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SAND2013-10709
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Printed December 2013

# Developments in digital in-line holography enable validated measurement of 3D particle field dynamics

Daniel Robert Guildenbecher

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico  87185 and Livermore, California  94550

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# Developments in digital in-line holography enable validated measurement of 3D particle field dynamics

Daniel Guildenbecher
Thermal/Fluid Experimental Sciences
Sandia National Laboratories
P.O. Box 5800
Albuquerque, New Mexico 87185-MS0840

**Abstract**

Digital in-line holography is an optical technique which can be applied to measure the size, three-dimensional position, and three-component velocity of disperse particle fields. This work summarizes recent developments at Sandia National Laboratories focused on improvement in measurement accuracy, experimental validation, and applications to multiphase flows. New routines are presented which reduce the uncertainty in measured position along the optical axis to a fraction of the particle diameter. Furthermore, application to liquid atomization highlights the ability to measure complex, three-dimensional structures. Finally, investigation of particles traveling at near sonic conditions prove accuracy despite significant experimental noise due to shock-waves.

# ACKNOWLEDGMENTS

This work was supported by a Laboratory Directed Research and Development (LDRD) grant. The author gratefully acknowledges this source of support.

Much of this work was performed in collaboration with Phillip L. Reu (1535) whose mentorship of the author throughout the LDRD grant is gratefully acknowledged.

Significant thanks also goes to collaborators at the Purdue University School of Mechanical Engineering including doctoral candidate Jian Gao and Professor Jun Chen. Their efforts were instrumental in much of the fundamental developments reported here.

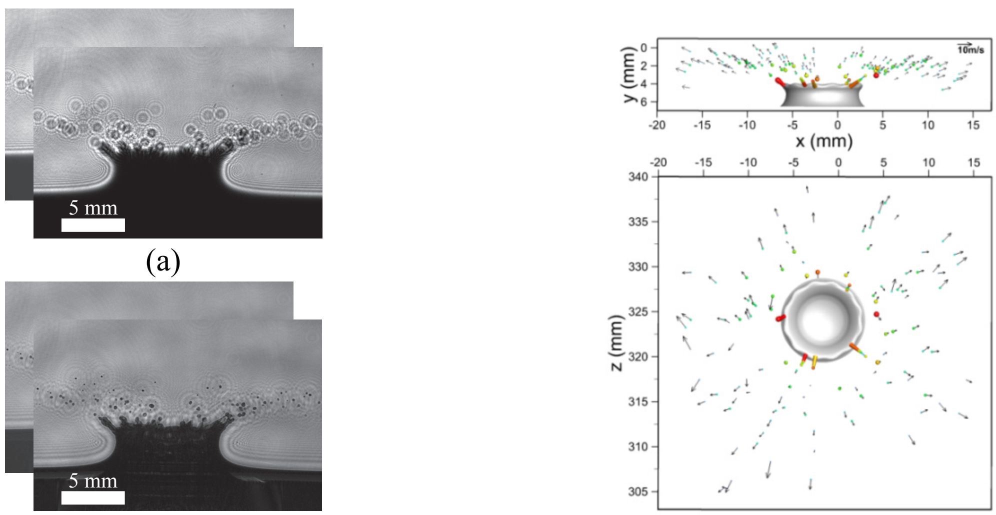
# CONTENTS



# FIGURES

# 1. INTRODUCTION

Digital in-line holography (DIH) is an optical technique in which a collimated laser illuminates an object field. The resulting diffraction patterns are digitally recorded, and numerically refocused via solution of the diffraction equations [1, 2]. For example, Figure 1(a) shows experimental holograms of the impact of a water drop on a thin film [3]. In Figure 1(b), these holograms are numerically refocused to a distance of $z$ = 320 mm along the optical axis. In-focus features of the secondary drops and the crown-morphology are revealed. Finally, by searching through a range of $z$-positions, algorithms are defined to automatically extract the in-focus features at each $z$, resulting in the three-dimensional (3D) reconstruction of the particle field shown in Figure 1(c). Here, velocities are obtained by matching detected particle locations from two holograms recorded with short inter-frame time. Full details on these results can be found in Guildenbecher *et al*. [3].




(a) (b) (c)

**Figure 1. (a) Digital in-line holograms of the impact of a drop on a thin film, (b) the corresponding amplitude images when the holograms are numerically refocused to *z* = 320 mm, and (c) the reconstructed 3D particle positions and velocities [3].**

This report summarizes developments in DIH obtained during the course of a Laboratory Directed Research and Development (LDRD) project at Sandia National Laboratories spanning the January 2011 to December 2013 timeframe. Following a brief explanation of the underlying principles and challenges, specific developments are presented with reference to a number of published works [3-13].

## 1.1. Numerical refocusing

Figure 2 shows the basic experimental configuration of DIH. A collimated laser propagates through a particle field. The portion of the collimated beam whose amplitude and phase is altered by the particle field is referred to as the object wave, with complex amplitude $E_o$. The remaining portion of the beam which passes through the particle field undisturbed is referred to as the reference wave with complex amplitude $E_r$. The hologram, $h$, forms from the interference between the object wave and reference wave,

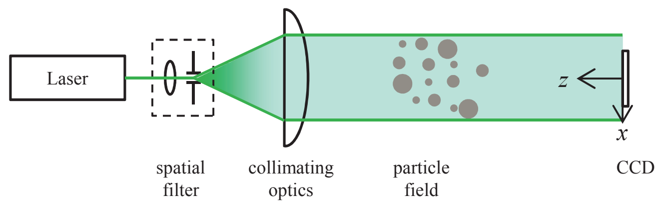



**Figure 2. Basic experimental configuration for digital in-line holography (DIH) of a 3D particle field.**

$$h = |E_r + E_o|^2 = |E_r|^2 + |E_o|^2 + E_o E_r^* + E_r E_o^*, \quad (1)$$

where all quantities are defined at the hologram plane ($z = 0$) and $^*$ signifies the conjugate. In the numerical reconstruction, the hologram is multiplied by the complex conjugate of the reference wave, $E_r^*$, resulting in an expression of complex amplitude

$$h \cdot E_r^* = \left[|E_r|^2 + |E_o|^2\right] E_r^* + E_r^{*2} E_o + |E_r|^2 E_o^*. \quad (2)$$

The first term on the right hand side of Eq. (2), $[|E_r|^2+|E_o|^2]E_r^*$, is referred to as the DC term and when numerically refocused results in the background intensity. The second term, $E_r^{*2}E_o$, is referred to as the virtual image term and when numerically refocused results in an out-of-focus virtual image. Finally, the third term, $|E_r|^2E_o^*$, is referred to as the real image term. This term is the complex conjugate of the original object wave multiplied by a constant. When numerically refocused, the real image term results in an in-focus real image of the particle at the actual particle $z$-position.

The complex amplitude in Eq. (2) is numerically propagated to any distance, $z$, through solution of the diffraction equation,

$$E(z) = h \cdot E_r^* \otimes g(z). \quad (3)$$

Here, $E(z)$ is the reconstructed complex amplitude at $z$; $g$ is the diffraction kernel, and $\otimes$ is the convolution operation. Eq. (3) can be numerically evaluated as

$$E(z) = \Im^{-1}\left\{\Im\left\{h \cdot E_r^*\right\} \cdot G(z)\right\}, \quad (4)$$

where $\Im$ and $\Im^{-1}$ denote the Fourier transform and inverse Fourier transform, respectively. These are typically numerically evaluated with the fast Fourier transform (FFT). $G(z)$ is the analytic expression of the diffraction kernel. For example, for the Rayleigh-Sommerfeld formulation of the diffraction equation,

$$G(z) = \exp\left\{jkz\sqrt{1-(\lambda m/M\Delta x)^2 - (\lambda n/N\Delta y)^2}\right\}. \quad (5)$$

Here $\lambda$ is the wavelength, $k$ is the wave number, $k = 2\pi/\lambda$; $m$ and $n$ are the pixel coordinates in the $x$- and $y$-directions, respectively; $M$ and $N$ are the total number of pixels in the $x$- and $y$-directions, respectively, and $\Delta x$ and $\Delta y$ are the size of the individual pixels in the $x$- and $y$-directions, respectively. Finally, once the reconstructed complex amplitude, $E(z)$ is obtained at depth $z$, the results can be visualized using the amplitude $A = |E|$ or intensity $I = |E|^2$. For example, Figure 1(b) shows the reconstructed amplitude when the hologram in Figure 1(a) is numerically refocused to $z = 320$ mm using Eq. (4). Further details on these numerical refocusing routines can be found in the book by Schnars and Jueptner [1].

## 1.2. Depth-of-focus problem

A major challenge in DIH is the depth-of-focus problem as discussed by Katz and Sheng [2]. The size of the angular aperture, $\Omega$, from which a particle is reconstructed can be estimated from the extent of the central diffraction lobe,

$$\Omega \approx 2\lambda/d\,, \tag{6}$$

where $d$ is the particle diameter. Traditionally, the depth-of-focus, $\delta$, is estimated based on the change in intensity within the Airy spot as

$$\delta \approx \lambda/4\Omega^2\,. \tag{7}$$

Combining Eqs. (6) and (7) gives as estimate of the depth-of-focus of reconstructed particle images in DIH [14]

$$\delta \approx d^2/\lambda\,. \tag{8}$$

For example, if a hologram of particle of diameter, $d = 465$ μm is recorded using a laser wavelength of $\lambda = 532$ nm, Eq. (8) estimates the depth-of-focus as $\delta \approx 400$ mm. This gives rise to high uncertainty in the measured in-focus depth, $z$. Consequently, minimization of the effects of this depth-of-focus problem and quantification of $z$-uncertainty are a major focuses of the efforts reported here.

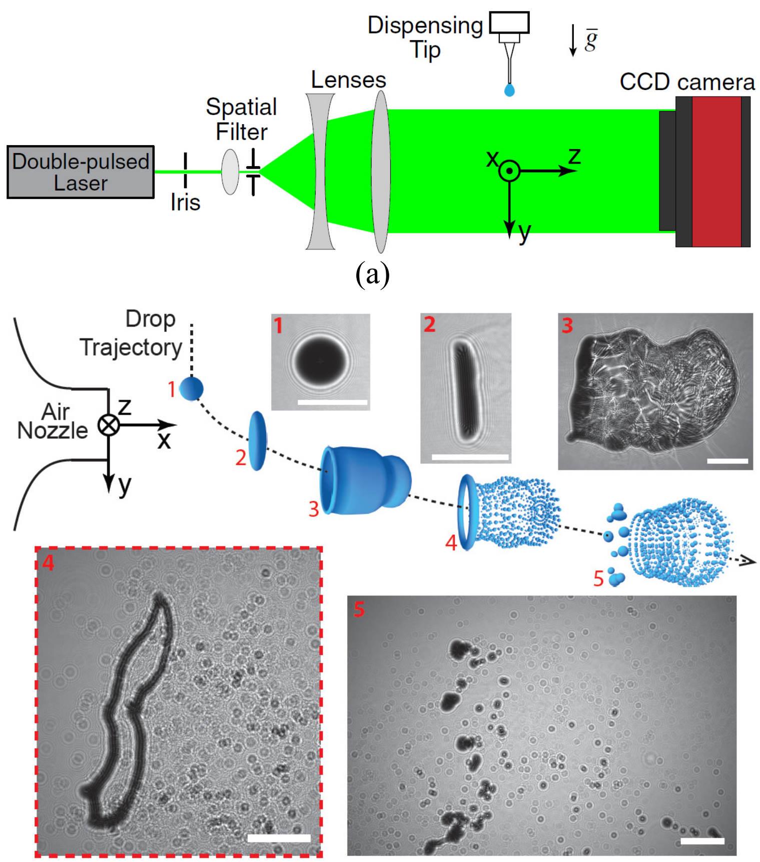
# 2. HYBRID METHOD OF PARTICLE DETECTION

The literature contains a number of methods which sweep through the expected particle $z$-positions using Eq. (4) and apply focus metrics to the numerically refocused images in order to extract the morphology and 3D location of in-focus particles (for examples see [4, 5, 15, 16]). The accuracy of a given method, especially with respect to the $z$-position, is a strong function of the chosen focus metric and other processing details. In Guildenbecher *et al*. [4] a series of simulations and experiments are reported which quantify the accuracy of such methods. As expected, it is shown that many methods suffer from high-uncertainty in the particle $z$-position. Furthermore, these methods often rely on user-tunable parameters to perform image thresholding and other processing necessary to extract particle morphologies. Consequently, accuracy of a given method is also a function of the selected parameters.

To minimize the use of user-tunable parameters and reduce the $z$-uncertainty, in Guildenbecher *et al*. [4] a new method of particle detection is proposed. This method is based on a combination of previously proposed methods [15, 16] and is therefore referred to as the *hybrid method* of particle detection. The method works by searching for regions in 3D space which display a minimum of intensity within the interior of the region and a maximum sharpness along outer edges of the region. This method was first proposed in a conference proceeding [6] and later published, with further details, in an *Applied Optics* article [4] and an *Optics Express* article [5]. For method specifics, readers are referred to those works.

Experimental quantification of method accuracy is especially vital given the large number of parameters expected to affect the uncertainty. For this reason, much of the work reported herein has been focused on experimental uncertainty quantification. In Guildenbecher *et al*. [4] a known, 3D particle field is created using polystyrene particles suspended in viscous silicone oil. Due to the high-viscosity, particles settle very slowly (on the order of μm/s); therefore, over sufficiently short periods, the particle field is essentially stationary. By placing the stationary particle field on a $z$-traverse, multiple holograms can be recorded wherein all particles undergo a constant $z$-displacement. Comparison between the $z$-displacement measured with holography and the actual $z$-displacement results in an experimental quantitation of uncertainty. For example, in Guildenbecher *et al*. [4] particles with a mean diameter, $\overline{d} = 465$ µm, are subjected to an actual displacement of $\Delta z = 2$ mm. Using the hybrid method of particle detection, measured displacements of $\Delta z = 1.91 \pm 0.81$ mm (mean ± standard deviation) are found. Notice, this is significantly less than the estimated depth-of-focus, $\delta \approx 400$ mm, as reported in § 1.2 for these conditions. Similar results are also reported in a few conference proceedings [6-9] as well as an in-depth article on uncertainty quantification published in *Optics Express* [5]. Again, readers are referred to these works for further details. Finally, by slowly stirring the particles within the silicone oil, the measured circular particle trajectories can be compared with theory to additionally derive an uncertainty in the overall measured $z$-positions. Initial results along the lines are reported in a recent conference proceeding [10].

DIH and the hybrid method have been applied to quantify a number of multiphase flow phenomena. For example, in Gao *et al*. [11], we report on the experimental quantification of the breakup of an ethanol drop in an air-stream. This phenomenon is a fundamental mechanism in the formation of fuel sprays and is of interest as a canonical problem for validation of multiphase simulations. Figure 3 shows the experimental configuration along with some example recorded holograms.




(b)

**Figure 3. DIH configuration for investigation of the breakup of an ethanol drop in an air-stream, (a) optical configuration and (b) orthogonal view illustrating the drop trajectory and breakup process along with recorded experimental holograms [11].**

Using the hybrid method of particle detection, the size and 3D position of individual secondary drops are measured. Furthermore, from two holograms recorded with short inter-frame time, matching between the measured 3D positions is used to derive the three-component (3C) velocity field. Finally, for hologram number 4 (circled in red in Figure 3(b)) the 3D morphology of the intact ring is measured from the *z*-position of maximum sharpness. This results in the measured 3D flow field shown in Figure 4.

The accuracy of the results in Figure 4 are validated by comparing the total measured volume of the ring and secondary drops to the volume of the initial drop, resulting in a discrepancy of 2.2%. Furthermore, the drop size distribution is independently measuring using a commercial phase Doppler anemometer (PDA) at a downstream location where the measured drops are spherical. The Sauter mean diameter measured with DIH and PDA agree to within 6%.

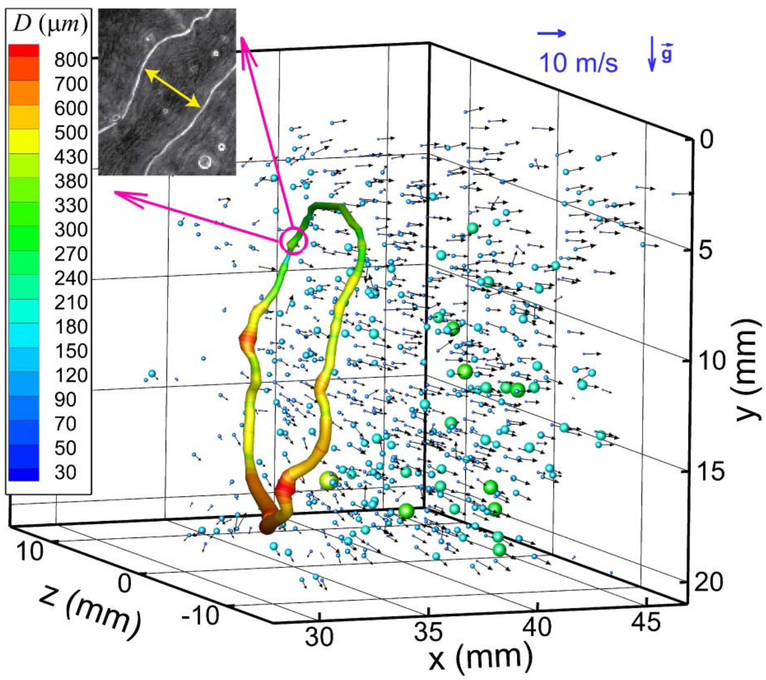
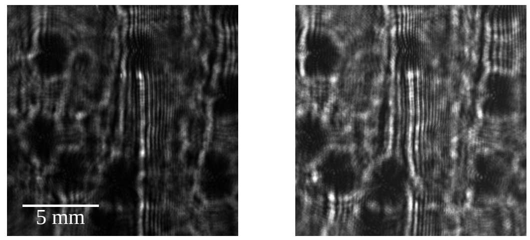



**Figure 4. Breakup of a drop in an air-stream as measured with DIH. Results correspond to hologram number 4 in Figure 3(b) [11].**

The experimental results shown in Figure 4 clearly demonstrate the ability of DIH and the hybrid method of particle detection to measure complex, 3D flow phenomena. Similar results are reported in a conference proceeding [12] and in an article in *Experiments in Fluids* [3] focused on the impact of water drop on a thin film, as illustrated in Figure 1. Again, the reader is referred to these works for further details.

## 3. CROSS-CORRELATION METHOD OF PARTICLE DETECTION

Although the results in Figure 1 and Figure 4 show that the hybrid method can be immensely useful for quantification of multiphase phenomena with accuracy previously unobtainable, for some applications sensitivity to experimental noise and uncertainty in the *z*-locations may nevertheless be unacceptable. For example, work is ongoing to investigate the application of DIH to high-speed particle fields. When particle speeds approach or exceed the sonic conditions of the surrounding gas, compressibility effects and shock-waves result in index of refraction gradients within the gas which greatly distort the hologram images. This is illustrated in Figure 5 which shows the holograms recorded from the pellets in a shotgun blast. The diffraction patterns from individual pellets (dark regions) are distorted by the presence of shock waves which result in the observed vertical bands.




(a) (b)

**Figure 5. Experimental holograms of $\overline{d}$ = 2mm shotgun pellets traveling at roughly 350 m/s, perpendicular to the collimated laser beam, from right to left in the images. Here the inter-frame time is 4 μs [13].**

When the results in Figure 5 are quantified with the hybrid method, the uncertainty in measured *z*-displacement is estimated to be on the order of 20 mean particle diameters. In comparison, for the calibration results of quasi-stationary particles reported in § 2, the uncertainty is on the order of 2 mean particle diameters. This order-of-magnitude increase in uncertainty can be attributed to significant experimental noise in Figure 5.

This challenge motivated the development of a second method of particle detection, referred to here as the *cross-correlation* method. In this method, it is recognized that when two holograms are recorded with short inter-frame time, the individual particle images from each hologram contain correlated information. By simultaneously searching throughout *z* in both holograms and calculating the cross-correlation of refocused particle images, an accurate measure of 3D particle positions and displacements can be obtained from the conditions where the cross-correlation coefficient maximizes. Details of this method are published in an *Optics Letters* article [13]. Using the same experimental data of quasi-stationary particles reported in § 2, measured displacements of $\Delta z = 1.996 \pm 0.072$ mm (mean ± standard deviation) are found using the cross-correlation method. In comparison to the hybrid method, this is an order-of-magnitude improvement in displacement uncertainty. Consequently, it is possible to accurately measure the size and 3C velocity of the pellets in the shotgun blast, as shown in Figure 6. Again, for full details on these results, the reader is referred to Guildenbecher *et al.* [13].

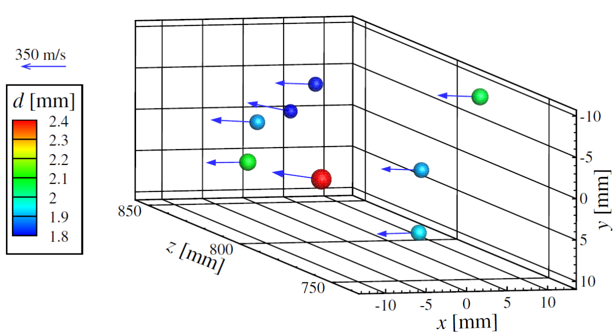



**Figure 6. Measured diameters, 3D positions, and 3C velocities from the processing of the hologram pair in Figure 5 with the cross-correlation method [13].**

# 3. CONCLUSIONS

Here, a brief summary of recent developments in digital in-line holography (DIH) at Sandia National Laboratories is reported. DIH is an optical method which allows for numerical refocusing of recorded holograms. For applications to three-dimensional (3D) particle fields, DIH provides quantification of particle sizes, 3D positions and three-component (3C) velocities. Consequently, the method has many potential applications; however, practical challenges arise due to the depth-of-focus problem and the resulting high-uncertainty in the measured particle $z$-positions along the optical axis. Here, new methods of particle detection are reported which reduce the depth uncertainty to a fraction of the particle diameter. These developments allow for measurement of complex, 3D flow phenomena with quantified uncertainty. For full details on these results, the reader is referred to the following journal articles which have been published as a result of this work:

Guildenbecher, D. R., Gao, J., Reu, P. L., and Chen, J., 2013, "Digital holography simulations and experiments to quantify the accuracy of 3D particle location and 2D sizing using a proposed hybrid method," Appl. Opt., 52(16), pp. 3790-3801.

Gao, J., Guildenbecher, D. R., Reu, P. L., Kulkarni, V., Sojka, P. E., and Chen, J., 2013, "Quantitative, 3D diagnostics of multiphase drop fragmentation via digital in-line holography," Opt. Lett., 38(11), pp. 1893-1895.

Gao, J., Guildenbecher, D. R., Reu, P. L., and Chen, J., 2013, "Uncertainty characterization of particle depth measurement using digital in-line holography and the hybrid method," Opt. Express, 21(22), pp. 26432-26449.

Guildenbecher, D. R., Reu, P. L., Stauffacher, H. L., and Grasser, T. W., 2013, "Accurate measurement of out-of-plane particle displacement from the cross-correlation of sequential digital in-line holograms," Opt. Lett., 38(20), pp. 4015-4018.

Guildenbecher, D. R., Engvall, L., Gao, J., Grasser, T. W., Reu, P. L., and Chen, J., 2014, "Digital in-line holography to quantify secondary droplets from the impact of a single drop on a thin film," Exp. Fluids, Accepted for publication.

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