SANDIA REPORT

SAND2012-5299
Unlimited Release
Printed June 2012

Contingency Contractor Optimization
Phase 2, Model Description and Formulation

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Abstract

The goal of Phase 2 the OSD ATL Contingency Contractor Optimization project is to create a prototype of a tool for the contingency contractor element of total force planning during the Support for Strategic Analysis (SSA). An optimization model was developed to determine the optimal mix of military, Department of Defense (DoD) civilians and contractors that accomplishes a set of user defined mission requirements at the lowest possible cost while honoring resource limitations and manpower use rules. Additional features allow the model to understand the variability of the Total Force Mix when there is uncertainty in mission requirements and recommend where investments in manpower capabilities might minimize the overall total force costs in future operations.

ACKNOWLEDGMENTS

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J4 Analysis and Resource Division (ARD)
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OSD CAPE Joint Data Support
OSD Policy
OSD Personnel & Readiness
OSD DASD Plans
SOUTHCOM
PACOM
CENTCOM
NORTHCOM
OSD Acquisition Test, & Logistics (ATL)

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NOMENCLATURE

DoD	Department of Defense
SSA	Support for Strategic Analysis
ATL	Acquisition, Technology and Logistics
OSD	Office of the Secretary of Defense

1. INTRODUCTION

The goal of the OSD ATL Contingency Contractor Optimization project is the development of a centralized strategic planning tool that allows senior decision makers to quickly and quantitatively assess the impacts, risks, and mitigation strategies associated with utilizing contract support. The intent of public law and congressional oversight of contractor operations in support of military contingencies is not supported by existing planning tools. As a consequence, the extent, duration, and resourcing needed to satisfy contingency needs for contracted capabilities in support of national military strategy objectives cannot currently be quantified.

During Phase 2 of the Contingency Contractor Optimization project a prototype of a tool for the contingency contractor element of total force planning during the Support for Strategic Analysis (SSA) was created. A key component of this tool was an optimization model which determines the least expensive Total Force Mix (military, Department of Defense (DoD) civilian, and contractor) that meets mission requirements. This document describes the concepts and mathematical formulation of this optimization model.

There are two major components to an optimization model, the objective and the constraints. The objective is a mathematical function of the decisions to be minimized or maximized. In the case of this model, the objective is to find the Total Force Mix (as defined as the number of person-months by personal type and skill category) that fulfills all of the mission requirements and minimizes the total personnel costs. Constraints are rules which limit the types of solutions that are allowed while achieving the goal of the objective. In this model, constraints consist of things such as limits on the number of military personnel available, treaties that prevent the use of contractors to fulfill certain mission requirements under one or more circumstances, and budget limitations. When solved, the optimization model recommends a Total Force Mix that minimizes the budget while still honoring all of these constraints.

An optimization model was selected because it contains several desirable properties for this application. First, optimization models are prescriptive, which means they provide recommendations on the best way to do something, including strategies to mitigate the impact of risks. This is applicable to total force planning, which is concerned with developing a Total Force Mix at the strategic level to support future operations. In contrast, modeling techniques, such as simulation, are descriptive in nature. This means that they represent how a system will perform given a set of decisions, whereas optimization models determine the best set of decisions. Optimization models have the added benefit that they are:

1) *rigorous*, as they can be written down in a precise mathematical language which facilitates understanding and validation,
2) *consistent*, since all analysis uses the same set of logic, different analyses can be readily compared, and
3) in this case, *tractable*, because there are well established algorithms for solving this model formulation.

The remainder of this report is divided into four sections. Section 2 contains a conceptual description of the model and the key capabilities it provides. Sections 3 and 4 provide the precise mathematical description of the model. Conclusions are presented in section 5. While sections 3

and 4 assume the reader is familiar with mathematical programming, the remainder of the document should be accessible to any reader familiar with the domain.

2. MODEL DESCRIPTION

This section provides a conceptual description of the model to illustrate the types of data it requires, the features of the model, and the types of output it provides. The Strategic Contractor Planning Tool prototype created during Phase 2 of the Contingency Contractor Optimization project provides a mechanism for both creating input data for solving and viewing the results of the model. The actual model that the prototype interacts with is written in a very generic form, which is discussed in sections 3 and 4. Since the purpose of this section is to understand the behavior of the model, it will be discussed in the context of the planning tool prototype.

2.1. Core Model Inputs and Behavior

Figure 1 illustrates some of the core ideas of the model. This figure is not intended to list all personnel groups or capabilities, but is illustrative of the core ideas. This figure is divided into three regions: supply, demand, and use rules. Mission requirements create a demand for personnel, which is satisfied by drawing on resources (personnel). The model's goal is to assign personnel resources to each mission in such a way that all of the mission requirements are fulfilled and the cost is minimized. Ideally, the costs could be minimized by using the cheapest resources for all mission requirements. In reality, there are constraints which complicate this task. Rules exist that prevent certain types of contractors from performing a required task or that require military personnel to be used for a task. This concept is illustrated with the use rules region of Figure 1, which shows the resources that are allowed to fulfill each task. There may be additional limitations, such as the availability of certain resources, which must be honored when determining assignments. Finally, there may be annual personnel-related budgets that limit spending.

The supply region of Figure 1 represents resources that are available to accomplish the workload. Resources are identified by the capability they provide and the personnel group to which they belong. In Figure 1, the capabilities are logistics and force support. The personnel groups are military, U.S. contractor, and local nation contractor. There are several additional pieces of data attached to each of these resources. Some resources have a limited number of personnel while others are effectively unlimited. There are also costs associated with using the resource. Finally, some resources may not be as efficient as others. For example, suppose for a particular element of the workload, contractors are 50% as efficient as military personnel. This means that two contractors are required to perform the same amount of work as a single person in uniform.

The demand region of Figure 1 illustrates how mission requirements are created. At the highest level, work is divided by mission scenarios. Each mission scenario is then divided into geographical regions such as bases. Since mission requirements can change over time, each base can then be divided into a series of phases, each with an associated duration. These phases then require a specific quantity of each capability. The compilation of all of the capability requirements by time makes up the total demand which is used by the model as input.

The use rules region of Figure 1 illustrates how use rules are implemented in the planning tool. In this example, only resources that are connected to a task can be used. The color of each line represents the personnel group. The solid and dashed lines are provided to help visually

distinguish each line. The most obvious rule in this example is that logistics and force support tasks can only be fulfilled by resources with the same capability. However, it is also important to notice that not all logistics resources can fulfill all logistics tasks. In the Phase 2 prototype, policies and manpower mix rules can be applied which further limit the resources that can be used. The optimization model must honor all use rules in its solution.

Figure 1 illustrates the structure of the problem that the model must solve. There are two important points which can be seen in this figure. First, both of the mission scenarios draw from the same set of resources. When assigning resources to tasks, the model must resolve the competition for resources. Second, in many cases a task can be fulfilled by several resources. When solved, the model will determine which resource or resources should be used for the task. The user creates a set of resource options, but the model selects which combination of resources to use. This is ultimately the value added by the model. If there are insufficient resources available given the use rules, the model identifies the shortages and the least expensive method to overcome them.

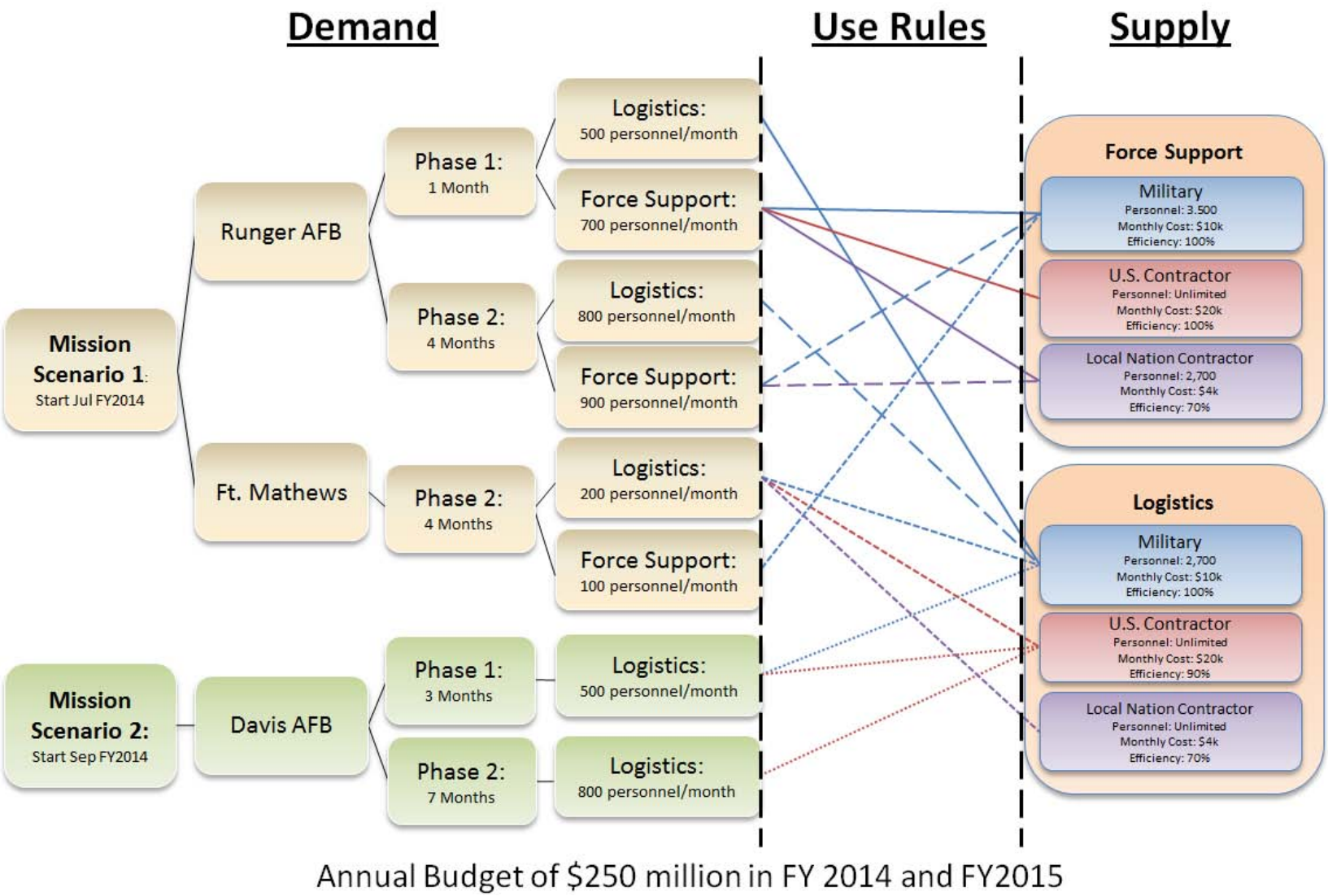

Figure 1. Conceptual Description of the model structure and data.

2.2. Key Model Outputs

The model provides three key outputs, referred to here as feasibility, manpower mix, and cost. In certain cases, the resources required may exceed what is available. When this occurs, the model provides outputs which show that it is not feasible to accomplish all of the missions given the current resources and rules. Manpower mix refers to the mixture of personnel groups that are used to accomplish the mission. Finally, cost outputs show how the mixture of different personnel groups affects the cost of the mission. The example below illustrates these three types of outputs.

Consider two hypothetical analysis cases. In the first case, there is a requirement to accomplish a single mission scenario. In the second case, there is a requirement to accomplish the original mission scenario and an additional mission scenario. The personnel requirements over time for these two cases are illustrated in the figure below. Notice that when both mission scenarios occur, they overlap. This will cause a large increase in total demand for personnel.

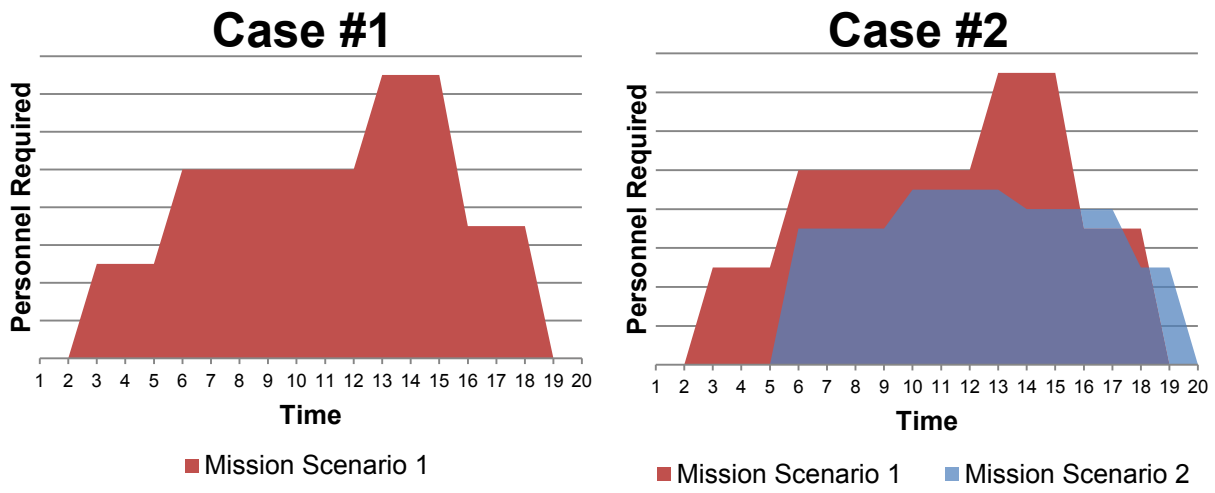

Figure 2. Resource demand profiles for two hypothetical cases.

As an example, consider two resource pools: force support and logistics. Assume that force support can only be performed by military personnel, whereas logistics can be performed by military, U.S. contractors, or local nation contractors. Assume that U.S. contractors are the most expensive option, followed by military, then local nation contractor, and that all three groups have the same efficiency. Also, assume that rules are in place which limits the number of local nation contractors that can be used. Finally, assume that military personnel are a limited resource while contractors are effectively an unlimited resource. When the model is solved, it attempts to accomplish all of the requested workload at the lowest cost possible.

Figure 3 below illustrates how the model outputs demonstrate the feasibility or infeasibility of a set of mission scenarios. In these two graphs, the blue bars show the number of military force support that were used in both cases. The black line represents the quantity of military force support that were available. The red bars indicate the quantity of additional military personnel

that would be required to complete all of the requested workload. In case #1, the requested workload does not exceed the quantity of resources that are available. However, when two scenarios occur simultaneously in case #2, there are insufficient resources to accomplish the workload. Two possible solutions to this problem would be to increase the size of the resource pool or to allow contractors to perform force support functions.

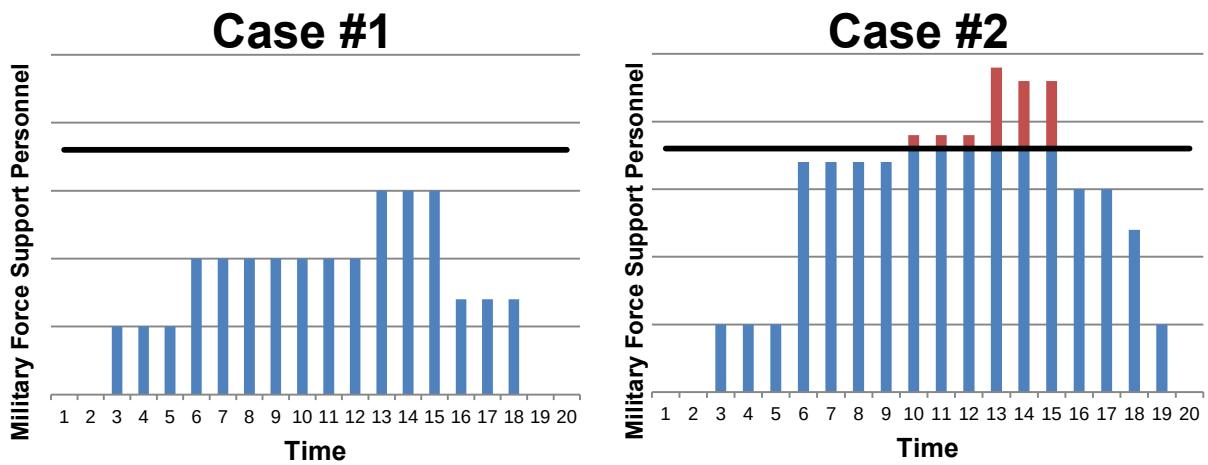

Figure 3. Comparison of military force support personnel assignments.

In the case of logistics personnel, both military and contractors are allowed to perform work. This means that all workload can be accomplished without any shortages. The figure below demonstrates the manpower mix outputs provided by the model. Both case #1 and case #2 include mission scenario 1. The pie charts below show the mixture of personnel groups that are used to satisfy the demand for the logistics capability for mission scenario 1. In case #1, about three quarters of the work is done by military personnel. In case #2, another mission scenario (mission scenario 2) is added that also requires military personnel. This means that mission scenario1 must share the military personnel with mission scenario 2 and use more U.S. contractors. Despite being the most expensive option, U.S. contractors are used to replace these personnel because there are limits on the number of local nation contractors that can be used.

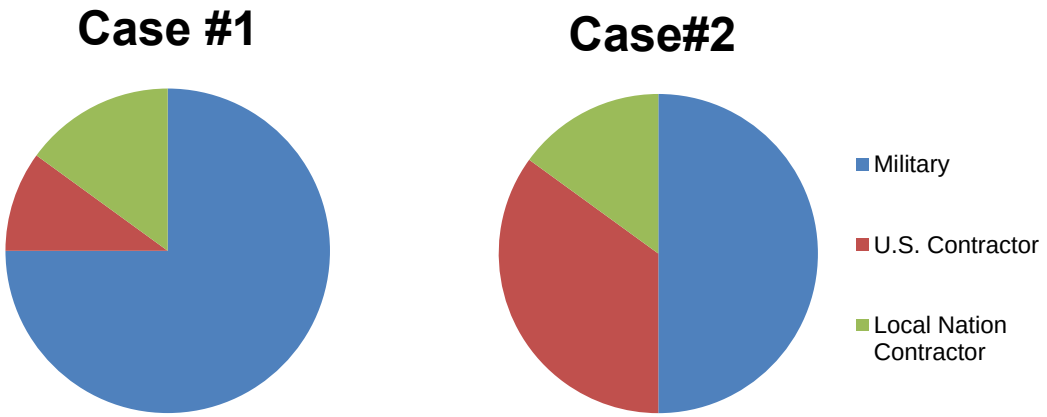

Figure 4. Comparison of manpower mix for the logistics capability in mission scenario 1.

Finally, Figure 5 below illustrates the type of outputs the model provides related to costs. Assume that U.S. contractors are more expensive than military personnel. The manpower mix example above demonstrated that a competition for resources can change the composition of the workforce. This in turn affects the cost of accomplishing the workload. In Figure 5, the cost of logistics personnel is higher for mission scenario #1 in the second case, since more U.S. contractors must be used.

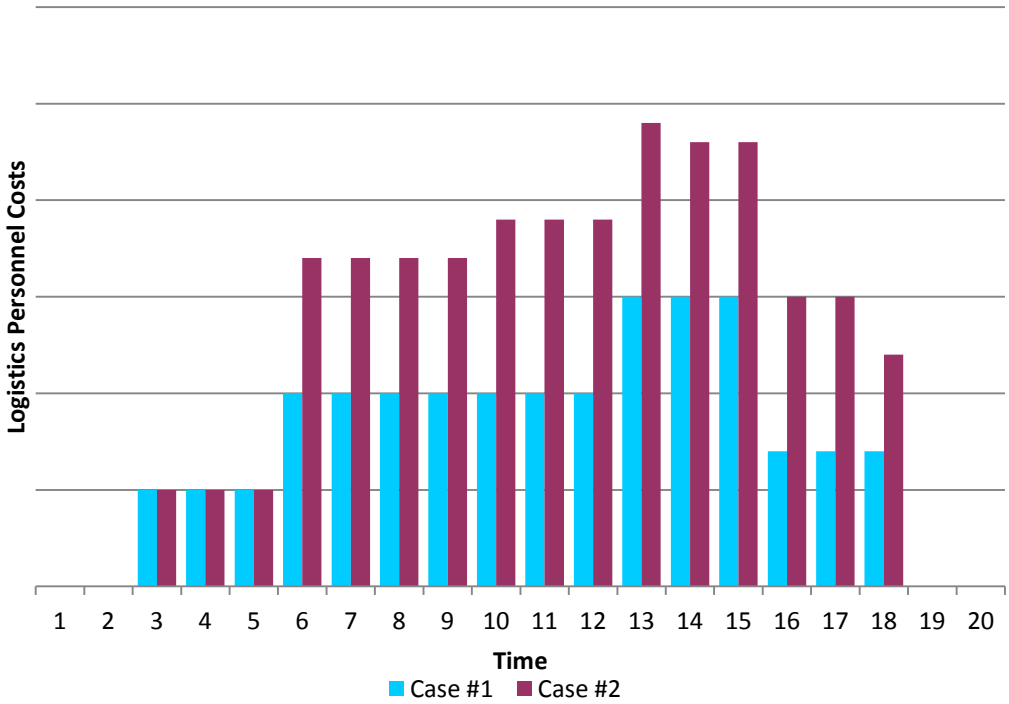

Figure 5. Comparison of logistics personnel cost for mission scenario #1.

2.3. Mission Scenario Uncertainty

When mission scenarios were discussed in section 2.1, the requirements for those scenarios were known with certainty. In reality, there is a large amount of uncertainty around the manpower requirements of a mission scenario. Characteristics like the start date of the mission scenario, its duration, and its personnel requirements will not be known with certainty. Since there is a large amount of uncertainty when planning for future missions, an additional feature is added to the model to address uncertainty.

This feature of the model focuses on uncertainty around mission scenario requirements, including the use rules associated with each mission scenario. The model is capable of handling any type of mission scenario uncertainty. In the planning tool prototype, users can add uncertainty to the durations of phases 3 and 4 for mission scenarios. For each phase, the user can specify the minimum and maximum possible duration. For the final planning tool, uncertainly could be expanded to any parameters requested. When there are several mission scenarios with uncertainty, there are a large number of outcomes that could occur. With uncertainty, there is the

additional concept of likelihood. Each of the possible outcomes may not be equally likely to occur. For example, a particular outcome of a mission scenario could require a lot of resources but it may be very unlikely. Both the outcome and its likelihood are considered by the model and are therefore in the results to be analyzed.

When uncertainty is added to the model, the outputs are no longer a single value. Instead, they can be represented by the minimum, maximum, a percentile, or the average. The figure below illustrates this idea. It shows how outputs are displayed when there is mission scenario uncertainty. Assume that there are several mission scenarios requiring logistics personnel but there is uncertainty around how many personnel will be required and when the mission scenarios will occur. The number of logistics personnel required is not a single value but a range of values. In this case, the minimums and maximums are displayed. Observe that the number of military logistics personnel required has a large range in some time periods.

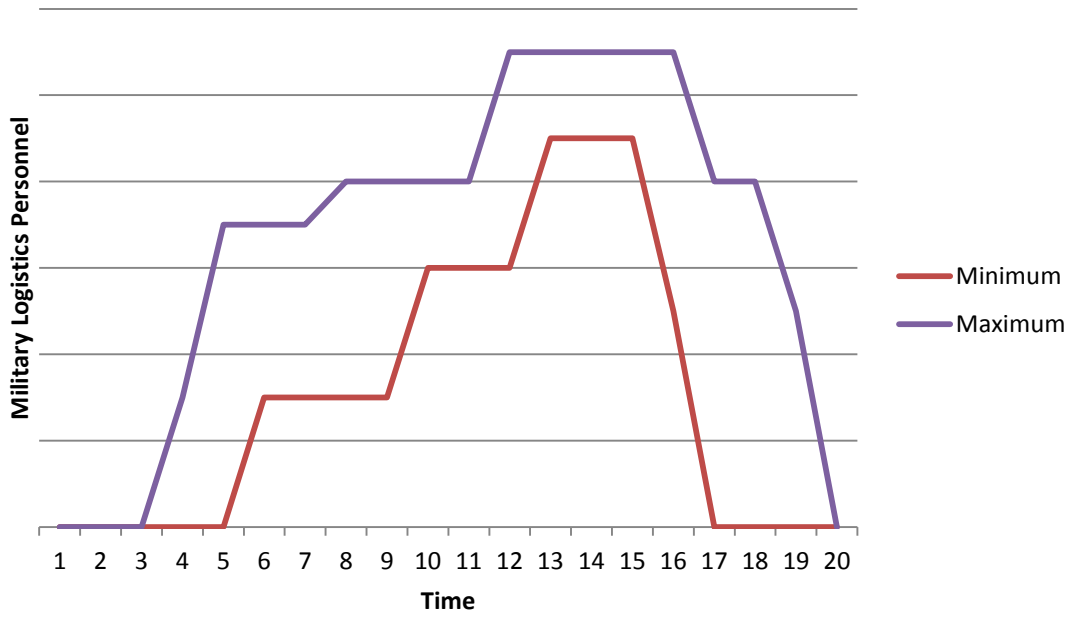

Figure 6. Personnel requirements for military logisticians under uncertainty.

The example above demonstrates how mission scenario uncertainty affects the manpower mix. However, uncertainty also affects feasibility and cost, the other two key model outputs. Since uncertainty affects the manpower mix, it affects manpower expenditures as well. Therefore graphs showing uncertain manpower costs will be very similar to Figure 6. Uncertainty can also answer feasibility questions such as, "What is the likelihood of not having enough resources to complete the mission?" Under mission scenario uncertainty, the outputs allow the analyst to assess levels of risk created given the uncertainty around mission scenario requirements.

2.4. Strategic Hiring Under Uncertainty

The model capabilities described previously determine the optimal manpower mix given (1) a fixed set of personnel resources and their availabilities over time; (2) annual budget limits; and

(3) uncertainty around mission scenario requirements. While the model provides insight into which resource pools should be used over time, it does not suggest how resources pools could be strategically resized to mitigate large personnel costs. With the previous constraints, the model behaves in a "reactive" manner. If contractors are needed to complete mission requirements, they are hired. Strategic hiring makes the model more "proactive". This means it recommends solutions that can be implemented to mitigate future costs.

Strategic hiring is a concept within the model that allows specific resources to be hired in anticipation of future demand. Strategic hiring is different than the just-in-time hiring that would be done to fulfill the demands of a specific mission scenario. It is used to *develop* resource pools because there is a sufficient likelihood that that resource will be required in the future. The mission scenarios may not occur which produce the demand that requires the full use of this resource. However, had this resource not been maintained and large demands for it had occurred, the result could be large personnel costs or mission failure. It also allows resource pools to be decreased over time when there is an insufficient level of anticipated demand.

Two examples are provided below that illustrate the concept of strategic hiring. In both cases, it is assumed that there are two types of contractors available: "long term" contractors and "short term" contractors. The "long term" contractor option requires that a modest initial investment be made to hire some number of contractors. Once hired, there is a lead time of three months before the personnel can be used. Once personnel are hired, modest maintenance costs must be paid to maintain their readiness whether or not they are used to support mission requirements. When no longer needed, the personnel can be released, and maintenance costs no longer have to be paid. The model includes the option to apply costs for terminating contractors, but they are ignored for this example. The "short term" contractors are not a resource pool that needs to be maintained. Instead they can be hired and released as needed with no lead time. The tradeoff for this flexibility is that they are very expensive to use when needed.

2.4.1. Strategic Hiring Without Uncertainty

Strategic hiring can be applied to cases with or without uncertainty. For simplicity, the first example considered does not include uncertainty. Assume that there are two mission scenarios which require 2,000 logistics personnel. Also, assume that the manpower use rules only allow the two contractor groups described above to be used. Each "long term" contractor costs $50,000 to hire and $2,000 dollars per month to maintain. Each "short term" contractor costs $9,500 for each month they are used. Finally, assume that there are two cases an analyst wants to consider; in the first case there is a 1 month gap between mission scenarios, and in the second case there is a 10 month gap between mission scenarios. The logistics manpower requirements for both cases are illustrated in Figure 7.

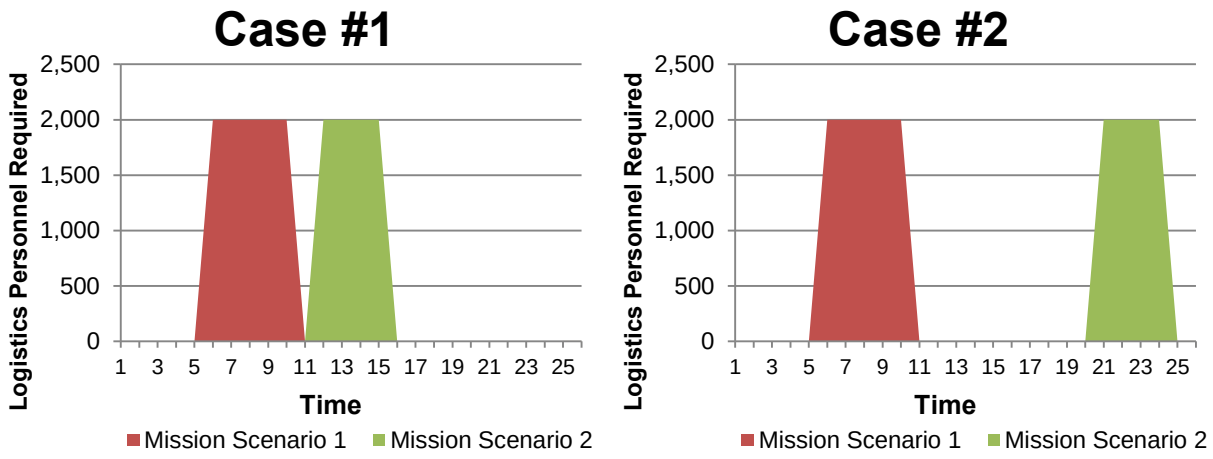

Figure 7. Comparison logistics personnel requirements.

When strategic hiring is applied to the cases above, the model will decide the best combination of the two contractor options. With strategic hiring, the objective is still to minimize the total costs, only now the costs of hiring and holding contractors must be considered. In the first case, the model would opt to hire 2,000 of the "long term" contractors and use them for both mission scenarios. Since there is only a one month gap between the missions, it is cheaper when the cumulative cost is considered to invest in the capability and maintain it throughout both missions. Since there is a three month lead time for these contractors, they would need to be hired in time period 3 to be ready for use in time period 6. Once both missions are completed, these contractors could be released. This means that each "long term" contractor will be hired and retained for thirteen months, producing a total cost of $76,000 per contractor. If "short term" contractors were used instead, there would be nine total months of demand for each contractor for a total of $85,500 per contractor. Using the "long term" contractors produces a total savings of $15 million over all 2,000 contractors.

In the second case, however, the model would choose to use the "short term" contractors. Since there is now a ten month gap between the mission scenarios it would be more expensive to maintain the "long term" contractor resource pool. Since "short term" contractors are used as needed, there is still only a total of nine months of demand for each contractor. Consequently, the cost is still $85,500 per contractor. Since the gap is now nine months longer, "long term" contractors would have to be maintained for twenty-two months, increasing their overall cost to $94,000 per contractor. In this case, using "long term" contractors would increase the costs by $17 million. Hiring "long term" contractors twice is not an option since the hiring costs alone would be $100,000 per contractor. Since there is not enough sustained demand for "long term" contractors, the "short term" contractors are the best option.

2.4.2. Strategic Hiring With Uncertainty

The second example illustrates the concept of strategic hiring under uncertainty. Assume that there is a single mission scenario that will last for one year. Again, assume that there is some subset of workload that can only be accomplished by the two contractors described above and that their costs are as described in the first example. There are two possible outcomes for the

mission scenario. There is a 90% chance that a neighboring country will allow the use of preexisting facilities for the logistics mission and a 10% chance they will not. If the first case occurs, only 2,000 logistics personnel will be required; in the second case, 4,000 logistics personnel will be required.

When a "long term" contractor is used in this example, they will be hired, wait the three month lead time and then be used for twelve months. This produces a total cost of $80,000 per contractor. In the case of a "short term" contractor, they will be used for twelve months for a total cost of $114,000. In this example, the model chooses to only hire 2,000 of the "long term" contractors and to use "short term" contractors to make up the additional demand for 2,000 personnel when it occurs. In both cases, at least 2,000 personnel will be required. Therefore selecting "long term" contractors is the cheapest option. However, since there is only a 10% chance of needing the additional 2,000 contractors, it is cheaper, on average, to use "short term" contractors to make up the difference in workload requirements. 2,000 additional "long term" contractors could be hired at a total cost of $160 million, but there is a 90% chance they would not be used. If "short term" contractors are used to make up the difference, the cost is $228 million. While this represents an increase of $68 million, it only has a 10% change of occurring. With strategic hiring under uncertainty, the model selects the manpower mix and hiring decisions that provide the lowest average cost.

3. MODEL NOTATION

As discussed in Section 1, an optimization model is a collection of mathematical equations. One equation is the objective and the remaining equations are the constraints. All of these equations contain a number of variables, each representing a decision to be made, whose optimal values are sought. This section provides a summary of the notation used in section 4, where all of the parameters and terms described below are explained.

The notation for the model formulation includes the following limits, indices, input data, decision variables, and "helper" variables. The model must determine the values for the decision variables that yield the optimal value for the objective function. The term "helper" variable is used to variables whose values can be determined from the decision variables. These variables are used for convenience purposes to simplify the notation.

3.1. Limits

I: number of workload elements
K: number of capabilities
P: number of personnel groups
R: number of mission scenario realizations
S: number of mission scenarios
T: number of time periods in the planning horizon
W: number of mission scenario packages
T: number of years in the planning horizon

3.2. Indices

$i = 1, \ldots, I$: index on workload elements
$k = 1, \ldots, K$: index on capabilities
$p = 1, \ldots, P$: index on personnel groups
$r = 1, \ldots, R$: index on mission scenario realizations
$s = 1, \ldots, S$: index on mission scenarios
$t = 1, \ldots, T$: index on time periods
$w = 1, \ldots, W$: index on mission scenario packages
$y = 1, \ldots, Y$: index on years

3.3. Input Data

a_{iskt}: manpower requirement of capability k for workload element i of mission scenario s in time period t
a^{r}_{iskt}: manpower requirement of capability k for realization r of workload element i of mission scenario s in time period t
B_y: annual budget available in year y
c_{spkt}: cost of using a resource from personnel group p with capability k for mission scenario s in time period t

h_{pkt}: manpower availability of personnel group *p* with capability *k* in time period *t*

H_{pkt}: per unit holding costs that must be paid for each unit of h_{pkt}

L_{pk}: number of time periods that must pass before a hired individual from personnel group *p* with capability *k* can be assigned to a mission scenario (lead time)

$m(w)$: set of workload element and mission scenario pairs <*i, s*> that belong to package *w*

$o(y)$: set of time periods *t* that are in year *y*

$P(i,s)$: set of personnel groups *p* that can be used for workload element *i* of mission scenario *s*

$P(i,s,r)$: set of personnel groups *p* that can be used for workload element *i* of mission scenario *s* in realization *r*

Q^s: importance factor of mission scenario *s* (larger values are more important)

q^w: probability of package *w*

W_{pkt}: cost of hiring an individual with capability *k* into personnel group *p* in time period *t*

$\bar{W}_{pkt}$: cost of removing an individual with capability *k* from personnel group *p* in time period *t*

γ: relative importance of accomplishing workload compared to minimizing personnel costs (typically γ>>1)

δ^r_{ws}: indicator parameter that takes on a value of 1 to indicate that realization *r* for mission scenario *s* is included in package *w*

ϵ_{spk}: efficiency factor when using a resource from personnel group *p* with capability *k* for mission scenario *s*

μ: number of units of resource availability added or removed to/from a resource pool for each individual that is hired or terminated

3.4. Decision Variables

g_{ispkt}: continuous variable representing the number of units that personnel group *p* with capability *k* is short for fulfilling the requirements of workload element *i* of mission scenario *s* in time period *t*

g^r_{wispkt}: continuous variable representing the number of units that personnel group *p* with capability *k* is short for fulfilling the requirements of workload element *i* of mission scenario *s* in time period *t* in realization *r*

I_{pkt}: continuous variable representing the quantity of hires into personnel group *p* with capability *k* in time period *t*

$\bar{I}_{pkt}$: continuous variable representing the quantity of terminations of personnel group *p* with capability *k* in time period *t*

x_{ispkt}: continuous variable representing the number of units of personnel group *p* with capability *k* that are assigned to workload element *i* of mission scenario *s* in time period *t*

x^r_{wispkt}: continuous variable representing the number of units of personnel group *p* with capability *k* that are assigned to workload element *i* of mission scenario *s* in time period *t* for realization *r*

3.5. "Helper" Variables

r_{pkt}: continuous variable representing the total manpower availability, *including* personnel that have been hired but are not yet available for use, of personnel group *p* with capability *k* in time period *t*

$\bar{r}_{pkt}$: continuous variable representing the total manpower availability, *excluding* personnel that have been hired but are not yet available for use, of personnel group *p* with capability *k* in time period *t*

4. MATHEMATICAL FORMULATION

The following sections describe the mathematical formulation. While there is ultimately a single version of the model, the sections below present the model in three stages: the core model, the core model with uncertainty, and the core model with uncertainty and strategic hiring. This approach is used since each version of the model builds on the previous version. This also mirrors the three modeling options that are available in the interface of the prototype.

4.1. Core Model

This section describes the core features of the mathematical formulation. In its most basic form, the model is formulated as a linear program. In the subsequent sections, the core formulation is expanded to include uncertainty and strategic hiring. However, the core assumptions and behavior described in this section will remain the same. At its core, the model behaves as follows. As input, the user provides manpower requirements for each mission scenario of interest by capability and time. The model then assigns personnel from various resource pools to fulfill this demand, subject to resource limitations, manpower use rules, and budget limitations. The objective of the model is to fulfill all of this demand at the lowest possible cost.

4.1.1. Workload Elements

The key model assumption is that there are one or more mission scenarios, each of which has well-defined resource requirements. The resource requirements from each mission scenario define the workload that needs to be accomplished during the timeframe of interest. Let a_{iskt} represent the number of units of capability *k* required in time period *t* to accomplish workload element *i* in mission scenario *s*. The term *workload element* is used to refer to a "piece" of the mission scenario. The concept of a workload element is a convenience for modeling, and it does not necessarily have a unique meaning in the mission scenario. It allows for a large mission scenario to be divided into more specific tasks. In the OCS prototype, a workload element is created for each capability, phase of war, and base for a given mission scenario. If desired, a workload element could be created for every task that must be accomplished during a mission scenario. It is assumed that each workload element *i* requires a specific capability *k*. Notice that a_{iskt} is indexed by time. This means that the manpower requirements for a given scenario and workload element are defined for each time period and can vary over time. This also implies that the set of mission scenarios and the manpower requirements for each are known in advance.

4.1.2. Resource Demand and Availability

The next set of input data and variables focuses on the resources that are available to fulfill the demand. For each capability *k* there are a set of personnel groups *p* that can be used to fulfill demand. The variable x_{ispkt} represents the number of units of personnel group *p* associated with capability *k* assigned to workload element *i* of mission scenario *s* in time period *t*. The number of units required can be defined in many ways (i.e. people or person-hours). In the prototype, the number of personnel (people) required per time period is used.

4.1.3. Resource Restrictions

It is assumed that restrictions exist on the personnel groups that may be used to satisfy that demand. Let the set $P(i,s)$ represent the set of personnel groups that are allowed to perform

workload element *i* of mission scenario *s.* In the OCS prototype, the user specifies which personnel groups are allowed to perform workload by adding policies to each base for each mission scenario.

4.1.4. Resource Efficiency

Next, define ϵ_{spk} as the efficiency of personnel group *p* with capability *k* when assigned to mission scenario *s.* Efficiency represents the rate at which work is performed relative to some standard. One approach for defining efficiency would be to use a specific personnel group as the standard. Assume that a task requires 400 personnel of military personnel for a given capability and that the same task would require 800 personnel if contractors were used. If military personnel were used as the standard for defining efficiency, then contractors would have an efficiency of 0.5. However, if the contractors were used as the baseline, then the military would have an efficiency of 2.0. The baseline personnel group would always have an efficiency of 1.0.

4.1.5. Model Constraints

(1) Workload Fulfillment Constraint

With these assumptions and concepts defined, the first constraint can be introduced.

$$a_{iskt} = \sum_{p \epsilon P(i,s)} \epsilon_{spk} \left[x_{ispkt} + g_{ispkt} \right] \qquad \forall i, s, k, t \qquad (1)$$

Constraint (1) states that the workload required for each mission scenario must be satisfied by one or more of the eligible personnel groups with the appropriate adjustment for efficiency. It is assumed that all workload must be fulfilled in the time period it is requested. The variable g_{ispkt} represents the number of units that personnel group *p* associated with capability *k* is short for fulfilling the requirements of workload element *i* of mission scenario *s* in time period *t.* Shortages could exist if there are budget or personnel limitations on the eligible personnel groups. Shortages are penalized in the objective function to encourage solutions that satisfy all demand.

It is important to notice that this equation is based on a critical assumption. This formulation cannot be used to determine how to optimally delay workload. It is assumed that all workload must be accomplished subject to the manpower use rules. If this is not possible given resource pool availabilities and budget limitations, the model reports what is the cheapest combination of additional resources that must be added to accomplish the workload in all periods as requested. In practice, based on this information, the analysts might alter the workload, but that alteration must be done by subject matter experts and not based on this formulation.

(2) Resource Pools Constraint

The two limiting factors in the model are resource pools and funding. For resource pools, it is assumed that there are a fixed number of units available by personnel group *p* and capability *k* for each time period *t.* This requirement leads to the following constraint.

$$\sum_{i,s} x_{ispkt} \leq h_{pkt} \qquad \forall p, k, t \qquad (2)$$

The input value $h_{pkt}$is the size of the resource pool of personnel group *p* with capability *k* in time *t*. Again, the size of the resource pool can be defined in many ways (i.e. people or person-

hours), but it should be the same as the manpower requirements a_{iskt}. Observe that for each personnel group, capability, and time period, the assignments are summed over all workload elements and mission scenarios. This implies that all mission scenarios draw on a common set of resources that must be shared to accomplish all of the mission requirements. For example, when two missions occur at the same time, they may have to divide the military resources to accomplish the inherently military tasks and use contractors for the additional workload.

(3) Funding Constraint
The second limiting resource is funding. By fiscal year, there are constraints on the funds that may be spent. The annual budget that can be spent is provided as input. Only the costs associated with using personnel are assessed as the funds spent. This leads to the following constraint.

$$\sum_{i,p,k,t\in o(y)} c_{spkt} x_{ispkt} \leq B_y \qquad \forall y \tag{3}$$

The set $o(y)$ is the collection of time periods, *t*, that occur in year *y*. c_{spkt} is the cost of using an individual from personnel group *p* with capability *k* for mission scenario *s* in time period *t*. B_y is the budget available in year *y*.

(4) Non-negativity Constraint
The final constraint requires that the two sets of decision variables be non-negative.

$$x_{ispkt},\ g_{ispkt} \geq 0 \qquad \forall i,s,p,k,t \tag{4}$$

4.1.6. Objective Function
The objective of the optimization is to determine the manpower mix that completes the required workload with minimal costs. The objective function is given in the expression below.

$$\min \sum_{pkit} c_{spkt} x_{ispkt} + \gamma \sum_{pkit} c_{spkt} Q^s g_{ispkt} \tag{5}$$

The objective function is divided into two terms. The first term represents the total manpower usage costs. It is very similar to the budget constraint (3) except that it determines the total costs over all years instead of on a per year basis. The second term is the penalty for not completing all of the required workload. The parameter γ is the relative importance of not completing workload compared to the manpower costs. Since fulfilling mission requirements takes priority over minimizing costs, γ should be large (γ>>1). The term Q^s is the relative importance of mission scenario *s*. It is useful to notice that the shortage penalty includes both the relative scenario importance and personnel group cost. These terms are included to encourage shortages of resources to be pushed onto mission scenarios that are relatively less important and onto scenarios, capabilities, and personnel groups with the lowest manpower use costs. It is useful to notice that the lowest manpower use costs are a combination of the cost of the personnel group and its efficiency at completing the workload element due to the influence of equation (1).

4.2. Mission Scenario Uncertainty

In the core model formulation it is assumed that the resource requirements for each mission scenario are known with certainty. In reality, there is uncertainty about the actual manpower

requirements that will be generated from mission scenarios. For example, the duration of a mission scenario or its overall manpower footprint may be unknown. This section describes an addition to the core model that allows uncertainty to be directly handled in the model.

When uncertainty is added to the model, it becomes a stochastic linear program. The key idea with the stochastic version of the model is that there are now a collection of possible *realizations* for each mission scenario. Realizations represent one possible outcome for the mission scenario and are indexed by the parameter *r*. The term a_{iskt}, which represented the manpower requirements in the core model, is updated to indicate the realization (a^r_{iskt}) associated with those particular requirements. This means that each realization of a given mission scenario can have its own manpower requirements.

An analysis can include multiple mission scenarios and each mission scenario now has a collection of realizations associated with it. Since mission scenarios compete for resources, it is important to consider the variety of resource demands that can be generated from different combinations of mission scenario realizations. For example, suppose that there are two mission scenarios: A and B. Assume that there are two realizations from the first mission scenario (A_1 and A_2) and three from the second mission scenario (B_1, B_2, and B_3). Also assume that the occurrence of the realization for mission scenario A is independent of the occurrence of the realization from mission scenario B (this is not a requirement, independence is used in this example for simplicity). Finally, suppose each of the realizations stemming from scenario A are equally likely, and the probabilities for the realizations from scenario B are 20%, 30% and 50%, respectively. This implies that there are six possible "*packages*" (or combinations) of realizations from the mission scenarios that could unfold. Those scenarios and their probabilities of occurrence are given in Table 1 below.

Table 1. Example "Packages" of Realizations

Package Number	Realization from Scenario A	Realization from Scenario B	Probability of Realization from A	Probability of Realization from B	Probability of "Package" of Realizations
1	A1	B1	50%	20%	10%
2	A1	B2	50%	30%	15%
3	A1	B3	50%	50%	25%
4	A2	B1	50%	20%	10%
5	A2	B2	50%	30%	15%
6	A2	B3	50%	50%	25%

Having defined the concepts of realizations and packages, the core model formulation can be expanded to include uncertainty. The index *w* is used to identify each package.

4.2.1. Model Constraints with Uncertainty

(1') Workload Fulfillment Constraint

The first constraint is modified as follows:

$$a_{iskt}^{r}\delta_{ws}^{r} = \sum_{p\epsilon P(i,s,r)} \epsilon_{spk}\delta_{ws}^{r}\left[x_{wispkt}^{r} + g_{wispkt}^{r}\right] \qquad \forall w, r, i, s, k, t \qquad (1')$$

The variables x_{wispkt}^{r} and g_{wispkt}^{r} now represent the number of units of resource capability *k* supplied (short) by personnel group *p* in period *t* for workload element *i* in mission scenario *s* under realization *r* and package *w*. The term δ_{ws}^{r} is an indicator parameter that takes on a value of one to indicate that realization *r* for mission scenario *s* is included in package *w*. When a realization for a given scenario is not part of a package (δ_{ws}^{r} equals zero), the entire constraint is removed. It is also important to notice that the definition of the set *P* has been modified in the expansion of this formulation to include uncertainty. Now the set P gives the set of personnel groups that can be used to accomplish workload element *i* under realization *r* for scenario *s*. This constraint requires that for each package the work load requirements are fulfilled or the shortages are identified.

(2') Resource Pools Constraint and (3') Funding Constraint

The modified versions of constraints representing resource limits, (2) and (3), are given below. In both cases the uncertainty around the mission scenarios does not affect the availability of resources. Therefore the resource pool size and budgets are the same for each for each package. The main modification to these constraints is that the resource limits must be honored for each package. The set $m(w)$ specifies which pairs of realizations and packages are part of package *w*.

$$\sum_{(r,s)\in m(w)} \sum_{i} x_{wispkt}^{r} \leq h_{pkt} \qquad \forall w, p, k, t \qquad (2')$$

$$\sum_{i,p,k,t\in o(y),(r,s)\in m(w)} c_{spkt}x_{wispkt}^{r} \leq B_{y} \qquad \forall y, w \qquad (3')$$

(4') Non-negativity Constraint

The non-negativity constraints remain essentially the same, but now include the realization and package indexes.

$$x_{wispkt}^{r}, g_{wispkt}^{r} \geq 0 \qquad \forall i, s, p, k, t, r \qquad (4')$$

4.2.2. Objective Function with Uncertainty

Finally, the objective is modified as given below. The expression inside of the brackets is fundamentally the same as (5). It represents the objective function that would be applied if the core model were used to solve a single package. The term q^{w} is the probability of package *w*. The aim of the modified objective is to minimize the expected resource shortages and manpower use costs.

$$\min \sum_{w,(r,s)\in m(w)} q^{w}\left[\sum_{pkit} c_{spkt}x_{wispkt}^{r} + \gamma \sum_{pkit} c_{spkt}Q^{s}g_{wispkt}^{r}\right] \qquad (5')$$

It is important to observe that the structure for addressing uncertainty is very generic and only requires a collection of packages with associated probabilities. In the example above, the realizations from each mission scenario were treated as independent. However, this structure could support correlation between mission scenario realizations.

It is also important to note that this version of the model is separable by package. When the problem is divided by package, the core model formulation can be used. So while this version of the model provides an organized framework for dealing with uncertainty, the same results could be achieved by solving the core model for each package and combining the results with the appropriate weighting. Both approaches allow probability functions to be built for the outputs of the model. The advantage of the modified version is that it provides a framework for making decisions under uncertainty through the use of variables and constraints that link each package. This is demonstrated in the next section.

4.3. Strategic Hiring Under Uncertainty

The final extension to the core model is strategic hiring. Strategic hiring allows for the hiring or termination of specific resources in anticipation of future demand. In constraints (2) and (2') the size of the resource pools (h_{pkt}) is given as input data. Strategic hiring allows the model to increase or decrease the size of the resource pools over time. When strategic hiring is combined with the stochastic version of the model, the result is a two-stage stochastic optimization model where the strategic hiring decision occurs in the upper level of the problem. In this context, strategic hiring can be thought of as a mitigation strategy that recommends resource pool sizes that can best respond to all possible combinations of mission scenario realizations.

Several assumptions are made with respect to strategic hiring. The first set of assumptions is concerned with how the resource pools can be changed. It is assumed that the baseline resource pools, represented by h_{pkt}, can be increased or decreased in size. Two new variables I_{pkt} and $\bar{I}_{pkt}$ are introduced to represent the number of personnel added to and removed from personnel group p for capability k in time period t, respectively. In reality, people are hired or terminated in discrete units. However, for the purposes of strategic planning, the resource pools will be sufficiently large that these variables can be treated as continuous, thereby easing the computational requirements. The concept of a lead time is also introduced with strategic hiring. This is motivated by the fact that some time may be required to pass between when a person is hired and when they are first available to be used. For example, specialized capabilities may require time consuming training.

4.3.1. Model Constraints with Strategic Hiring Under Uncertainty

(2a'') Current Resource Pool Size

With these assumptions a set of constraints that replace (2) and (2') can be introduced. The constraint below illustrates how strategic hiring is addressed in the model.

$$\bar{r}_{pkt} = h_{pkt} + \mu\left[\sum_{t*=1}^{t-L_{pk}} I_{pkt*} - \sum_{t*=1}^{t} \bar{I}_{pkt*}\right] \qquad \forall p, k, t \qquad \text{(2a'')}$$

The term $\bar{r}_{pkt}$ represents the size of personnel group p for capability s in time period t that can be used to fulfill mission scenario requirements. μ is the number of units of manpower availability that are added to or removed from a resource pool for each individual that is hired or terminated. L_{pk} is the lead time for personnel group p for capability k. This constraint states the current resource pool size is equal to the original resource pool size, plus any hires outside of the lead time, minus any terminations up to and including the current time period.

(2b'') Total Resource Pool Size That Can Be Employed
The constraint above represents the current size of a resource pool that can be assigned to a mission scenario. Since it includes a lead time, there may be personnel who have been hired but cannot be used. The constraint below represents the total resource pool size including personnel who are within the hiring lead time.

$$r_{pkt} = h_{pkt} + \mu \sum_{t*=1}^{t} I_{pkt*} - \bar{I}_{pkt*} \qquad \forall w, p, k, t \qquad \text{(2b'')}$$

The only difference between the two constraints above is that the (2b") does not include the lead time. Notice that total resource pool size r_{pkt} will always be greater than or equal to the available resource pool size $\bar{r}_{pkt}$.

(2c'') Available Resource Pool Size Based on Hiring and Termination
Since the resource pool sizes can be changed, the right hand side of constraints (2) and (2') must be updated to reflect the available resource pool size based on hiring and termination.

$$\sum_{(r,s)\in m(w)} \sum_{p,i} x^{r}_{wispkt} \leq \bar{r}_{pkt} \qquad \forall w, p, k, t \qquad \text{(2c'')}$$

Notice that since the left hand side of the constraint above cannot be negative. This implies that both r_{pkt} and $\bar{r}_{pkt}$ cannot be negative either.

(3'') Funding Constraint
The assumptions and constraints above specify how the resource pools can be changed. The next set of assumptions addresses the costs associated with hiring and termination. In the core and mission scenario uncertainty versions of the model, only costs associated with using personnel for mission scenario tasks was counted against the budget. Since the size of resource pools could not change, they were treated as a sunk cost. Since this assumption no longer holds with strategic hiring, three new costs are introduced: hiring, termination and holding. The first two costs, hiring and termination, are costs that are paid each time a person is hired or terminated. Holding costs are the costs that must be paid to maintain the current resource pool size. These costs are separate from the costs associated with using a resource and must be paid whether or not that resource is used for a mission scenario. The constraint below shows how these costs are incorporated into the budget constraint.

$$\sum_{i,p,k,t\in o(y),(r,s)\in m(w)} c_{spkt} x^{r}_{wispkt} + \sum_{p,k,t\in o(y)} \left[W_{pkt} I_{pkt} + \bar{W}_{pkt} \bar{I}_{pkt} + H_{pkt} r_{pkt} \right] \leq B_y \qquad \forall y, w \qquad \text{(3'')}$$

The terms W_{pkt} and $\bar{W}_{pkt}$ represent the costs associated with hiring and terminating an individual from personnel group *p* with capability *k* in time period *t.* The term H_{pkt} represents the per period cost to maintain personnel group *p* for capability *k* in time period *t.* Observe that holding costs are applied to the total resource pool, which includes personnel that have been hired but are still within the lead time. This constraint states that the sum of per-use costs and resource pool maintenance costs cannot exceed the available budget.

(4'') Non-negativity Constraint
With the introduction of the hiring and termination variables, the non-negativity constraints are updated as follows:

$$x^r_{wispkt}, g^r_{wispkt} \geq 0 \qquad \forall i, s, p, k, t, r$$
$$I_{pkt}, \bar{I}_{pkt} \geq 0 \qquad \forall p, k, t \tag{4''}$$

(5'') Objective Function
Finally, the total costs associated with resource pool maintenance are included in the objective function.

$$\min \sum_{w,(r,s)\in m(w)} q^w \left[\sum_{pkit} c_{spkt} x^r_{wispkt} + \gamma \sum_{pkit} c_{spkt} Q^s g^r_{wispkt} \right] + \sum_{pkt} \left[W_{pkt} I_{pkt} + \bar{W}_{pkt} \bar{I}_{pkt} + H_{pkt} r_{pkt} \right] \tag{5''}$$

Observe that in constraints (3'') and (5'') the resource pool maintenance costs do not vary by package. Recall that strategic hiring is used as a mitigation strategy. Given a set of possible outcomes, the model will recommend the correct size for each resource pool. Recall the examples given in section 2.4, where the model could choose to use strategic hiring to maintain a pool of "long term" contractors instead of "short term" contractors if there was going to be sustained demand and the "short term" contractors were more expensive in the long run. Conversely, when there were limited spikes in demand for contractors, the model chose recommend using the "short term" contractors instead of using a portion of the budget to make resource pool investments in "long term" contractors.

4.4. Comments on Model Implementation

The model was implemented in the OPL mathematical programming language. Several additional features were added to assist with analysis and produce more intuitive answers. First, a feature was added that allows the user to indicate whether or not to enforce the budget constraints. When budgets are not enforced, constraint (3) is removed from the optimization. Since the model is concerned with fulfilling military mission requirements, a decision not to complete workload due to budget limitations might not be an option. Instead, the user can disable budget constraints and have the model return the cheapest possible manpower mix. Additionally, parameters are included that allow the user to disable hiring and/or terminations by personnel group and capability. This allows the user to focus hiring and terminations to a specific set of personnel groups and capabilities. In both cases, the same result could be achieved by entering extremely large values for budgets or costs, but providing an explicit "do not use" option for the user simplifies this process.

When strategic hiring is used and there are non-zero holding costs, the model will always elect to hire as late as possible and terminate as soon as possible. Otherwise, unnecessary holding costs would be incurred. However, when holding costs are not used, there is no incentive to hire as late as possible and terminate as soon as possible. While there is technically no issue with this, it may lead to non-intuitive hiring and termination recommendations. In order to prevent this, two exponential terms are added to hiring and termination costs in the objective function. The hiring costs are written as $e^{-\alpha t} W_{pkt} I_{pkt}$ and the termination costs are written as $e^{-\alpha(T-t)} \overline{W}_{pkt} \bar{I}_{pkt}$, where T is the maximum time period index. The parameter α is set sufficiently small so that the costs are not reduced drastically.

Finally, only the third version of the model was implemented. This is because it can be reduced to the other two versions with appropriate input data. The mission scenario uncertainty version occurs when hiring and terminations are disabled for all personnel groups and capabilities. When strategic hiring is disabled and only one package is provided to the model, the behavior is the same as the core formulation.

5. CONCLUSION

The previous sections describe the model that was implemented in Phase 2 of the Contingency Contractor Optimization project. The goal of the model is to assign resources to mission scenarios so that all requested workload is completed at the lowest possible cost while honoring all of the use rules. As input, the model receives mission scenarios which create a demand for personnel, resource pools which can be used to fulfill this demand, and rules governing how these resources can be used. As output, the model identifies whether or not the workload can be accomplished within the specified limits, the recommended Total Force Mix, and the cost of that mixture. The uncertainty feature of the model allows these outputs to be assessed in a probabilistic sense, to better reflect reality. The strategic hiring feature provides recommendations on the correct size of resource pools to help minimize the overall personnel costs through better long term planning.

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