | Shear stress |
| τ | Auto-correlation time separation |
| θ | Momentum thickness |

**Subscripts/Superscripts**

| | |
|---|---|
| inc | Incompressible |
| FT | Fully Turbulent |
| max | Maximum |
| os | Laminar-turbulent pressure “over-shoot” |

| | |
|---|---|
| pp | Pressure PSD |
| rms | Root Mean Square (RMS) |
| s | Steady |
| turb | Turbulent |
| t, tran | Transition |
| T | Turbulent |
| vehicle | Reentry vehicle |
| w | Wall |
| $\infty$ | Steady free-stream constant |

# I. INTRODUCTION

Estimating heat transfer near the nose or leading edge of aerodynamic bodies is a classical and essential problem for aerodynamic vehicle design, White (2006). Stagnation point flows are necessarily laminar at the exact stagnation location. However, free stream turbulence can significantly modify the laminar flow with a corresponding increase in skin friction and heat transfer. The enhancement of the laminar heat transfer by free stream turbulence is a very well know problem. The classical analysis by Smith and Kuethe (1966) and the detailed discussions by Hoshizaki et. al. (1975) provide an excellent overview of the problem. Additional well established computational studies such as Traci and Wilcox (1975) and Ibrahim (1987) demonstrate successful numerical modeling approaches for the enhanced laminar problem.

While certainly stagnation point flows are usually laminar at the stagnation location, rapid transition to turbulent behavior is possible (due to roughness and free-stream turbulence). A coarse estimate of the transition behavior near a stagnation zone can be ascertained by the flat plate model of Van Driest and Blumen (1963): $\mathrm{Re}_{x\_tr}^{1/2} \approx \dfrac{\sqrt{1+132500I^2}-1}{39.2I^2}$ (here the turbulence intensity I is reported as a fraction as opposed to a percent). Considering an actual flight (Reidel and Sitzmann (1998)) with a high free-stream turbulence (cloud encounter), i.e. turbulence intensity I≈0.05 and flight speed on the order of 93 m/s (180 kn) one can estimate the transition Reynolds number to be on the order of $3\text{x}10^4$ which suggests that for the flight speed 93 m/s (180 kn) that the extent of the laminar zone may be on the order of $L_{lam} \approx \dfrac{3E4}{(93)1E5} \approx 0.3cm$ which is small indeed. Using a more traditional transition Reynolds number, say $\mathrm{Re}_{x\_tr}$=5E5 (I<<1) one would expect a laminar zone for this flight condition on the order of 5cm.

This discussion suggests that for some cases, a turbulent stagnation point model may be of use. The most well-known closed form analytical (but implicit) model for turbulent stagnation point flow associated with blunt body behavior is Van Driest (1958)). Indeed, when most studies need to estimate a turbulent stagnation point behavior (skin friction or heat transfer) the Van Driest study is mentioned. Additional studies regarding the quantities for blunt body turbulent stagnation flow are less available. The related problem associated with jet impingement has a broader study basis. The review by Jambunathan et. al. (1992) provides a comprehensive assessment.

Here we derive an approximate turbulent self-similar model for a class of favorable pressure gradient wedge-like flows, focusing on the stagnation point limit. While the self-similar model provides a useful gross flow field estimate this approach must be combined with a near wall model is to determine skin friction and by Reynolds analogy the heat transfer coefficient. The combined approach is developed in detail for the stagnation point flow problem where turbulent skin friction and Nusselt number results are obtained. Comparison to the classical Van Driest (1958) result suggests overall reasonable agreement. Though the model is only valid near the stagnation region of cylinders and spheres it nonetheless provides grossly adequate model for overall cylinder and sphere heat transfer. The enhancement effect of free stream turbulence upon the laminar flow is used to derive a similar expression which is valid for turbulent flow. Examination of free stream enhanced laminar flow is then suggests that the rather than enhancement of a laminar flow behavior free stream disturbance results in early transition to turbulent stagnation point behavior. Excellent agreement is shown between enhanced laminar flow and turbulent flow behavior for high levels, e.g. 5% of free stream turbulence. Finally the blunt body turbulent stagnation results are shown to provide realistic heat transfer results for turbulent jet impingement problems.

# II. ANALYSIS/RESULTS

## A. Self-Similar Flow Model

Consider the turbulent boundary layer equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-U\frac{dU}{dx}=\frac{\partial}{\partial y}\left(\nu_T\left(\frac{\partial u}{\partial y}\right)\right) \qquad (1)$$

where $\nu_T$ is a simple turbulent viscosity expression that we will define subsequently. Assuming that $u\frac{\partial u}{\partial x} << v\frac{\partial u}{\partial y}$ and introducing the linearization $u\frac{\partial u}{\partial x}\approx U\frac{\partial u}{\partial x}$ we can write the reduced expression:

$$U\frac{\partial u}{\partial x}-U\frac{dU}{dx}=\frac{\partial}{\partial y}\left(\frac{\nu_T}{u}\left(\frac{\partial w}{\partial y}\right)\right) \qquad (2)$$

The appropriateness of the linearization assumption is ultimately determined as we compare analytical results to experimental/empirical approaches.

To proceed, let's examine the turbulent viscosity model. We suggest that the appropriate length scale spans the boundary layer thickness δ and the displacement thickness $\delta^*$. We approximate the appropriate length scale as 2 $\delta^*$ . Following Clauser et. al (see White (2005)) and using our length scale we can approximate:

$$\nu_T = 2CU\delta^* \equiv KU\delta \tag{3}$$

where $\delta^*$ and δ are the displacement thickness and boundary layer thickness, respectively. Thus we have:

$$\frac{\partial u}{\partial x} - \frac{dU}{dx} = K\delta \frac{\partial^2 u}{\partial y^2} \tag{4}$$

Let's examine the possibility of a similarity solution for equation (4). We let $f(\xi) = \frac{u}{U}$ ; $\xi = \frac{y}{\delta}$ .

Here l(x) is a (currently) unspecified length scale. Using these variables we can readily formulate:

$$K\frac{d^2 f}{d\xi^2} + \left(\frac{d\delta}{dx}\right)\xi \frac{df}{d\xi} - \left(\frac{\delta}{U}\frac{dU}{dx}\right)(f-1) = 0 \tag{5}$$

For a self-similar solution we require that:

$$\frac{d\delta}{dx} = const_1$$
$$\frac{\delta}{U}\frac{dU}{dx} = const_2 \tag{6}$$

Let's introduce a model for the boundary layer thickness as: $\delta = \delta_0 x$ Using $\delta = \delta_0 x$ and assuming that $const_1 = \delta_0$. In a similar manner we analyze: $\frac{\delta}{U}\frac{dU}{dx} = const_2$ and write:

$$\delta_0 x \frac{1}{U}\frac{dU}{dx} = const_2 \tag{7}$$

To proceed we need to impose a definition for U(x). Consider the traditional power-law expression: $U = Ax^m$ which then gives: $const_2 = \delta_0 m$. Obviously this expression for U(x) is directly related to the classical "Faulkner-Skan power-law parameter" (See White 2006). Using this value equation (5) then can be written:

$$\frac{d^2 f}{d\eta^2} + \eta \frac{df}{d\eta} - m(f-1) = 0 \tag{8}$$

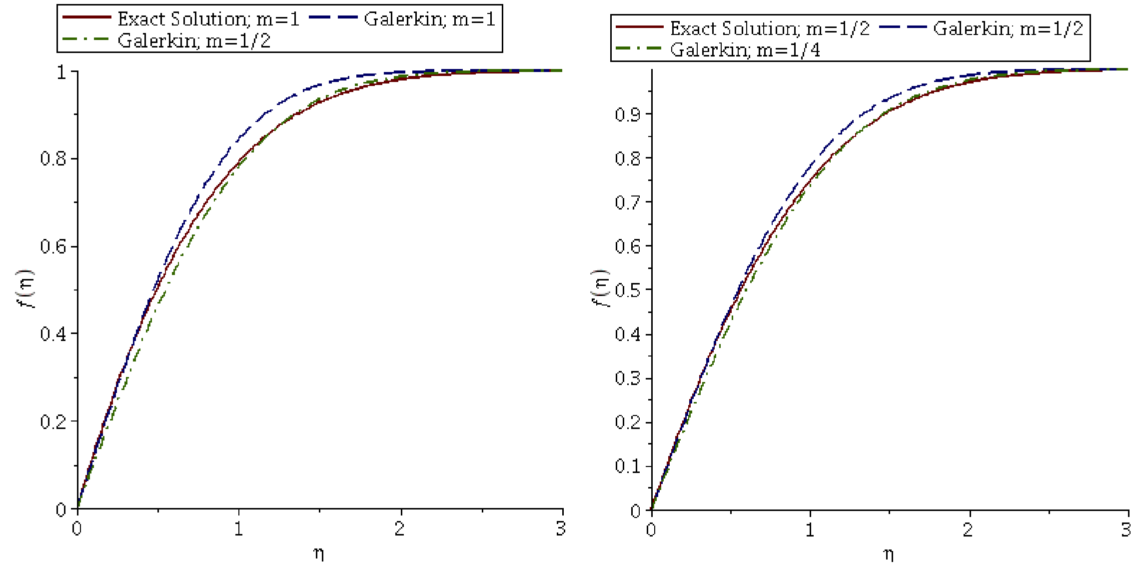
where we have introduced the change of variables: $\eta = \delta_0^{1/2}(K)^{-1/2}\xi$. Boundary conditions for equation (8) are f(0)=0 and f(∞)=1. Though a linear expression, equation (8) can be rather difficult to solve in closed form for arbitrary values of m. A simple solution which corresponds to the stagnation point flow $U = Bx$ to equation (8) follows for m=1 whereby:

$$f_{m=1} = \sqrt{\frac{\pi}{2}}\eta\left(1 - erf(\frac{\eta}{\sqrt{2}})\right) + 1 - \exp(-\frac{1}{2}\eta^2) \quad (9)$$

analogous solutions are possible for m=0,1,2,3,.. Solutions are also possible in terms of Bessel functions for m=n/2 with n=1,3,5,... However, for m=1/3 (say) solutions are not readily available analytically or are (at minimum) only possible in terms of rather more exotic special functions, e.g. Kummer's function.

Since it is our intention to use these models for engineering purposes access to simpler, approximate solution would be of value. A particularly simple solution is found for m=0 (which is a simple flat plate approximation) whereby $f_{m=0} = erf(\frac{\sqrt{2}}{2}\eta)$. Let's consider the use of this solution as a trial function for a traditional Galerkin approximation as: $f_t = erf(a\eta)$ where C is an unknown parameter. The traditional Galerkin approach computes a residual by substituting the trial function into the governing equation and computing a residual (Fletcher, 1984). The residual is that integrated over the full domain of the problem and an expression for the parameter "a" can be computed. Using this procedure we arrive at the solution for "a" as: $f \approx erf(a\eta)$ ; $a = \frac{\sqrt{2}}{2}\sqrt{m+1}$. We will have particular use for the wall derivative which is found to be: $f'(0) = \sqrt{\frac{2}{\pi}}\sqrt{m+1}$. We emphasize that m<<1 the flat plate result yields: $f'(0) = \sqrt{\frac{2}{\pi}} = 0.798$ whereas the stagnation point result gives: $f'(0) = \frac{2}{\sqrt{\pi}} = 1.128$

Since we have several exact solutions to the exact linear problem we can readily determine the viability of this approximation. Let's examine the solution




(b) $U \approx x$ (b) $U \approx x^{1/2}$

Figure 1. Comparison between approximate Galerkin solution and exact analytical solutions to equation (8) for (a) m=1 and (b) m=1/2.

Remarkably, modifying the “m” value to $m \rightarrow \frac{m}{2}$ provides a better fit over much of the curve. The near wall behavior, however, is better represented by the original closure approach since the exact solution: $\left.\frac{df}{d\eta}\right|_{m=1}(0) = 1.253$; the Galerkin approximation: $\left.\frac{df}{d\eta}\right|_{m=1}(0) = 1.128$; and the modified solution, $m \rightarrow \frac{m}{2} : \left.\frac{df}{d\eta}\right|_{m=1}(0) = 0.977$.

## B. Mean Profiles, Skin Friction and Stanton Number

With access to a flow field solution one can readily estimate overall quantities. Let’s focus on m=1 which supports the stagnation point problem. Using equation (9) we explicitly write the velocity model as:

$$u = U\left(\sqrt{\frac{\pi}{2}}\eta\left(1 - erf\left(\frac{\eta}{\sqrt{2}}\right)\right) + 1 - \exp\left(-\frac{1}{2}\eta^2\right)\right) \tag{10}$$

or using the Galerkin solution:

$$u = U erf(\eta) \tag{11}$$

Equation (10) (or the more approximate form, equation (11) is of direct interest since it directly applies to turbulent stagnation point flow behavior.

To utilize a result such as equation (11) it is necessary to estimate the constants that have been introduced into the formulation. Specifically we need to determine the turbulence and length scale constants. A plausible supposition for K is that it is directly related to the Clauser constant, i.e. C=0.016 and $\frac{\delta^*}{\delta} \approx \frac{1}{8}$ so that $K = \frac{2}{8}C = \frac{2}{8}(0.016) = 0.004$. We further need to estimate $\delta_0$ and c where $\delta = \delta_0 x$ Let's examine equation (11). For η=2.05 we find that u/U=1 implying that y=δ so that:

$$2.05 = \delta_0^{1/2}(K)^{-1/2}\frac{\delta}{\delta} \rightarrow \delta_0 = K(2.05)^2 \approx 0.0168 \qquad (12)$$

which is virtually the Clauser constant i.e. C=0.016. Note, that if we used equation (10) (the exact result) the value for η would be slightly modified as η=2.3 introducing a small variation in the value for $\delta_0 \approx 0.0212$.

The expression for the boundary layer thickness is of interest since it can be broadly compared to the boundary layer growth achieved for a flat plate boundary layer with $\mathrm{Re}_\delta = 0.16\,\mathrm{Re}_x^{6/7}$ with our result taking the form: $\mathrm{Re}_\delta = 0.017\,\mathrm{Re}_x$. Plotting these two results yields:

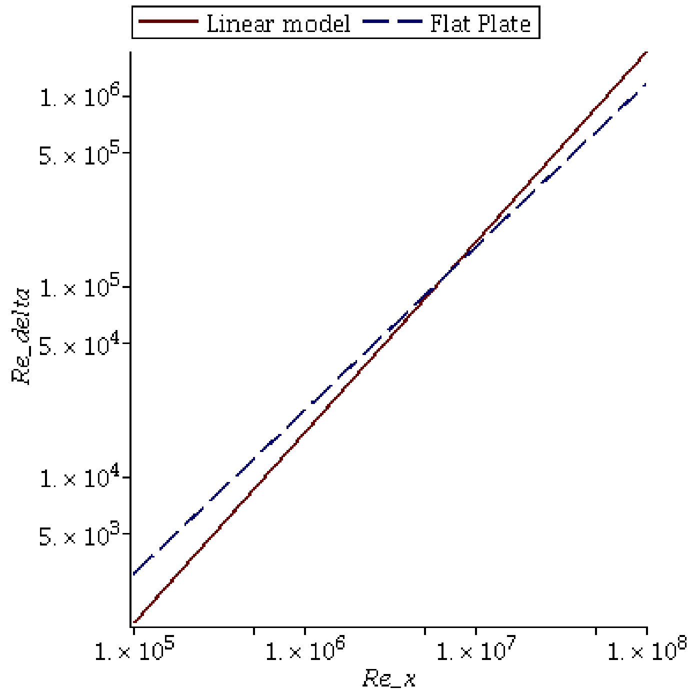



Figure 2. Comparison between linear boundary layer thickness model and classical flat plate result..

Comparison between linear boundary layer thickness model $\mathrm{Re}_\delta = 0.017\,\mathrm{Re}_x$ and classical flat plate result $\mathrm{Re}_\delta = 0.16\,\mathrm{Re}_x^{6/7}$. With appropriate constants available we turn to estimate the stagnation point skin friction and through the Reynolds analogy the Stanton number. Using a simple turbulence model based upon "outer" flow behavior, the preceding model has provided a useful estimate for the boundary layer thickness. It will, not, however, be directly applicable to the near wall problem that governs the skin friction. To solve this problem we will need to introduce a near wall turbulence model.

By definition the boundary layer approximation for the skin friction:

$$\frac{\tau / \rho}{\frac{1}{2} U^2} = \frac{2\nu_T \left.\frac{\partial u}{\partial y}\right|_{y=0}}{U^2} \tag{13}$$

Using the estimate the near wall turbulent viscosity as: $\nu_T = (\kappa y)^2 \frac{du}{dy} \approx \kappa^2 (2\delta)(2\delta^*)\left(\frac{W}{\delta}\right) = W\kappa^2\delta^*$ where κ is the Von Karman constant with κ=0.41 and the displacement thickness is estimated in terms of the boundary layer thickness $\delta^* \approx \frac{1}{8}\delta$. The velocity scale W cannot be simply the outer scale U but must include viscous information. We choose it as $W=(U_0U)^{1/2}$ (geometric average results) where $U_0$ is a viscous velocity scale based upon $U_0=\sqrt{B\nu}$ . Using the near wall effective viscosity model: and the change of variables: $\frac{\partial}{\partial y}=\frac{d}{d\eta}\frac{d\eta}{d\xi}\frac{\partial\xi}{\partial y}=\delta_0^{1/2}(K)^{-1/2}\frac{1}{\delta}\frac{d}{d\eta}$ so that we can write $C_f=\frac{\tau/\rho}{\frac{1}{2}U^2}$ :

$$C_f=\frac{\frac{1}{2}\kappa^2 W\delta}{U^2}\left(\delta_0^{1/2}(K)^{-1/2}\right)\frac{U}{\delta}(f'(0))=\frac{\kappa^2 W}{U}(f'(0)) \tag{14}$$

where $\sqrt{\frac{\delta_0}{K}}\approx\sqrt{\frac{C}{C/4}}=2$. Evaluation of f'(0) is trivial as: $f'(0))=\left(\frac{\pi}{2}\right)^{1/2}=1.25$ for the exact solution and $f'(0))=1.13$ using the Galerkin approximation. Using the Galerkin approximation we then can write the skin friction as:

$$C_f=\frac{1.13\kappa^2}{\left(\frac{(Bx)x}{\nu}\right)^{1/4}}=\frac{0.19}{\mathrm{Re}_x^{1/4}} \tag{15}$$

Where we have used $U_0=(B\nu)^{1/2}$ and $U=Bx$ with $W=(U_0U)^{1/2}$. Subsequently we will relate B to a freestream velocity and an appropriate length scale.

The same formalism that was used to for the stagnation flow result may as well be applicable for a broader range of external flow fields. For example the velocity scale in the turbulence model can be generalized as $W\propto(\frac{U_\infty\nu}{a})^{(1-\alpha)/2}(U_\infty(\frac{x}{a})^m)^\alpha$ and we expect that 0<α<1. For the stagnation point problem we have m=1 and α=1/2. An obvious and important special case is the flat plate boundary layer with m=0. However, the skin friction result $C_f=\frac{\kappa^2 W}{U}(f'(0))$ will imply a constant value for the skin friction a result which is simply not consistent with observation.

To capture anticipated skin friction result we propose that we apply a simple, semi-empirical ansantz where we modify the coefficient and exponent associated with equation (15). Consider modifying the exponent as: $(\frac{1}{4}-\frac{1}{7})m+\frac{1}{7}=\frac{3m+4}{28}$ and the coefficient as: $(1-\frac{1}{7})m+\frac{1}{7}=\frac{6m+1}{7}$. This very simple modification then gives: $C_f = \frac{6m+1}{7}\frac{0.19}{\mathrm{Re}_x^{3/28m+1/7}}$. Obviously for "m"=1 we simply recapitulate equation (15). Of rather more interest for "m"=0 we have $C_{f\_plate} = \frac{1}{7}\frac{0.19}{\mathrm{Re}_x^{1/7}} = \frac{0.0271}{\mathrm{Re}_x^{1/7}}$ which (likely by serendipity since this extension is so crude) is almost exactly the same as classical power law model (White 2006) is written as: $C_{f\_plate} = \frac{0.027}{\mathrm{Re}_x^{1/7}}$.

With the skin friction available to us we can readily use the Reynolds analogy to estimate the Stanton number and thereby the Nusselt number as:

$$St = \frac{1}{2}(\mathrm{Pr})^{-2/3} C_f = \frac{1}{2}(\mathrm{Pr})^{-2/3}\frac{0.19}{\mathrm{Re}_x^{1/4}} = 0.095\,\mathrm{Re}_x^{-1/4}\,\mathrm{Pr}^{-2/3}$$
$$Nu = \mathrm{Pr}\,\mathrm{Re}_x\,St = 0.095\,\mathrm{Re}_x^{3/4}\,\mathrm{Pr}^{1/3} \tag{16}$$

Data is available to compare to the expressions represented for the skin friction and the Nusselt numbers. A convenient way to compare between experimental data sets and implicit analytical models is to perform regressions (curve fits) for the results that can then directly compared to equations (15) or (16). Classical analytical results have been computed by Van Driest (1958). The models are implicit in the skin friction variable and are presented graphically. A simple curve fit for the 2-d Van Driest skin friction (cylinder) results yields:

$$C_{f\_VD} \approx 0.072\,\mathrm{Re}_x^{-0184} \tag{17}$$

Analogous experimental measurements have been performed by Nagamatsu and Duffy (1984). A regression for their measurements (cylinder) as:

$$C_{f\_ND} \approx 0.092\,\mathrm{Re}_x^{-0.191} \tag{18}$$

There is value in pointing out equation (18) was actually obtained from Nusselt number measurements and recovered using $\frac{1}{2}\mathrm{Pr}^{-2/3} C_f = St = \mathrm{Re}_x^{-1}\,Nu$. The Nagamatsu and Duffy (1984) measurements are of particular interest in that they clearly demonstrate transition to turbulent stagnation point behavior for $\mathrm{Re}_x$=5x10$^4$. Figure 2. provides comparisons between equation (15) and the regressions (17) and (18):

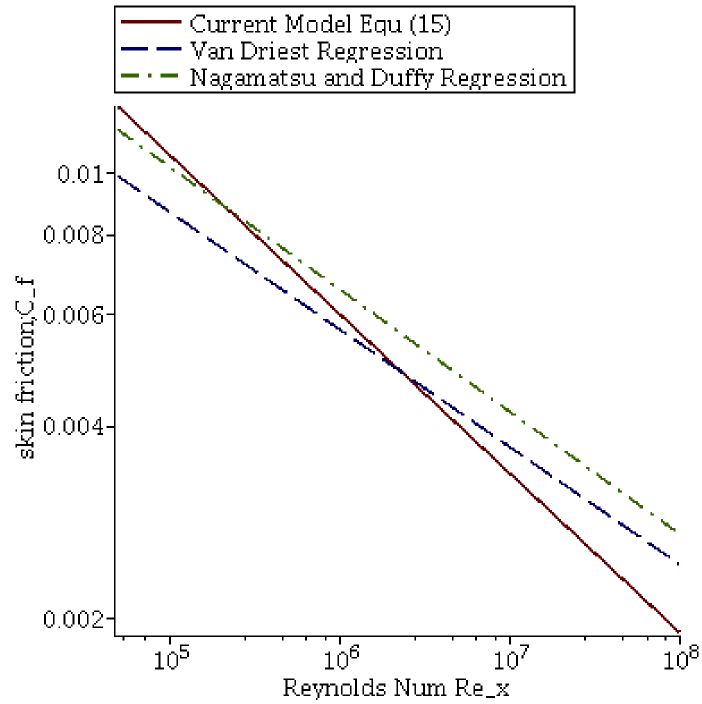



Figure 3. Comparison between current analytical model, the Van Driest analytical model represented by a regression and Nagamatsu and Duffy (1984) measurements also represented by the regression.

Figure 3 provides a comparison between current analytical model: $C_f = 0.188\,\mathrm{Re}_x^{-0.25}$, Van Driest analytical model represented by regression: $C_{f\_V} \approx 0.072\,\mathrm{Re}_x^{-0184}$ and Nagamatsu and Duffy (1984) measurements represented by the regression: $C_{f\_N} \approx 0.092\,\mathrm{Re}_x^{-0.191}$. Examination of figure 2 or the power law model exponents suggests broad agreement between the models, though certainly the ¼ exponent associated with the current analytical model is 30% too large. From figure 2, however, it is apparent that this deficiency is of limited concern for the Reynolds number regime spanned by: $5\text{x}10^5<\mathrm{Re}_x<5\text{x}10^6$ where the current model tends to be consistent with both of the regression expressions.

Consistency with other modeling approaches is suggested by power-law variation inherent to the Reshotko and Tucker boundary layer integral model applied to a fully-turbulent stagnation point. From their report we glean that a 2-d Stanton number estimate varies with $C_{f\_N} \propto St \propto \mathrm{Re}^{-0.22}$ which is similar to the analytical model.

## C. Application: Heat Transfer for 2-d and 3-d Blunt Bodies

Application of this model is typically focused on heat transfer from cylinders (and by extension spherical bodies). For a cylinder one approximates the local velocity using: $U \approx 2U_\infty \dfrac{x}{a} \rightarrow B = \dfrac{4U_\infty}{D}$. We note, however that the classical inviscid solution for flow over a sphere is not experimentally recovered (White (2006)) and is to first order: $U \approx 1.81 U_\infty \dfrac{x}{a}$ whereby $B = \dfrac{3.62 U_\infty}{D}$ For a sphere we have: $U \approx \dfrac{3}{2} U_\infty \dfrac{x}{a} \rightarrow B = \dfrac{3U_\infty}{D}$. The overall stagnation heat transfer for a cylinder is typically written in terms of a Reynolds number based on diameter as opposed to the local Reynolds number result that has been previously. To estimate an overall heat transfer behavior a typical approximation is evaluate the local result for "x' is some fraction of the diameter D. We choose x=a/2 or x=D/4 for x=D so as to give $\mathrm{Re}_{x\_cyl}(x = \dfrac{D}{4}) = 0.905 \dfrac{U_\infty D}{\nu} \equiv 0.905 \, \mathrm{Re}_D$ for a cylinder, while a sphere yields $\mathrm{Re}_{x\_sphere}(x = \dfrac{D}{4}) = \dfrac{3U_\infty \dfrac{D}{4}}{\nu} \equiv \dfrac{3}{4} \mathrm{Re}_D$.

The preceding results can then be used to estimate overall cylinder heat transfer near the stagnation point. Though the difference between cylinder and sphere stagnation point heat transfer are small, we nonetheless can write:

$$
\begin{aligned}
Nu_{D\_cyl} &= 0.088 \, \mathrm{Re}_D^{3/4} \, \mathrm{Pr}^{1/3} \\
Nu_{D\_sphere} &= 0.076 \left( \frac{5}{4} \right) \mathrm{Re}_D^{3/4} \, \mathrm{Pr}^{1/3} = 0.095 \, \mathrm{Re}_D^{3/4} \, \mathrm{Pr}^{1/3}
\end{aligned}
\qquad (19)
$$

To ascertain the viability of equation (22) for zero turbulence intensity we can compare to the classical empirical expression by Hilpert (see Incropera and DeWitt (1981)) which for high Reynolds number is written as:

$$
Nu_{D\_cyl} = C \, \mathrm{Re}_D^m \, \mathrm{Pr}^{1/3} \quad \begin{cases} C = 0.193 \quad m = 0.618 \quad 4000 < \mathrm{Re}_D < 40000 \\ C = 0.027 \quad m = 0.805 \quad 40000 < \mathrm{Re}_D < 400000 \end{cases} \qquad (20)
$$

We note reasonable trend agreement between the basic solution and (in particular) the higher Reynolds number regime. McAdams (1954) gives the sphere to gas (Pr=1) heat transfer result:

$$Nu_{D\_sphere} = 0.37\,\mathrm{Re}_D^{0.6} \tag{21}$$

Let's plot these results for a range of Reynolds numbers.

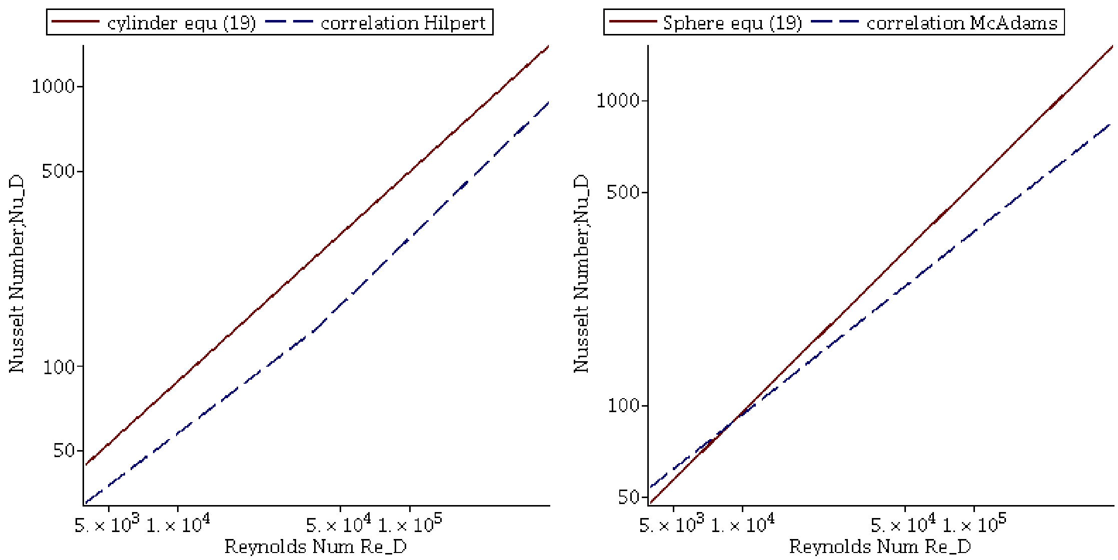



Figure 4. Mean/average Nusselt number estimates for spheres and cones using equation (19) and classical correlations for cylinders (Hilpert) and sphere (McAdams). While it is not expected that the stagnation model will capture the overall heat transfer

Though figure 4 suggests only gross comparison between equation (19) and the referenced correlations we note, that the correlations estimate mean or overall heat transfer for cylinders and spheres in cross flow includes separation behavior and is unlikely to be adequately modeled by the stagnation point expressions alone.

A well know effect associated with stagnation point heat transfer is the enhancement of the basic laminar flow heat transfer by free-stream turbulence where the free-stream turbulence is characterized by the turbulence intensity. The analysis and measurements suggested by Smith and Kuethe (1966) and the review by Hoshizaki et. al. (1975) suggests that the laminar Nusselt number result, i.e. $Nu_{lam} \propto \mathrm{Re}_D^{1/2}$ is enhanced according to:

$$\frac{Nu_{lam}}{\mathrm{Re}^{1/2}} = 1 + f(I, \mathrm{Re}_D) \tag{22}$$

Where $I \equiv \frac{u'}{U_\infty}$ the turbulence intensity and $\mathrm{Re}_D = \frac{U_\infty D}{\nu}$. A typical (semi-empirical) result is written:

$$\frac{Nu_{lam}}{\mathrm{Re}_D^{1/2}} = 0.945 + 0.0348 I\,\mathrm{Re}_D^{1/2} - 3.99x10^{-4} I^2\,\mathrm{Re}_D \tag{23}$$

where we emphasize that I is reported as a fraction. A similar (linear) result follows from Smith and Kuethe (1966) which would be approximately $\frac{Nu_{lam}}{\mathrm{Re}_D^{1/2}} = 1 + 0.0348 I\,\mathrm{Re}_D^{1/2}$ . It is important to note that maximum turbulence enhancement occurs for $I\,\mathrm{Re}_D^{1/2} = 43.6$ or $\mathrm{Re}_D = \frac{1901}{I^2}$. Remarkably this expression has a similar form to the by-pass transition Reynolds number of Schmid and Henningson (2001) who find that $\mathrm{Re}_{x\_tr} = \frac{K^2}{I^2}$ with K≈1301 and I measure in percent, though clearly, the maximum laminar turbulence enhancement Reynolds number is much larger (two orders of magnitude) than the transition Reynolds number value.

There is value in examining this result to better understand the modeling approach utilized by Smith and Kuethe for laminar flow heat transfer enhancement by free-stream turbulence. The appropriate modification follows from the skin friction whereby we can approximately write: $C_f = \frac{\tau/\rho}{\frac{1}{2}U^2} = \frac{2\nu \left.\frac{\partial u}{\partial y}\right|_{y=0}}{U^2}$ but replace the laminar viscosity with $\nu_{eff} = \nu(1+\frac{\kappa y u'}{\nu}) \approx \nu(1+const\frac{u'\delta}{\nu})$ so one can approximate:

$$C_{f\_lam} \propto \frac{(1+const\frac{u'}{U_\infty}\left(\frac{U_\infty}{U}\right)\mathrm{Re}_\delta)}{U^2}\mathrm{Re}_\delta^{-1} \tag{24}$$

For a laminar stagnation point flow, the boundary layer thickness Reynolds number is $\frac{\delta}{x} = \mathrm{Re}_x^{-1/2}$ and we can approximate: $\frac{U_\infty}{U} = O(1)$ so that we can write:

$$C_{f\_lam} \propto (1+const\frac{u'}{U_\infty}\mathrm{Re}_x^{1/2})\mathrm{Re}_x^{-1/2} \tag{25}$$

Ignoring constants, Prandtl number effects, and using $Nu_x = \Pr \mathrm{Re}_x St_x \to Nu_d = \Pr \mathrm{Re}_d St_d$ we can write:

$$
\begin{aligned}
&Nu_d \propto (1 + const\,\mathrm{Re}_d^{1/2} I)\,\mathrm{Re}_d^{1/2} \\
&\frac{Nu_d}{\mathrm{Re}_d^{1/2}} \propto (1 + const\,\mathrm{Re}_d^{1/2} I)
\end{aligned}
\tag{26}
$$

Obviously, the result $\frac{Nu_d}{\mathrm{Re}_d^{1/2}} \propto (1 + const\,\mathrm{Re}_d^{1/2} I)$ mimics the equation (20) for I<<1 and suggests the formalism used to include the effect of the free-stream turbulence.

Freestream turbulence will also be important as it enhances turbulent flow behavior. The preceding analysis can be readily modified where the effective viscosity fluctuation scale is modified as: $\kappa y \frac{du}{dy} \approx U_\infty I + \kappa\delta\left(\frac{U}{\delta}\right)$. Using the same length scale, closure approximations used to achieve equation (15), the cylinder Reynolds number approximation and the Reynolds analogy the Nusselt number expression is found to be:

$$
Nu_D = \Pr \mathrm{Re}_D St_D = 0.088\,\mathrm{Re}_D^{3/4} \Pr^{1/3}\left(1 + \frac{5}{8} I\,\mathrm{Re}_D^{1/4}\right) \tag{27}
$$

Equation (27) provides an estimate of the turbulent Nusselt number with enhancement caused by free-stream turbulence effects. Notice that as compared to the laminar flow enhancement term which grows as $\mathrm{Re}_D^{1/2}$ (this is the actual growth rate of the basic laminar Nusselt number itself), the turbulent enhancement term grows relatively slowly as $\mathrm{Re}_D^{1/4}$. The dichotomy between enhanced laminar stagnation point heat transfer and the turbulent stagnation point behavior suggests that while laminar enhancement is provides a lower Reynolds number heat transfer prediction mechanism, at higher Reynolds numbers, the heat transfer process is effectively turbulent and should be modeled using fully turbulent values.

Let's examine this hypothesis. Let's utilize the classical Smith and Kuethe (1966) which would be approximately $Nu_{lam\_enhanced} = \mathrm{Re}_D^{1/2} + 0.0348 I\,\mathrm{Re}_D$ and then compare to the unenhanced turbulence expression, i.e. equation (19) for several free stream turbulence intensities.

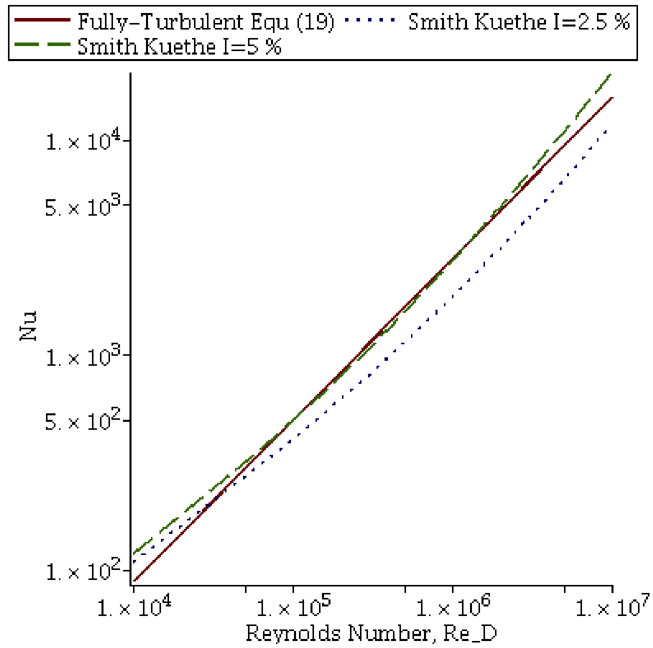



Figure 5. Comparison between free stream turbulence enhanced laminar heat transfer coefficient using Smith and Kuethe (1966) approximation with turbulence intensities I=2.5% and 5.0% and the fully turbulent Nusselt number approximation, equation (19).

Figure 5. is a comparison between free stream turbulence enhanced laminar heat transfer coefficient using Smith and Kuethe (1966) approximation: $\frac{Nu_{lam}}{\mathrm{Re}_D^{1/2}} = 1 + 0.0348 I \,\mathrm{Re}_D^{1/2}$ turbulence intensities I=2.5% and 5.0% and the fully turbulent Nusselt number approximation, equation (19). Figure 4. suggests that a free stream turbulence enhanced laminar stagnation point flow heat transfer coefficient estimate with a sufficiently high Reynolds number may actually be full turbulent and is equivalently or better modeled using a fully turbulent heat transfer expression. Further, we note that Reynolds number at which the stronger I=5% free stream disturbance heat transfer is equivalent to the fully turbulent model is approximately $\mathrm{Re}_D$=5.7x$10^4$ which is similar to the transition behavior Reynolds number noted by Nagamatsu and Duffy (1984). Nagamatsu and Duffy (1984) state that:

> "Indications are that the level of observed turbulence intensity is not sufficient to explain the high measured heat transfer."

A turbulent heat transfer model or even a free stream enhanced turbulence expression such as equation (27), might better explain this observed higher heat transfer behavior.

Our discussion has been concentrated on external flow blunt body heat transfer behavior, however, an important class of stagnation problems follows from circular jet impingement problems. An excellent review of heat transfer behavior for this problem is provided by Jambunathan et. al. (1992). A particularly relevant class of jet impingement problem is found for jets positioned a large distance, e.g. z/D>10 from the impingement plate whereby the entire flow field is turbulent. The heat transfer behavior for this configuration can be modeled via:

$$Nu_D = K \,\mathrm{Re}_D^a \tag{28}$$

Where for z/D=10 and x/D=0 ( X/D is the distance from stagnation point). Under these conditions measurements suggest that K=0.075 while a=0.75. Obviously, these values are in reasonable agreement with the spherical model in equation (19) $Nu_{D\_sphere} = 0.095\,\mathrm{Re}_D^{3/4}\,\mathrm{Pr}^{1/3}$ suggesting the current analysis may be useful for the fully turbulent jet impingement problem.

# III. CONCLUSIONS

.

The focus of this report was to derive an approximate turbulent self-similar model for a class of favorable pressure gradient wedge-like flows, focusing on the stagnation point limit. Mean profiles were recovered by the self-similar model while a near wall model was utilized to determine skin friction and by Reynolds analogy the heat transfer coefficient. Comparison to the classical Van Driest (1958) result suggests overall reasonable agreement for turbulent skin friction and Stanton number estimates. Though the model is only valid near the positive pressure gradient stagnation region of cylinders and spheres it nonetheless provides a reasonable model for overall cylinder and sphere heat transfer. The enhancement effect of free stream turbulence upon the laminar flow is used to derive a similar expression which is valid for turbulent flow. Examination of free stream enhanced laminar flow suggests that the rather than enhancement of a laminar flow behavior free stream disturbance results in early transition to turbulent stagnation point behavior. Excellent agreement is shown between enhanced laminar flow and turbulent flow behavior for high levels, e.g. 5% of free stream turbulence implying that the the fully turbulent model developed here may be appropriate. Finally, consistent with experimental observation the blunt body turbulent stagnation results were shown to provide realistic heat transfer results for turbulent jet impingement problems.

# IV. REFERENCES


1. Fletcher, C. A. J., *Computational Galerkin Methods*, Springer-Verlag, NY, 1984.
2. Hoshizaki, H., Chou, Y. S., Kulgein, N. G., Meyer, J. W., "Critical Review of Stagnation Point Heat Transfer Theory," AFFDL-TR-75-85, ADA018885, www.dtic.mil/cgi-bin/GetTRDoc?AD=ADA018885, 1975.
3. Ibrahim, M. B. "A Turbulence Model for the Heat Transfer Near Stagnation Point of a Circular Cylinder, *Applied Scientific Research*, Volume 44, Issue 3, pp 287-302, 1987.
4. Incropera, F. P., and De Witt, D. P., *Fundamentls of Heat and Mass Transfer*, 3rd ed. J. Wiley, NY, 1981.
5. Jambunathan, K., Lai, E., Moss, M. A., Button, B. L., "A Review of Heat Transfer Data for a Single Cicrcular Impingement Jet," Int. Journal of Heat and Fluid Flow, 13, 2, pp. 106-115, 1992.
6. Kondjoyan, A., Frédéric Péneau, F., Boisson, H, "Effect of High Free Stream Turbulence on Heat Transfer Between Plates and Air Flows A review of Existing Experimental Results," *Int. J. Thermal Sciences*, pp. 1-16, 2002.
7. McAdams, W. H., Heat Transmission, McGraw-Hill, NY, 1954.
8. Nagamatsu, H. T., Duffy, R. E., "Investigation of the Effects of Pressure Gradient, Temperature, and Wall Temperature Ratio on the Stagnation Point Heat Transfer for Circular Cylinders and Gas Turbine Vanes,", NASA CR 174667, 1984.
9. Reidel, H., Sitzmann, M., "In-Flight Investigation of Atmospheric Turbulence," Aerospace Science and technology, V. 2, 5, pp. 301-319, 1998.
10. Reshotko, E. Tucker, M., "Approximate Solution of the Compressible Turbulent Boundary Layer with Heat Transfer and Arbitrary Pressure Gradient," NACA TN 4154 1957.
11. Schmid, P. J., Henningson, D. S., *Stability and Transition in Shear Flows*, Springer, New York, 2001.
12. Smith, M. C., Kuethe, A. M. "Effects of Turbulence on Laminar Skin Friction and Heat Transfer," *Physics of Fluids*, 9, 12, 2337-2344, 1966.
13. Traci, R. M., Wilcox, D. C. "Freestream Turbulence Effects on Stagnation Point Heat Transfer", *AIAA Journal*, Vol. 13, No. 7, pp. 890-896, 1975.
14. Van Driest, E. R., "On the Aerodynamic Heating of Blunt Bodies," *ZAMP*, 17, Vol. 9, issue 5-6, pp. 233-248. 1958.
15. White, F. M., *Viscous Fluid Flow* 3rd ed., McGraw-Hill, NY, 2006.

# V. DISTRIBUTION

| | | | |
|---|---|---|---|
| 1 | MS0825 | Srinivasan Arunajatesan | 1515 (electronic copy) |
| 1 | MS0825 | Lawrence DeChant | 1515 (electronic copy) |
| 1 | MS0346 | Dean Dobranich | 1514 (electronic copy) |
| 1 | MS0346 | Richard Field | 1526 (electronic copy) |
| 1 | MS0828 | Adam Hetzler | 1544 (electronic copy) |
| 1 | MS0346 | Mikhail Mesh | 1523 (electronic copy) |
| 1 | MS0825 | Jonathan Murray | 1515 (electronic copy) |
| 1 | MS0825 | Jeff Payne | 1515 (electronic copy) |
| 1 | MS0825 | Justin Smith | 1515 (electronic copy) |
| 1 | MS9018 | Central Technical Files | 8944 (electronic copy) |
| 1 | MS0899 | Technical Library | 4536 (electronic copy) |

Sandia National Laboratories