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SAR Impulse Response with Residual Chirps

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SAR Impulse Response with Residual Chirps

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Abstract

A residual chirp waveform is a quadratic phase error that broadens the Impulse Response (IPR) and diminishes its peak value in a predictable manner. This report qualitatively and quantitatively analyzes the effects of a residual chirp on the IPR.

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Foreword

It is well known that quadratic phase errors defocus or blur energy in a SAR image. A question has arisen “What is the impact of interfering signals with only partial or residual Linear FM chirp characteristics in a SAR image?” This necessitated an analysis to quantify these effects.

1 Introduction

A Linear Frequency-Modulated (LFM) chirp is a function with unit amplitude and quadratic phase characteristic. In a focused Synthetic Aperture Radar (SAR) image, a residual chirp is undesired for targets of interest, as it coarsens the manifested resolution. However, for undesired spurious signals, a residual chirp is often advantageous because it spreads the energy and thereby diminishes its peak value.

In either case, a good understanding of the effects of a residual LFM chirp on a SAR Impulse Response (IPR) is required to facilitate system analysis and design. This report presents an analysis of the effects of a residual chirp on the IPR.

As reference, there is a rich body of publications on various aspects of LFM chirps. A quick search reveals a plethora of articles, going back to the early 1950s. We mention here purely as trivia one of the earlier analysis papers on this waveform by Klauder, et al.[1]

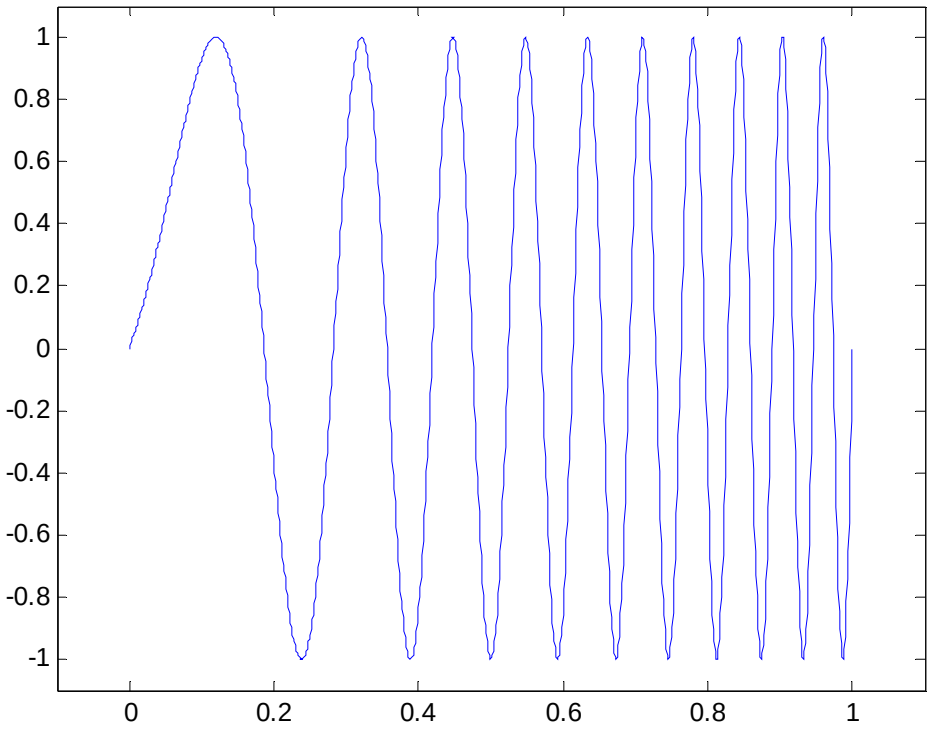


Figure 1. A Linear FM chirp waveform. Frequency changes linearly with time.

2 The Impulse Response of a Residual Chirp

Consider a time-dependent signal with a quadratic phase function described by

$$x(t) = \mathrm{rect}\left(\frac{t}{T}\right)\exp\left\{ j\left(\phi_{res} + \omega_{res}t + \frac{\gamma_{res}}{2}t^2\right)\right\} \tag{1}$$

where

ϕ_{res} = residual phase,
ω_{res} = residual frequency,
γ_{res} = residual chirp rate,
T = pulse width,

and

$$\mathrm{rect}(z) = \begin{cases} 1 & |z| \le 1/2 \\ 0 & else \end{cases}. \tag{2}$$

This report is concerned with the effects of the quadratic component of the phase function. Without loss of generality, for this report we will assume

$$\phi_{res} = 0 \text{, and}$$
$$\omega_{res} = 0 . \tag{3}$$

We readily calculate the residual chirp bandwidth in Hz as

$$B_c = \frac{\gamma_{res}T}{2\pi}. \tag{4}$$

In addition, we can calculate the peak quadratic phase deviation from the center of the pulse as

$$\Phi_q = \frac{\gamma_{res}}{8}T^2 . \tag{5}$$

Combining these two results allows us to relate the quadratic phase deviation to the chirp bandwidth as

$$\Phi_q = \frac{\pi}{4} T B_c . \tag{6}$$

This relationship is stated in terms of the residual chirp bandwidth, that is, the difference between chirp instantaneous start and stop frequencies. With some foresight, a more interesting bandwidth is the actual signal bandwidth, which for large time-bandwidth signals approaches the chirp bandwidth. That is, the signal bandwidth

$$B \approx B_c . \tag{7}$$

When this is accurate, we can calculate

$$\Phi_q \approx \frac{\pi}{4} T B . \tag{8}$$

The quantity TB is the time-bandwidth product. It identifies the broadening of the bandwidth of the signal over the ideal. In general

$$TB \geq 1 . \tag{9}$$

The equivalence $B \approx B_c$ is fairly accurate for large time-bandwidth products, where

$$TB >> 1 , \tag{10}$$

however it offers reduced accuracy as the time-bandwidth approaches one. This, of course, depends on several factors, including just how exactly bandwidth is defined, and whether any amplitude tapering (windowing) is performed prior to transformation to the frequency domain.

Of interest to us in this report is the Fourier Transform of $x(t)$, which will ultimately correspond to the Impulse Response (IPR) of the radar with a residual phase error function corresponding to $x(t)$.

2.1 Uniform Taper

Consider our signal model to be

$$x(t) = \mathrm{rect}\left(\frac{t}{T}\right)\exp\left\{ j\frac{\gamma_{res}}{2}t^2 \right\} \tag{11}$$

where

$$\gamma_{res} = \frac{8\Phi_q}{T^2} \tag{12}$$

and we calculate

$$\Phi_q \approx \frac{\pi}{4}TB. \tag{13}$$

We now examine several time-bandwidth products as examples. Figure 2 through Figure 5 plot the Fourier Transform of $x(t)$, exemplifying TB = 2000, 200, 20, and 2 respectively. Note how for the large TB the band edge in the spectrum of $x(t)$ is at approximately $TB/2$ resolution units away from the center of the spectral response, as desired. However as TB decreases, the band edge is less accurately predicted by $TB/2$, but still reasonably approximate for all but the $TB = 2$ case.

We now define the IPR width in resolution units as calculated to be

$$n_\rho \approx TB. \tag{14}$$

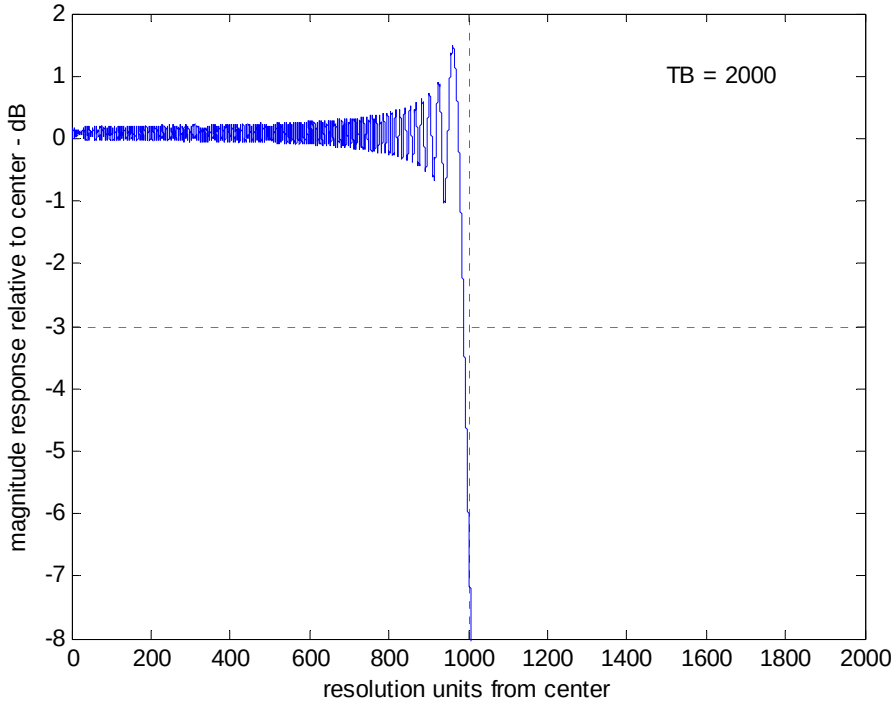


Figure 2. Impulse Response for residual chirp with *TB* = 2000, and uniform weighting.

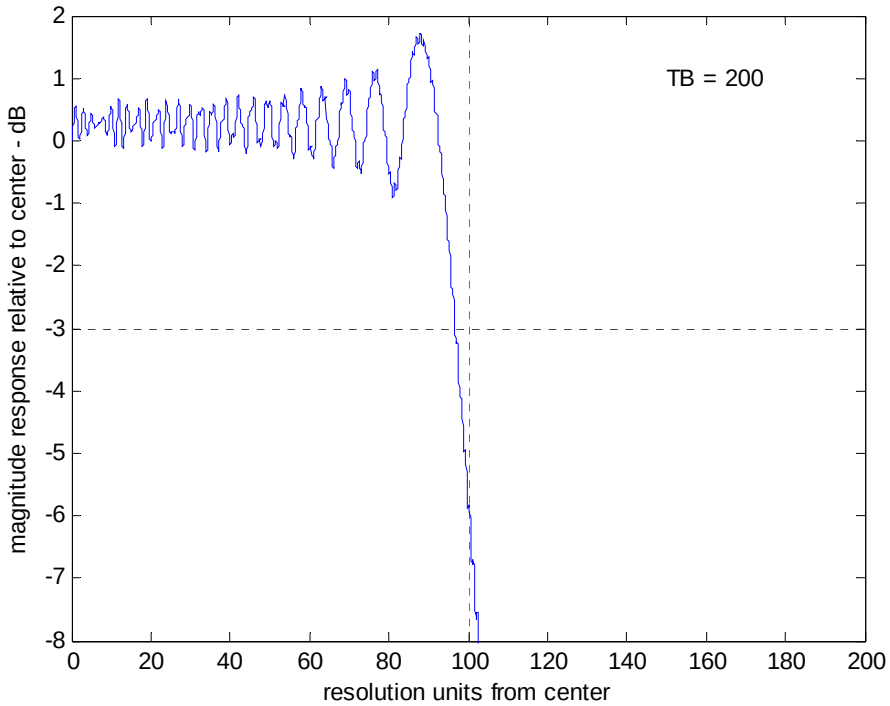


Figure 3. Impulse Response for residual chirp with *TB* = 200, and uniform weighting.

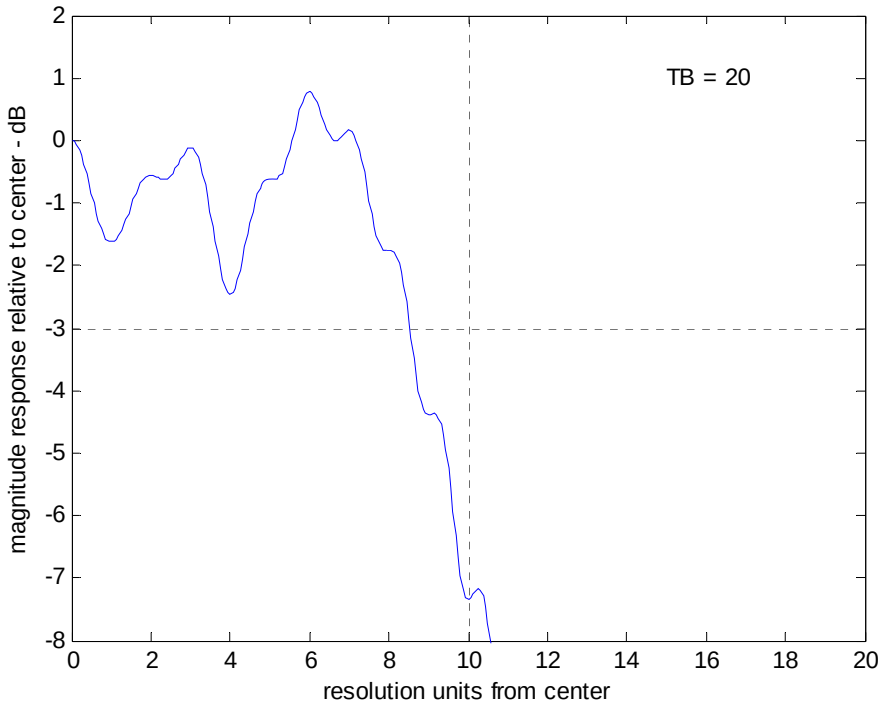


Figure 4. Impulse Response for residual chirp with *TB* = 20, and uniform weighting.

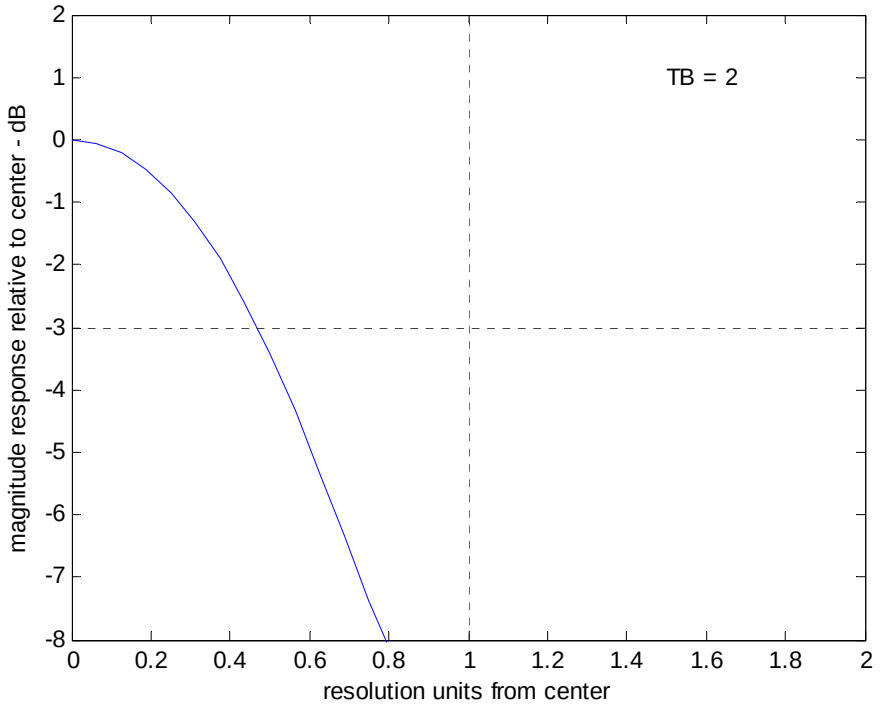


Figure 5. Impulse Response for residual chirp with *TB* = 2, and uniform weighting.

2.2 Taylor Taper

Consider now our signal model to be modified to

$$x(t) = \mathrm{rect}\left(\frac{t}{T}\right)\mathrm{w}\left(\frac{t}{T}\right)\exp\left\{ j\frac{\gamma_{res}}{2}t^2 \right\} \tag{15}$$

where

$$\gamma_{res} = \frac{8\Phi_q}{T^2} \tag{16}$$

but now we employ a taper, or weighting, to the data identified by

$$\mathrm{w}(z) = \begin{cases} \textit{weighting function} & |z| \le 1/2 \\ 0 & \textit{else} \end{cases}. \tag{17}$$

While any of a number of weighing functions may be employed, we shall presume that $\mathrm{w}(z)$ is a Taylor weighting with –35 dB sidelobes, and $\bar{n} = 4$. This weighting is illustrated in Figure 6. Its Impulse Response (IPR), calculated as its Fourier Transform, is illustrated in Figure 7.

Recall that the signal is changing in frequency linearly with time, so that the window function is applied effectively over frequency. That is, the taper of Figure 6 represents a passband transfer function. This implies that the taper effectively reduces the effective bandwidth of the signal. Let the bandwidth now be defined as the noise bandwidth of the window taper. That is

$$B \approx B_c b, \tag{18}$$

where the relative bandwidth factor is given by

$$b = \frac{\int_{-1/2}^{1/2} w^2(x)dz}{w^2(0)}. \tag{19}$$

For the –35 dB Taylor window ($\bar{n} = 4$), we identify

$$b \approx 0.4472. \tag{20}$$

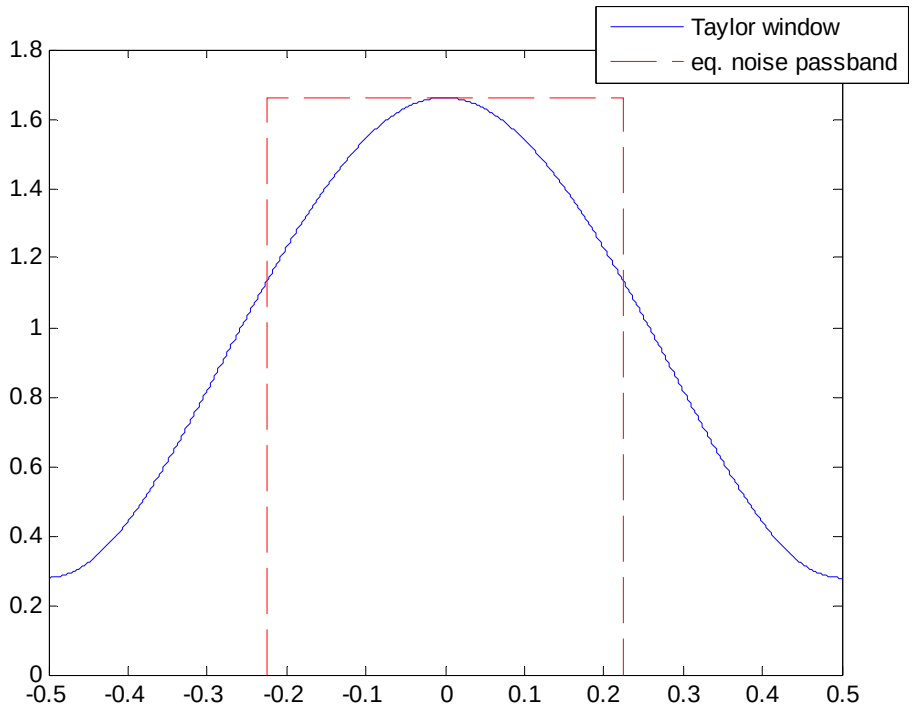
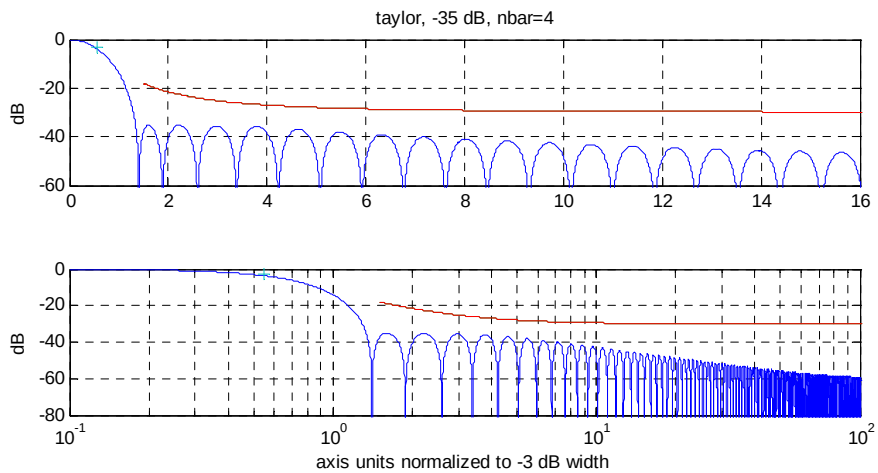


Figure 6. Taylor weighting with –35 dB sidelobes, and $\bar{n} = 4$.



window IPR -3 dB bin bandwidth = 1.1822 (corresponds to sinc width of 0.8845)
window IPR -18 dB rel bandwidth = 2.2087 (normalized to -3 dB width)
window IPR first null = 1.4076 (normalized)
window SNR gain = -0.91408 dB
window PSL = -35.1665 dBc
window ISL = -27.1331 dBc from 1.4076 outward

Figure 7. Characteristics of –35 dB Taylor window. The red curve is a common peak sidelobe specification.

This relative noise passband for the Taylor window is also illustrated in Figure 6. Note that for a uniform taper, nominally $b = 1.0$.

In any case, we calculate the required chirp bandwidth to achieve this noise bandwidth as

$$B_c \approx \frac{B}{b}. \tag{21}$$

Combining this with earlier results yields the required quadratic phase deviation as

$$\Phi_q \approx \frac{\pi}{4}\left(\frac{TB}{b}\right). \tag{22}$$

Putting this all together yields the set of equations for generating $x(t)$ as

$$x(t) = \mathrm{rect}\left(\frac{t}{T}\right)\mathrm{w}\left(\frac{t}{T}\right)\exp\left\{ j\frac{\gamma_{res}}{2}t^2 \right\} \tag{23}$$

where

$$\gamma_{res} = \frac{8\Phi_q}{T^2}, \tag{24}$$

and

$$\Phi_q \approx \frac{\pi}{4}\left(\frac{TB}{b}\right). \tag{25}$$

We now examine several time-bandwidth products as examples. Figure 8 through Figure 11 plot the Fourier Transform of $x(t)$, exemplifying TB = 2000, 200, 20, and 2 respectively. Note how the band edge in the spectrum of $x(t)$ is at approximately $TB/2$, as desired, for all values of TB.

The IPR width in resolution units is then calculated to be

$$n_\rho \approx TB = TbB_c. \tag{26}$$

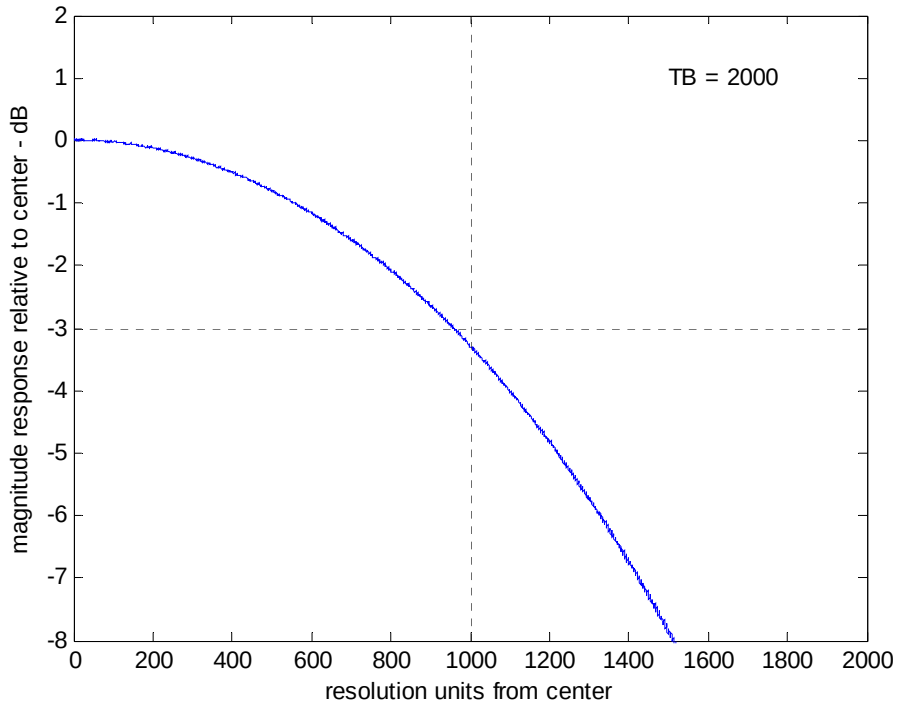


Figure 8. Impulse Response for residual chirp with *TB* = 2000, and Taylor weighting.

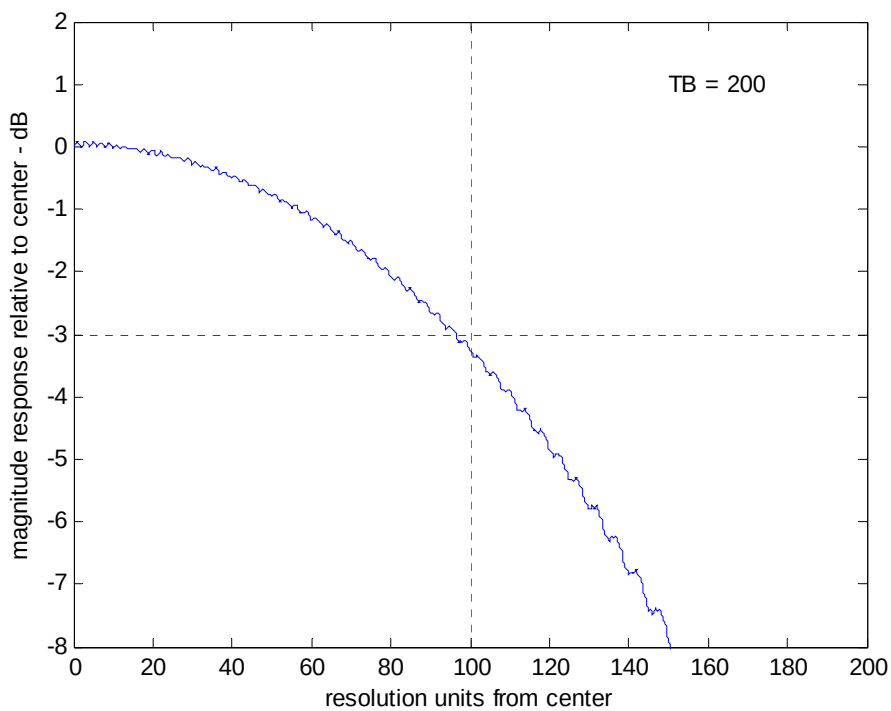


Figure 9. Impulse Response for residual chirp with *TB* = 200, and Taylor weighting.

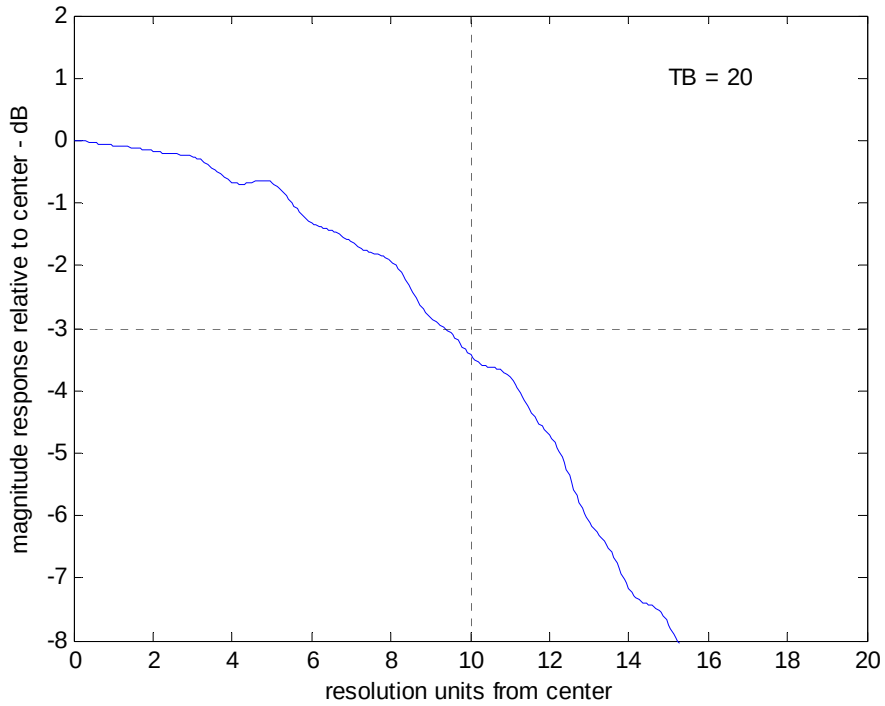


Figure 10. Impulse Response for residual chirp with *TB* = 20, and Taylor weighting.

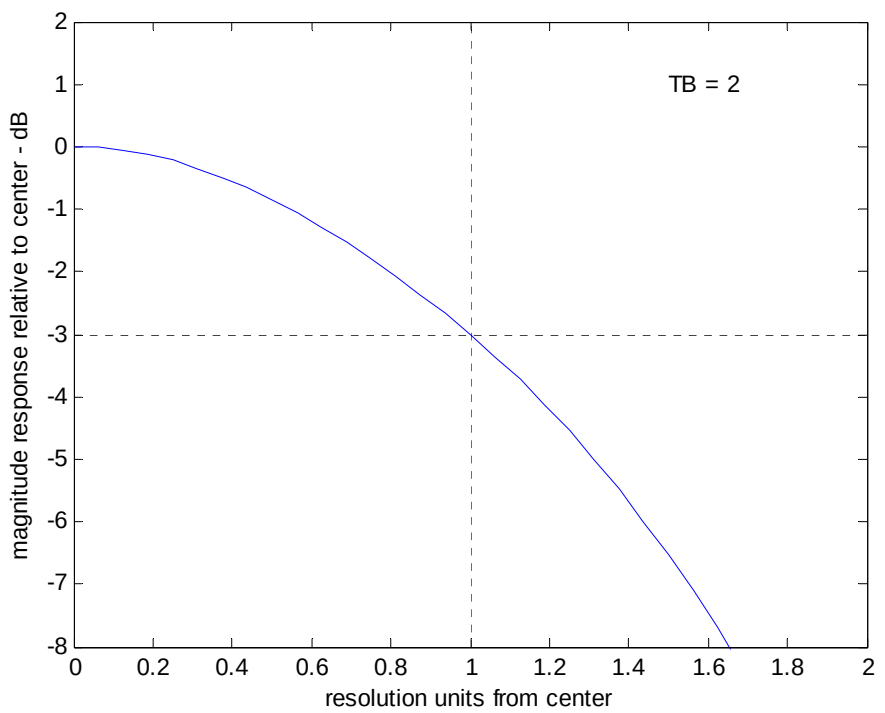


Figure 11. Impulse Response for residual chirp with *TB* = 2, and Taylor weighting.

2.3 Conservation of Energy

The energy of the signal is finite and must be conserved, regardless of the phase error function introduced. Consequently, as the energy of a signal is smeared across multiple resolution cells, the peak value must be diminished accordingly, in fact proportionately. That is

$$IPR_{peak} \propto \frac{1}{n_\rho}. \tag{27}$$

This is illustrated in Figure 12. Each factor of 10 in *TB* yields a 10 dB reduction in peak value of the respective IPR.

The implication is that any signals for which we wish do diminish their peak value, we can do so by operating in a manner to provide them with a residual chirp.

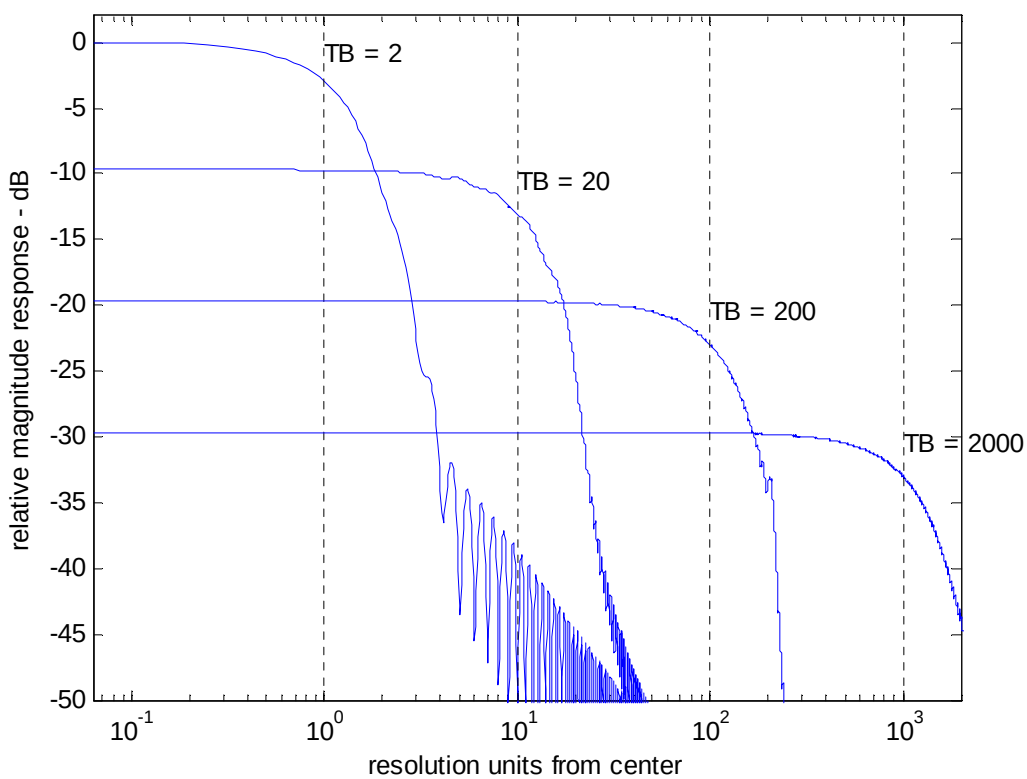


Figure 12. Impulse Response comparison for residual chirps with various *TB*, and Taylor weighting.

"Energy can be transformed (changed from one form to another), but it can neither be created nor destroyed." – First Law of Thermodynamics

3 Procedure for Precisely Smearing Range IPR

We will assume the following.

- We wish to smear the range IPR in a range-Doppler map by a relatively precise amount.
- The video signal has been de-chirped using stretch processing.
- We wish to smear the range response by adding back a slight residual chirp, that is, a quadratic phase function to each pulse echo data. Equivalently, this can be part of the de-chirping process.
- The IPR is ultimately generated using a Taylor weighting with –35 dB sidelobes, and $\overline{n} = 4$.

3.1 Specified Number of Resolution Units

We begin with a specified number of resolution cells to smear across, that is, n_ρ.

Recall that

$$TB = n_\rho, \tag{28}$$

and that we calculate the corresponding peak quadratic phase deviation as

$$\Phi_q \approx \frac{\pi}{4}\left(\frac{TB}{b}\right), \tag{29}$$

where $b \approx 0.4472$ for the chosen Taylor window.

Furthermore, we calculate the necessary chirp rate for the perturbation signal as

$$\gamma_{res} = \frac{8\Phi_q}{T^2}. \tag{30}$$

Combining all these yields

$$\gamma_{res} = \frac{2\pi}{b}\left(\frac{n_\rho}{T^2}\right). \tag{31}$$

3.2 Specified Percentage of Resolution Units

The total number of independent range dimension resolution cells in the SAR image is limited to

$$N_\rho = \frac{TB_{IF}}{a_w} . \tag{32}$$

An image will have somewhat less than this due to the passband filter bandwidth needing to be less than the sampling frequency to limit aliasing. A typical number for usable resolution cells in an image might be 85% of this number.

The ratio of IPR smearing to the maximum number of resolution cells is calculated to be

$$r_\rho = \frac{n_\rho}{N_\rho} = a_w b \frac{B_c}{B_{IF}} . \tag{33}$$

where $b \approx 0.4472$ for the chosen Taylor window. This can be rearranged to

$$B_c = \frac{1}{a_w b} r_\rho B_{IF} . \tag{34}$$

This in turn allows the required chirp rate to be calculates as

$$\gamma_{res} = 2\pi \frac{B_c}{T} = \frac{2\pi}{a_w b} r_\rho \frac{B_{IF}}{T} . \tag{35}$$

Note that the residual chirp will force some target energy outside of the data passband that otherwise would be available. In fact, this will occur for n_ρ resolution cells, split between the near edge and far edge of the image. To avoid an excessive amount of this, r_ρ should be limited to something acceptable, say a few percent or so.

4 Conclusions

The following points are worth repeating.

- A residual chirp waveform is a quadratic phase error.
- The quadratic phase error broadens the Impulse Response in a predictable manner. The equations for this are presented herein.
- The quadratic phase error also reduces the peak value of the Impulse Response.

“The difference between chirping out of turn and a faux pas depends on what kind of a bar you're in” -- Wilson Mizner (Playwright, 1876-1933)

Reference

[1] Klauder, J.R., Price, A.C., Darlington, S. and Albersheim, W.J., 'The theory and design of chup radars'; BSTJ., Vo1.39, 1960, No.4, pp745-809; reprinted in Radars - vo1.3, Pulse Compression, D.K. Barton ed., Artech House, 1975.

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