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Contingency Contractor Optimization Phase 3, Model Description and Formulation Contingency Contractor Optimization Tool - Prototype

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Abstract

The goal of Phase 3 the OSD ATL Contingency Contractor Optimization (CCO) project is to create an engineering prototype of a tool for the contingency contractor element of total force planning during the Support for Strategic Analysis (SSA). An optimization model was developed to determine the optimal mix of military, Department of Defense (DoD) civilians, and contractors that accomplishes a set of user defined mission requirements at the lowest possible cost while honoring resource limitations and manpower use rules. An additional feature allows the model to understand the variability of the Total Force Mix when there is uncertainty in mission requirements.

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NOMENCLATURE

DoD	Department of Defense
SSA	Support for Strategic Analysis
ATL	Acquisition, Technology and Logistics
OSD	Office of the Secretary of Defense
CCO	Contingency Contractor Optimization

1. INTRODUCTION

The goal of the OSD ATL Contingency Contractor Optimization (CCO) project is the development of a centralized strategic planning tool that allows senior decision makers to quickly and quantitatively assess the impacts, risks, and mitigation strategies associated with utilizing contract support. The intent of public law and congressional oversight of contractor operations in support of military contingencies is not supported by existing planning tools. As a consequence, the extent, duration, and resourcing needed to satisfy contingency needs for contracted capabilities in support of national military strategy objectives cannot currently be quantified.

During Phase 3 of the Contingency Contractor Optimization project, an engineering prototype of a tool for the contingency contractor element of total force planning was created. A key component of this tool was an optimization model which determines the least expensive Total Force Mix (military, Department of Defense (DoD) civilian, and contractor) that meets mission requirements. This document describes the concepts and mathematical formulation of this optimization model.

There are two major components to an optimization model, the objective and the constraints. The objective is a mathematical function of the decisions to be minimized or maximized. In the case of this model, the objective is to find the Total Force Mix (as defined as the number of person-time periods by personal type and skill category) that fulfills all of the mission requirements and minimizes the total personnel costs. Constraints are rules which limit the types of solutions that are allowed while achieving the goal of the objective. In this model, constraints consist of things such as limits on the number of military personnel available, treaties that prevent the use of contractors to fulfill certain mission requirements under one or more circumstances, and budget limitations. When solved, the optimization model recommends a Total Force Mix that minimizes the budget while still honoring all of these constraints.

An optimization model was selected because it contains several desirable properties for this application. First, optimization models are prescriptive, which means they provide recommendations on the best way to do something, including strategies to mitigate the impact of risks. This is applicable to total force planning, which is concerned with developing a Total Force Mix at the strategic level to support future operations. In contrast, modeling techniques such as simulation are descriptive in nature. This means that they represent how a system will perform given a set of decisions, whereas optimization models determine the best set of decisions. Optimization models have the added benefit that they are:

- 1) *rigorous*, as they can be written down in a precise mathematical language which facilitates understanding and validation,
- 2) *consistent*, since all analysis uses the same set of logic, different analyses can be readily compared, and
- 3) in this case, *tractable*, because there are well established algorithms for solving this model formulation.

The remainder of this report is divided into four sections. Section 2 contains a conceptual description of the model and the key capabilities it provides. Sections 3 and 4 provide the precise

mathematical description of the model. Conclusions are presented in section 5. While sections 3 and 4 assume the reader is familiar with mathematical programming, the remainder of the document should be accessible to any reader familiar with the domain.

2. MODEL DESCRIPTION

This section provides a conceptual description of the model to illustrate the types of data it requires, the features of the model, and the types of output it provides. All input data and results shown in this section are notional and are included for demonstration purposes only. The Contingency Contractor Optimization Tool engineering prototype created during Phase 3 of the Contingency Contractor Optimization project provides a mechanism for creating input data, solving and viewing the results of the model. The actual model that the prototype interacts with is written in a very generic form, which is discussed in sections 3 and 4. Since the purpose of this section is to understand the behavior of the model, it will be discussed in the context of the planning tool prototype.

2.1. Core Model Inputs and Behavior

Figure 1 illustrates some of the core ideas of the model. This figure is not intended to list all personnel groups or capabilities, but is illustrative of the core ideas. This figure is divided into three regions: supply, demand, and use rules. Mission requirements create a demand for personnel, which is satisfied by drawing on resources (personnel). The model's goal is to assign personnel resources to each mission in such a way that all of the mission requirements are fulfilled and the cost is minimized. Ideally, the costs could be minimized by using the cheapest resources for all mission requirements. In reality, there are constraints which complicate this task. Rules exist that prevent certain types of contractors from performing a required task or that require military personnel to be used for a task. This concept is illustrated with the use rules region of Figure 1, which shows the resources that are allowed to fulfill each task. There may be additional limitations, such as the availability of certain resources, which must be honored when determining assignments. Finally, there may be annual personnel-related budgets that limit spending.

The supply region of Figure 1 represents resources that are available to accomplish the workload. Resources are identified by the capability they provide and the personnel group to which they belong. In Figure 1, the capabilities are logistics and force support. The personnel groups are military, U.S. contractor, and local nation contractor. There are several additional pieces of data attached to each of these resources. Some resources have a limited number of personnel while others are effectively unlimited. There are also costs associated with using the resource. Finally, some resources may not be as efficient as others. For example, suppose for a particular element of the workload, contractors are 50% as efficient as military personnel. This means that two contractors are required to perform the same amount of work as a single person in uniform.

The demand region of Figure 1 illustrates how mission requirements are created. At the highest level, work is divided by mission scenarios. Each mission scenario is then divided into geographical regions such as bases. Since mission requirements can change over time, each base can then be divided into a series of phases, each with an associated duration. These phases then require a specific quantity of each capability. The compilation of all of the capability requirements by time makes up the total demand which is used by the model as input.

The use rules region of Figure 1 illustrates how use rules are implemented in the planning tool. In this example, only resources that are connected to a task can be used. The color of each line represents the personnel group. The solid and dashed lines are provided to help visually distinguish each line. The most obvious rule in this example is that logistics and force support tasks can only be fulfilled by resources with the same capability. However, it is also important to notice that not all logistics resources can fulfill all logistics tasks. In the Phase 3 engineering prototype, policies and manpower mix rules can be applied which further limit the resources that can be used. The optimization model must honor all use rules in its solution.

Figure 1 illustrates the structure of the problem that the model must solve. There are two important points which can be seen in this figure. First, both of the mission scenarios draw from the same set of resources. When assigning resources to tasks, the model must resolve the competition for resources. Second, in many cases a task can be fulfilled by several resources. When solved, the model will determine which resource or resources should be used for the task. The user creates a set of resource options, but the model selects which combination of resources to use. This is ultimately the value added by the model. If there are insufficient resources available given the use rules, the model identifies the shortages and the least expensive method to overcome them.

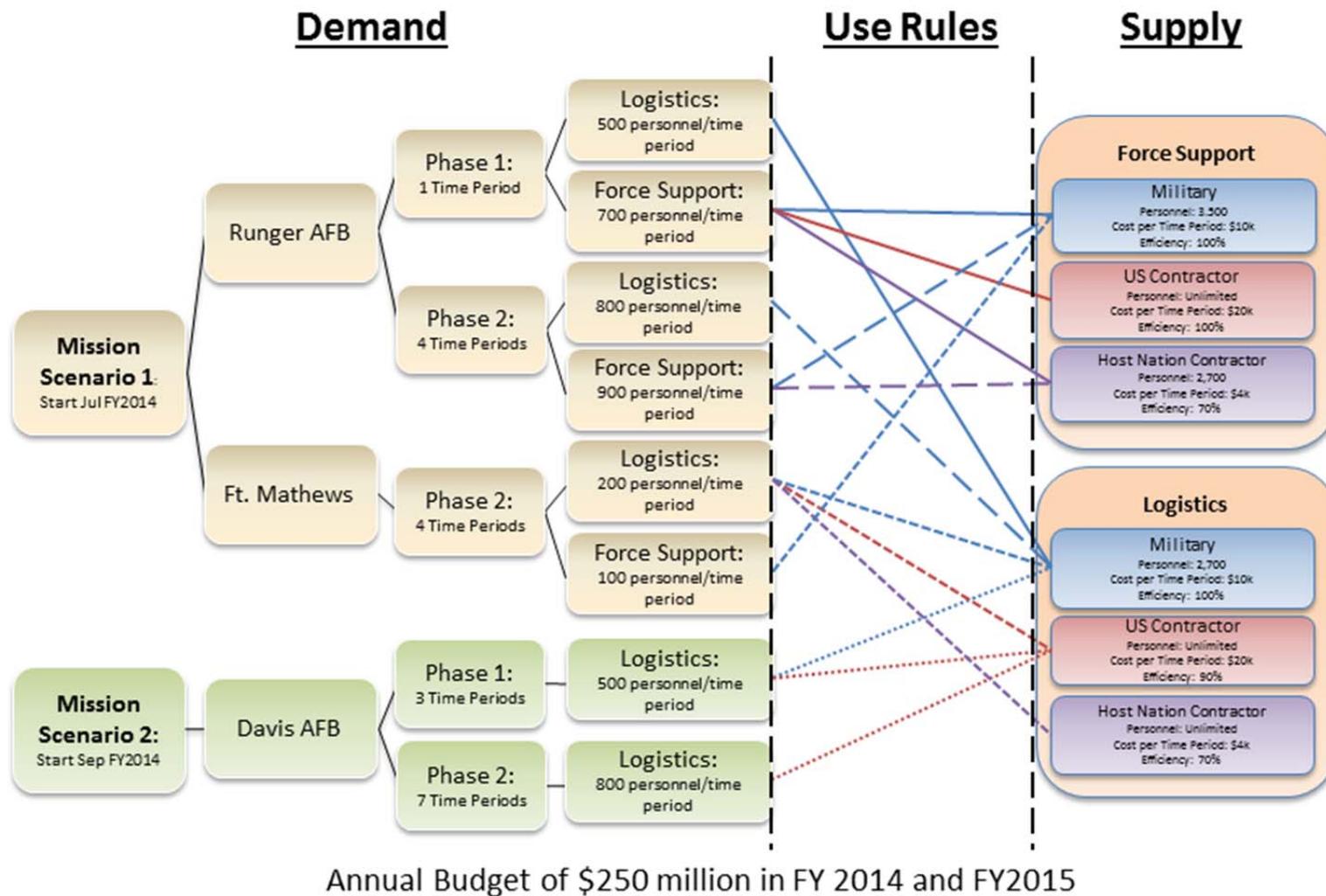


Figure 1. Conceptual Description of the model structure and data.

2.2. Key Model Outputs

The model provides three key outputs, referred to here as feasibility, manpower mix, and cost. In certain cases, the resources required may exceed what is available. When this occurs, the model provides outputs which show that it is not feasible to accomplish all of the missions given the current resources and rules. Manpower mix refers to the mixture of personnel groups that are used to accomplish the mission. Finally, cost outputs show how the mixture of different personnel groups affects the cost of the mission. The example below illustrates these three types of outputs.

Consider two hypothetical analysis cases. In the first case, there is a requirement to accomplish a single mission scenario. In the second case, there is a requirement to accomplish the original mission scenario and an additional mission scenario. The personnel requirements over time for these two cases are illustrated in the figure below. Notice that when both mission scenarios occur, they overlap. This will cause a large increase in total demand for personnel.

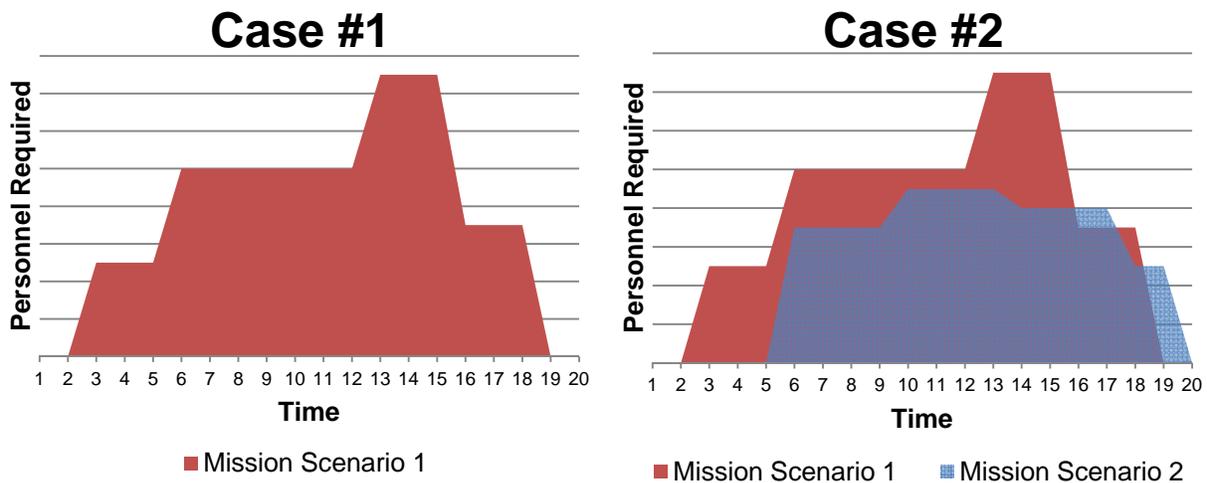


Figure 2. Resource demand profiles for two hypothetical cases.

As an example, consider two resource pools: force support and logistics. Assume that force support can only be performed by military personnel, whereas logistics can be performed by military, U.S. contractors, or local nation contractors. Assume that U.S. contractors are the most expensive option, followed by military, then local nation contractor, and that all three groups have the same efficiency. Also, assume that rules are in place which limits the number of local nation contractors that can be used. Finally, assume that military personnel are a limited resource while contractors are effectively an unlimited resource. When the model is solved, it attempts to accomplish all of the requested workload at the lowest cost possible.

Figure 3 below illustrates how the model outputs demonstrate the feasibility or infeasibility of a set of mission scenarios. In these two graphs, the blue bars show the number of military force support that were used in both cases. The black line represents the quantity of military force support that were available. The red bars indicate the quantity of additional military personnel

that would be required to complete all of the requested workload. In case #1, the requested workload does not exceed the quantity of resources that are available. However, when two scenarios occur simultaneously in case #2, there are insufficient resources to accomplish the workload. Two possible solutions to this problem would be to increase the size of the resource pool or to allow contractors to perform force support functions.

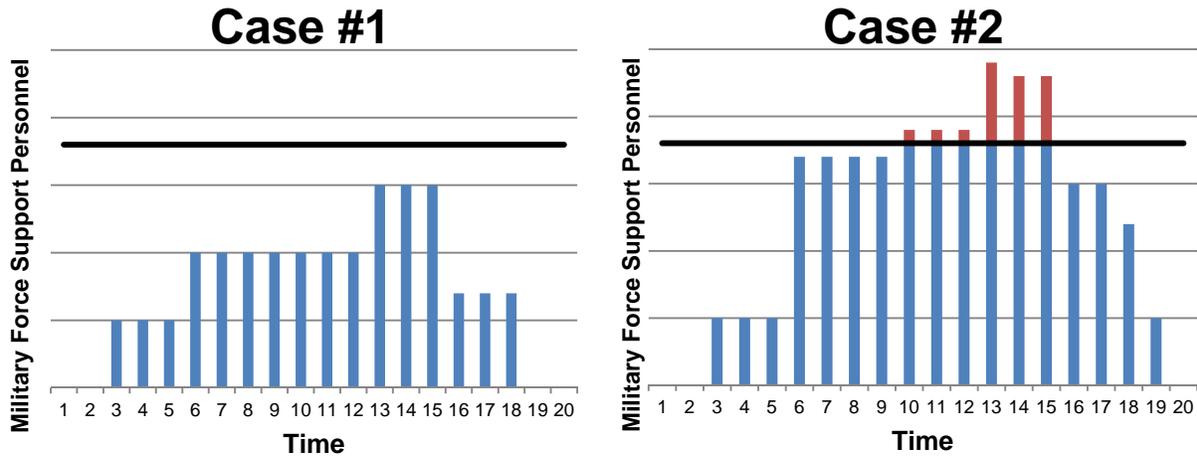


Figure 3. Comparison of military force support personnel assignments.

In the case of logistics personnel, both military and contractors are allowed to perform work. This means that all workload can be accomplished without any shortages. The figure below demonstrates the manpower mix outputs provided by the model. Both case #1 and case #2 include mission scenario 1. The pie charts below show the mixture of personnel groups that are used to satisfy the demand for the logistics capability for mission scenario 1. In case #1, about three quarters of the work is done by military personnel. In case #2, another mission scenario (mission scenario 2) is added that also requires military personnel. This means that mission scenario 1 must share the military personnel with mission scenario 2 and use more U.S. contractors. Despite being the most expensive option, U.S. contractors are used to replace these personnel because there are limits on the number of local nation contractors that can be used.

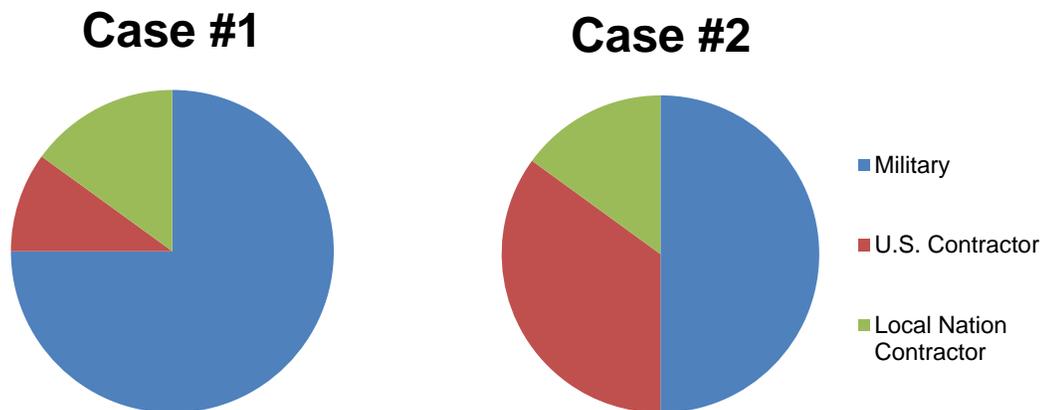


Figure 4. Comparison of manpower mix for the logistics capability in mission scenario 1.

Finally, Figure 5 below illustrates the type of outputs the model provides related to costs. Assume that U.S. contractors are more expensive than military personnel. The manpower mix example above demonstrated that a competition for resources can change the composition of the workforce. This in turn affects the cost of accomplishing the workload. In Figure 5, the cost of logistics personnel is higher for mission scenario #1 in the second case, since more U.S. contractors must be used.

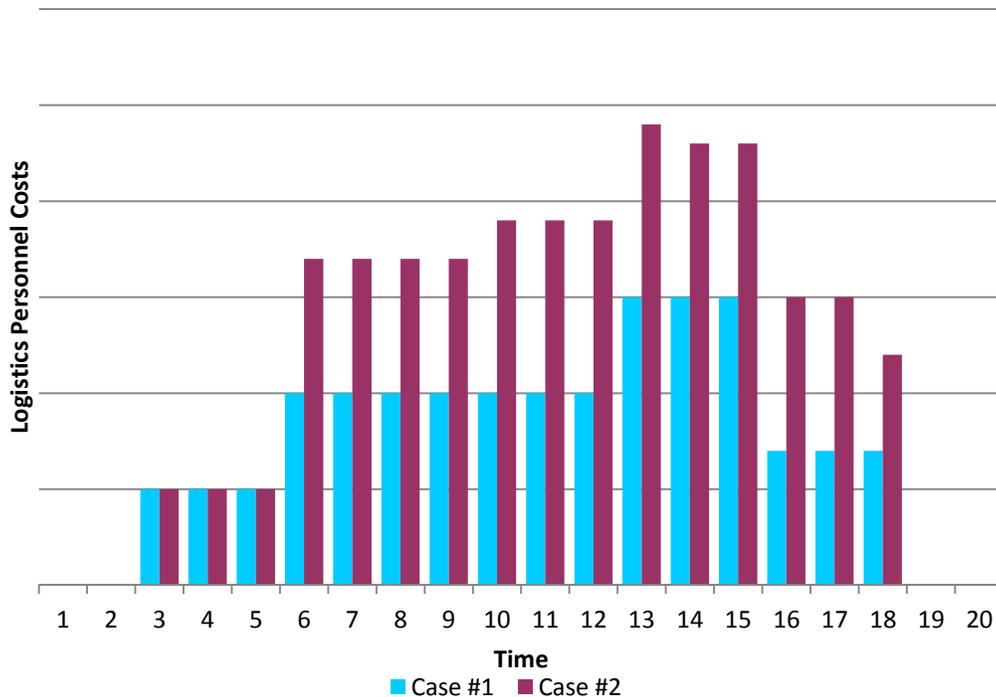


Figure 5. Comparison of logistics personnel cost for mission scenario #1.

2.3. Mission Scenario Uncertainty

When mission scenarios were discussed in section 2.1, the requirements for those scenarios were known with certainty. In reality, there is a large amount of uncertainty around the manpower requirements of a mission scenario. Characteristics like the start date of the mission scenario, its duration, and its personnel requirements will not be known with certainty. Since there is a large amount of uncertainty when planning for future missions, an additional feature is added to the model to address uncertainty.

This feature of the model focuses on uncertainty around mission scenario requirements, including the use rules associated with each mission scenario. The model is capable of handling any type of mission scenario uncertainty. When there are several mission scenarios with uncertainty, there are a large number of outcomes that could occur. With uncertainty, there is the additional concept of likelihood. Each of the possible outcomes may not be equally likely to occur. For example, a particular outcome of a mission scenario could require a lot of resources

but it may be very unlikely. Both the outcome and its likelihood are considered by the model and are therefore in the results to be analyzed.

When uncertainty is added to the model, the outputs are no longer a single value. Instead, they can be represented by the minimum, maximum, a percentile, or the average. The figure below illustrates this idea. It shows how outputs are displayed when there is mission scenario uncertainty. Assume that there are several mission scenarios requiring logistics personnel but there is uncertainty around how many personnel will be required and when the mission scenarios will occur. The number of logistics personnel required is not a single value but a range of values. In this case, the minimums and maximums are displayed. Observe that the number of military logistics personnel required has a large range in some time periods.

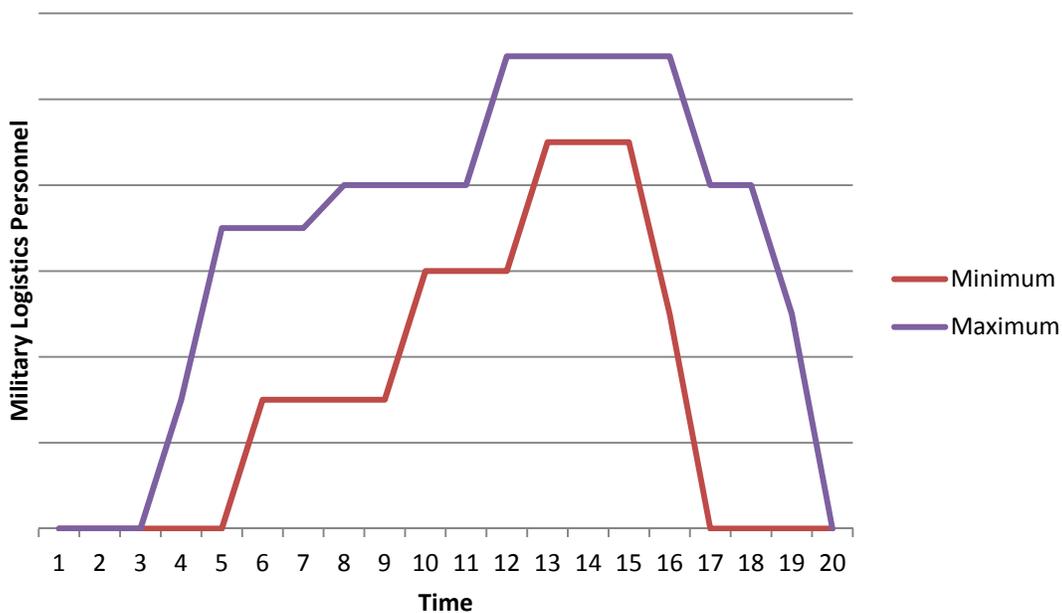


Figure 6. Personnel requirements for military logisticians under uncertainty.

The example above demonstrates how mission scenario uncertainty affects the manpower mix. However, uncertainty also affects feasibility and cost, the other two key model outputs. Since uncertainty affects the manpower mix, it affects manpower expenditures as well. Therefore, graphs showing uncertain manpower costs will be very similar to Figure 6. Uncertainty can also answer feasibility questions such as, “What is the likelihood of not having enough resources to complete the mission?” Under mission scenario uncertainty, the outputs allow the analyst to assess levels of risk created given the uncertainty around mission scenario requirements.

3. MODEL NOTATION

As discussed in Section 1, an optimization model is a collection of mathematical expressions. One expression is the objective and the remaining expressions are the constraints. All of these expressions contain a number of variables, each representing a decision to be made, whose optimal values are sought. This section provides a summary of the notation used in section 4, where all of the parameters and terms described below are explained.

The notation for the model formulation includes the following limits, indices, input data, and decision variables. The model must determine the values for the decision variables that yield the optimal value for the objective function.

3.1. Limits

I :	number of workload elements
K :	number of capabilities
P :	number of personnel groups
R :	number of mission scenario realizations
S :	number of mission scenarios
T :	number of time periods in the planning horizon
W :	number of mission scenario packages
Y :	number of years in the planning horizon

3.2. Indices

$i = 1, \dots, I$:	index on workload elements
$k = 1, \dots, K$:	index on capabilities
$p = 1, \dots, P$:	index on personnel groups
$r = 1, \dots, R$:	index on mission scenario realizations
$s = 1, \dots, S$:	index on mission scenarios
$t = 1, \dots, T$:	index on time periods
$w = 1, \dots, W$:	index on mission scenario packages
$y = 1, \dots, Y$:	index on years

3.3. Input Data

a_{iskt} :	manpower requirement of capability k for workload element i of mission scenario s in time period t
a_{iskt}^r :	manpower requirement of capability k for realization r of workload element i of mission scenario s in time period t
B_y :	annual budget available in year y
c_{spkt} :	cost of using a resource from personnel group p with capability k for mission scenario s in time period t
h_{pkt} :	manpower availability of personnel group p with capability k in time period t
$m(w)$:	set of workload element and mission scenario pairs $\langle i, s \rangle$ that belong to package w

$o(y)$:	set of time periods t that are in year y
$P(i, s)$:	set of personnel groups p that can be used for workload element i of mission scenario s
$P(i, s, r)$:	set of personnel groups p that can be used for workload element i of mission scenario s in realization r
Q^s :	importance factor of mission scenario s (larger values are more important)
q^w :	probability of package w
γ :	relative importance of accomplishing workload compared to minimizing personnel costs (typically $\gamma \gg 1$)
δ_{ws}^r :	indicator parameter that takes on a value of 1 to indicate that realization r for mission scenario s is included in package w
ϵ_{spk} :	efficiency factor when using a resource from personnel group p with capability k for mission scenario s

3.4. Decision Variables

g_{ispkt} :	continuous variable representing the number of units that personnel group p with capability k is short for fulfilling the requirements of workload element i of mission scenario s in time period t
g_{wispkt}^r :	continuous variable representing the number of units that personnel group p with capability k is short for fulfilling the requirements of workload element i of mission scenario s in time period t in realization r
x_{ispkt} :	continuous variable representing the number of units of personnel group p with capability k that are assigned to workload element i of mission scenario s in time period t
x_{wispkt}^r :	continuous variable representing the number of units of personnel group p with capability k that are assigned to workload element i of mission scenario s in time period t for realization r

4. MODEL FORMULATION

The following sections describe the mathematical formulation. While there is ultimately a single version of the model, the sections below present the model in two stages: the core model and the core model with uncertainty. This approach is used since the second version of the model builds on the first version. This also mirrors the two modeling options that are available in the interface of the prototype.

4.1. Core Model

This section describes the core features of the mathematical formulation. In its most basic form, the model is formulated as a linear program. In the subsequent section, the core formulation is expanded to include uncertainty. However, the core assumptions and behavior described in this section will remain the same. At its core, the model behaves as follows. As input, the user provides manpower requirements for each mission scenario of interest by capability and time. The model then assigns personnel from various resource pools to fulfill this demand, subject to resource limitations, manpower use rules, and budget limitations. The objective of the model is to fulfill all of this demand at the lowest possible cost.

4.1.1. Workload Elements

The key model assumption is that there are one or more mission scenarios, each of which has well-defined resource requirements. The resource requirements from each mission scenario define the workload that needs to be accomplished during the timeframe of interest. Let a_{iskt} represent the number of units of capability k required in time period t to accomplish workload element i in mission scenario s . The term *workload element* is used to refer to a “piece” of the mission scenario. The concept of a workload element is a convenience for modeling, and it does not necessarily have a unique meaning in the mission scenario. It allows for a large mission scenario to be divided into more specific tasks. In the CCO engineering prototype, a workload element is created for each capability, phase of war, and base for a given mission scenario. If desired, a workload element could be created for every task that must be accomplished during a mission scenario. It is assumed that each workload element i requires a specific capability k . Notice that a_{iskt} is indexed by time. This means that the manpower requirements for a given scenario and workload element are defined for each time period and can vary over time. This also implies that the set of mission scenarios and the manpower requirements for each are known in advance.

4.1.2. Resource Demand and Availability

The next set of input data and variables focuses on the resources that are available to fulfill the demand. For each capability k there are a set of personnel groups p that can be used to fulfill demand. The variable x_{ispkt} represents the number of units of personnel group p associated with capability k assigned to workload element i of mission scenario s in time period t . The number of units required can be defined in many ways (i.e. people or person-hours). In the engineering prototype, the number of personnel (people) required per time period is used.

4.1.3. Resource Restrictions

It is assumed that restrictions exist on the personnel groups that may be used to satisfy that demand. Let the set $P(i, s)$ represent the set of personnel groups that are allowed to perform workload element i of mission scenario s . In the CCO engineering prototype, the user specifies which personnel groups are allowed to perform workload by adding policies to each base for each mission scenario.

4.1.4. Resource Efficiency

Next, define ϵ_{spk} as the efficiency of personnel group p with capability k when assigned to mission scenario s . Efficiency represents the rate at which work is performed relative to some standard. One approach for defining efficiency would be to use a specific personnel group as the standard. Assume that a task requires 400 personnel of military personnel for a given capability and that the same task would require 800 personnel if contractors were used. If military personnel were used as the standard for defining efficiency, then contractors would have an efficiency of 0.5. However, if the contractors were used as the baseline, then the military would have an efficiency of 2.0. The baseline personnel group would always have an efficiency of 1.0.

4.1.5. Model Constraints

(1) Workload Fulfillment Constraint

With these assumptions and concepts defined, the first constraint can be introduced.

$$a_{iskt} = \sum_{p \in P(i,s)} \epsilon_{spk} [x_{ispkt} + g_{ispkt}] \quad \forall i, s, k, t \quad (1)$$

Constraint (1) states that the workload required for each mission scenario must be satisfied by one or more of the eligible personnel groups with the appropriate adjustment for efficiency. It is assumed that all workload must be fulfilled in the time period it is requested. The variable g_{ispkt} represents the number of units that personnel group p associated with capability k is short for fulfilling the requirements of workload element i of mission scenario s in time period t . Shortages could exist if there are budget or personnel limitations on the eligible personnel groups. Shortages are penalized in the objective function to encourage solutions that satisfy all demand.

It is important to notice that this equation is based on a critical assumption. This formulation cannot be used to determine how to optimally delay workload. It is assumed that all workload must be accomplished subject to the manpower use rules. If this is not possible given resource pool availabilities and budget limitations, the model reports what is the cheapest combination of additional resources that must be added to accomplish the workload in all periods as requested. In practice, based on this information, the analysts might alter the workload, but that alteration must be done by subject matter experts and not based on this formulation.

(2) Resource Pools Constraint

The two limiting factors in the model are resource pools and funding. For resource pools, it is assumed that there are a fixed number of units available by personnel group p and capability k for each time period t . This requirement leads to the following constraint.

$$\sum_{i,s} x_{ispkt} \leq h_{pkt} \quad \forall p, k, t \quad (2)$$

The input value h_{pkt} is the size of the resource pool of personnel group p with capability k in time t . Again, the size of the resource pool can be defined in many ways (i.e. people or person-hours), but it should be the same as the manpower requirements a_{iskt} . Observe that for each personnel group, capability, and time period, the assignments are summed over all workload elements and mission scenarios. This implies that all mission scenarios draw on a common set of resources that must be shared to accomplish all of the mission requirements. For example, when two missions occur at the same time, they may have to divide the military resources to accomplish the inherently military tasks and use contractors for the additional workload.

(3) Funding Constraint

The second limiting resource is funding. By fiscal year, there are constraints on the funds that may be spent. The annual budget that can be spent is provided as input. Only the costs associated with using personnel are assessed as the funds spent. This leads to the following constraint.

$$\sum_{i,p,k,t \in o(y)} c_{spkt} x_{ispkt} \leq B_y \quad \forall y \quad (3)$$

The set $o(y)$ is the collection of time periods, t , that occur in year y . c_{spkt} is the cost of using an individual from personnel group p with capability k for mission scenario s in time period t . B_y is the budget available in year y . In the implementation of this model, applying funding constraints is optional.

(4) Non-negativity Constraint

The final constraint requires that the two sets of decision variables be non-negative.

$$x_{ispkt}, g_{ispkt} \geq 0 \quad \forall i, s, p, k, t \quad (4)$$

4.1.6. Objective Function

The objective of the optimization is to determine the manpower mix that completes the required workload with minimal costs. The objective function is given in the expression below.

$$\min \sum_{spkit} c_{spkt} x_{ispkt} + \gamma \sum_{spkit} c_{spkt} Q^s g_{ispkt} \quad (5)$$

The objective function is divided into two terms. The first term represents the total manpower usage costs. It is very similar to the budget constraint (3) except that it determines the total costs over all years instead of on a per year basis. The second term is the penalty for not completing all of the required workload. The parameter γ is the relative importance of not completing workload compared to the manpower costs. Since fulfilling mission requirements takes priority over minimizing costs, γ should be large ($\gamma \gg 1$). The term Q^s is the relative importance of mission scenario s . It is useful to notice that the shortage penalty includes both the relative scenario importance and personnel group cost. These terms are included to encourage shortages of resources to be pushed onto mission scenarios that are relatively less important and onto scenarios, capabilities, and personnel groups with the lowest manpower use costs. It is useful to notice that the lowest manpower use costs are a combination of the cost of the personnel group and its efficiency at completing the workload element due to the influence of equation (1).

4.2. Mission Scenario Uncertainty

In the core model formulation it is assumed that the resource requirements for each mission scenario are known with certainty. In reality, there is uncertainty about the actual manpower requirements that will be generated from mission scenarios. For example, the duration of a mission scenario or its overall manpower footprint may be unknown. This section describes an addition to the core model that allows uncertainty to be directly handled in the model.

When uncertainty is added to the model, it becomes a stochastic linear program. The key idea with the stochastic version of the model is that there are now a collection of possible *realizations* for each mission scenario. Realizations represent one possible outcome for the mission scenario and are indexed by the parameter r . The term a_{iskt} , which represented the manpower requirements in the core model, is updated to indicate the realization (a_{iskt}^r) associated with those particular requirements. This means that each realization of a given mission scenario can have its own manpower requirements.

An analysis can include multiple mission scenarios, and each mission scenario now has a collection of realizations associated with it. Since mission scenarios compete for resources, it is important to consider the variety of resource demands that can be generated from different combinations of mission scenario realizations. For example, suppose that there are two mission scenarios: A and B. Assume that there are two realizations from the first mission scenario (A_1 and A_2) and three from the second mission scenario (B_1 , B_2 , and B_3). Also assume that the occurrence of the realization for mission scenario A is independent of the occurrence of the realization from mission scenario B (this is not a requirement, independence is used in this example for simplicity). Finally, suppose each of the realizations stemming from scenario A are equally likely, and the probabilities for the realizations from scenario B are 20%, 30% and 50%, respectively. This implies that there are six possible “*packages*” (or combinations) of realizations from the mission scenarios that could unfold. Those scenarios and their probabilities of occurrence are given in Table 1 below.

Table 1. Example “Packages” of Realizations

Package Number	Realization from Scenario A	Realization from Scenario B	Probability of Realization from A	Probability of Realization from B	Probability of "Package" of Realizations
1	A1	B1	50%	20%	10%
2	A1	B2	50%	30%	15%
3	A1	B3	50%	50%	25%
4	A2	B1	50%	20%	10%
5	A2	B2	50%	30%	15%
6	A2	B3	50%	50%	25%

In practice, the number of packages that need to be solved could be very large. Since solving each package may not be practical due to time limitations, a sampling approach is used to

estimate uncertainty outputs. A description of the sampling strategy used is provided in Appendix A. Uncertainty Sampling Procedure.

Having defined the concepts of realizations and packages, the core model formulation can be expanded to include uncertainty. The index w is used to identify each package.

4.2.1. Model Constraints with Uncertainty

(1') Workload Fulfillment Constraint

The first constraint is modified as follows:

$$a_{iskt}^r \delta_{ws}^r = \sum_{p \in P(i,s,r)} \epsilon_{spk} \delta_{ws}^r [x_{wispkt}^r + g_{wispkt}^r] \quad \forall w, r, i, s, k, t \quad (1')$$

The variables x_{wispkt}^r and g_{wispkt}^r now represent the number of units of resource capability k supplied (short) by personnel group p in period t for workload element i in mission scenario s under realization r and package w . The term δ_{ws}^r is an indicator parameter that takes on a value of one to indicate that realization r for mission scenario s is included in package w . When a realization for a given scenario is not part of a package (δ_{ws}^r equals zero), the entire constraint is removed. It is also important to notice that the definition of the set P has been modified in the expansion of this formulation to include uncertainty. Now the set P gives the set of personnel groups that can be used to accomplish workload element i under realization r for scenario s . This constraint requires that for each package the work load requirements are fulfilled or the shortages are identified.

(2') Resource Pools Constraint and (3') Funding Constraint

The modified versions of constraints representing resource limits, (2) and (3), are given below. In both cases the uncertainty around the mission scenarios does not affect the availability of resources. Therefore the resource pool size and budgets are the same for each for each package. The main modification to these constraints is that the resource limits must be honored for each package. The set $m(w)$ specifies which pairs of realizations and packages are part of package w .

$$\sum_{(r,s) \in m(w)} \sum_i x_{wispkt}^r \leq h_{pkt} \quad \forall w, p, k, t \quad (2')$$

$$\sum_{i,p,k,t \in o(y), (r,s) \in m(w)} c_{spkt} x_{wispkt}^r \leq B_y \quad \forall y, w \quad (3')$$

(4') Non-negativity Constraint

The non-negativity constraints remain essentially the same, but now include the realization and package indexes.

$$x_{wispkt}^r, g_{wispkt}^r \geq 0 \quad \forall i, s, p, k, t, r \quad (4')$$

4.2.2. Objective Function with Uncertainty

Finally, the objective is modified as given below. The expression inside of the brackets is fundamentally the same as (5). It represents the objective function that would be applied if the core model were used to solve a single package. The term q^w is the probability of package w . The aim of the modified objective is to minimize the expected resource shortages and manpower use costs.

$$\min \sum_{w,(r,s) \in m(w)} q^w \left[\sum_{pkt} c_{pkt} x_{wispkt}^r + \gamma \sum_{pkt} c_{pkt} Q^s g_{wispkt}^r \right] \quad (5')$$

It is important to observe that the structure for addressing uncertainty is very generic and only requires a collection of packages with associated probabilities. In the example above, the realizations from each mission scenario were treated as independent. However, this structure could support correlation between mission scenario realizations.

It is also important to note that this version of the model is separable by package. When the problem is divided by package, the core model formulation can be used. So while this version of the model provides an organized framework for dealing with uncertainty, the same results could be achieved by solving the core model for each package and combining the results with the appropriate weighting. Both approaches allow probability functions to be built for the outputs of the model. The advantage of the modified version is that it provides a framework for expanding the model to a two-stage stochastic optimization model, where strategic decisions could be made in the first stage and evaluated against mission scenario realizations in the second stage.

5. CONCLUSION

The previous sections describe the model that was implemented in Phase 3 of the Contingency Contractor Optimization project. The goal of the model is to assign resources to mission scenarios so that all requested workload is completed at the lowest possible cost while honoring all of the use rules. As input, the model receives mission scenarios which create a demand for personnel, resource pools which can be used to fulfill this demand, and rules governing how these resources can be used. As output, the model identifies whether or not the workload can be accomplished within the specified limits, the recommended Total Force Mix, and the cost of that mixture. The uncertainty feature of the model allows these outputs to be assessed in a probabilistic sense, to better reflect reality.

REFERENCES

- [1] J. D. Durfee, K. L. Adair, A. Bandlow, T. R. Brounstein, J. L. Gearhart, K. A. Jones, N. Martin, "Contingency Contractor Optimization Phase 3, oftware Design Document - Contingency Contractor Optimization," Sandia National Laboratories, October 2013.

APPENDIX A. UNCERTAINTY SAMPLING PROCEDURE

Since the possible number of packages to analyze can be large, a sampling procedure is used to estimate the uncertainty outputs. A stratified sampling approach is used. This approach was chosen over simple random sampling because it encourages diversity in the sampled durations from each of the mission scenarios. Initial analysis showed that coverage of the range of possible mission scenario durations was essential for obtaining good estimates of uncertainty outputs. When a simple random sampling approach was used, the outputs showed unacceptable levels of variability. While this variability can be reduced by increasing the number of samples, this approach was not chosen since there was a desire to keep the solution time to tens of minutes.

The core idea of the of the stratified sampling procedure is to divide the total durations of each mission scenario into bins of approximately equal size. Once the width of each bin is determined, the probability a drawing a sample from each bin can be calculated. Once this has been done, “hyperbins” are created by taking all possible combinations of bins from each of the mission scenarios. The probability of drawing a sample from each of these “hyperbins” can be determined from the joint distribution of mission scenario durations. Finally, a sampling budget can be allocated to each “hyperbin” based its associated probability. For each “hyperbin,” samples are drawn without replacement. If the sampling budget is larger than the number of possible outcomes, only the number of samples needed to exhaust the sampling space are used.

Three parameters are used for this sampling procedure. The first is a sampling budget. This parameter defines the total number of samples to be drawn. The second is a limit on the number of bins to create for each mission scenario. This parameter ensures that there are enough bins to cover the range of durations but not so many bins that the benefits of this approach are not realized. The final parameter is the minimum width of each bin (number of time periods). This is used to prevent cases where bins are created that have widths that are too narrow for practical analysis needs. These parameters are set in the uncertainty.properties files which is described in [1].

To illustrate this sampling procedure, consider an example where two mission scenarios have uncertainty around phases 3, 4, and 5. Assume that the uncertainty around each phase is represented by a uniform distribution, the maximum number of bins is three, the minimum width of each bin is three time periods, and there is a sample budget of twenty. The durations of each phase are shown in Table 2.

Table 2. Example Mission Scenario Durations (Number of Time Periods)

Mission Scenario	Phase 0	Phase 1	Phase 2	Phase 3	Phase 4	Phase 5
1	1	2	3	1-3	5-7	5-6
2	2	5	6	5-9	3-7	4-7

For mission scenario 1 there are 18 possible outcomes, all ranging from 17 to 22 time periods in duration. Since there are only six possible durations and each bin must be at least three time

periods wide, only two bins are created. The bins and their associated details are summarized in Table 3.

Table 3. Example Bins For Mission Scenario 1

	“Short”	“Long”
Duration (Number of Time Periods)	17-19	20-22
Number of Sample	6	6
Probability	0.5	0.5

For mission scenario 2 there are 100 possible outcomes, all ranging from 25 to 36 time periods in duration. Since there are twelve possible durations and each bin must be at least three time periods wide, three bins are created. The bins and their associated details are summarized in Table 4.

Table 4. Example Bins For Mission Scenario 2

	“Short”	“Medium”	“Long”
Duration (Number of Time Periods)	25-28	29-32	33-36
Number of Sample	20	60	20
Probability	0.2	0.6	0.2

Once the bins have been created for each mission scenario, the “hyperbins” and associated joint probabilities can be created. Since the first and second mission scenarios have two and three bins, respectively, a total of six (2x3) “hyperbins” are created. Table 5 summarizes the “hyperbins” for this example. The number of outcomes in each “hyperbin” is shown in the table, along with the associated probability of each “hyperbin” (shown in parenthesis).

Table 5. Example “Hyperbins” with Number of Possible Outcomes and Associated Probabilities

		Mission 2 Bins		
		“Short”	“Medium”	“Long”
Mission 1 Bins	“Short”	120 (0.1)	360 (0.3)	120 (0.1)
	“Long”	120 (0.1)	360 (0.3)	120 (0.1)

Finally, the sampling budget can be allocated to each bin. Two samples are allocated to each of the four bins that have a probability of 0.1, and six samples are allocated to the two bins that have a probability of 0.3. This is determined by multiplying the probability of the “hyperbin” by the sampling budget and rounding to the nearest whole number. For each “hyperbin” the allocated number of samples is drawn from the total number of samples without replacement.

APPENDIX B. PHASE 3 MODEL IMPLEMENTATION

The model formulation presented in the previous sections is written generically and could be implemented in many ways. For Phase 3 of the OSD ATL CCO project, the following implementation decisions and assumptions were made:

- Weeks are used as the units for time periods.
- Manpower assignments, requirements, and availability are expressed in full-time equivalents.
- Budgets and costs are expressed in thousands of dollars.
- Active military personnel are used as the reference for manpower efficiency.
- Uncertainty is only implemented for the durations of phases 3, 4, and 5 of mission scenarios.
- Phase duration uncertainty is modeled using uniform distributions.
- Phase durations are assumed to be independent within a mission scenario and between mission scenarios.
- By default the sampling budget is set to 60, the minimum bin width is set to 4 weeks, and the maximum number of bins is set to 4. These settings should be sufficient for analyzing two mission scenarios, where each mission scenario has several months of uncertainty. With additional uncertainty, these default values may need to be updated.

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