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Explorations in Combining Cognitive Models of Individuals and Systems Dynamics Models of Groups

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Abstract

This report documents a demonstration model of interacting insurgent leadership, military leadership, government leadership, and societal dynamics under a variety of interventions. The primary focus of the work is the portrayal of a token societal model that responds to leadership activities. The model also includes a linkage between leadership and society that implicitly represents the leadership subordinates as they directly interact with the population. The societal model is meant to demonstrate the efficacy and viability of using System Dynamics (SD) methods to simulate populations and that these can then connect to cognitive models depicting individuals. SD models typically focus on average behavior and thus have limited applicability to describe small groups or individuals. On the other hand, cognitive models readily describe individual behavior but can become cumbersome when used to describe populations. Realistic security situations are invariably a mix of individual and population dynamics. Therefore, the ability to tie SD models to cognitive models provides a critical capability that would be otherwise be unavailable.

Table of Contents

<i>Introduction</i>	7
<i>Model Equations</i>	9
Societal Behavior	9
Economic Development.....	10
Cognitive-SD Interface	11
Cognitive Insurgent Leader.....	12
Cognitive Government Leader.....	14
Cognitive Military Leader.....	14
<i>Demonstrative Examples of Model Implications</i>	15
<i>Conclusion</i>	23
<i>References</i>	25
<i>Appendix 1: Listing on Model Variable and Parameters</i>	27
<i>Appendix 2: Initial and demonstrative model variable/parameter values</i>	29

Table of Figures

Figure 1: Basic Response.....	15
Figure 2: Base Run with Added Insurgency Costs and “Wake” Expectations.....	16
Figure 3: Base response with Intensified Security Intervention.	16
Figure 4: Societal Counter Response to Excessive Coercion.	17
Figure 5: Added Security impacts on the Population.	18
Figure 6: Added Leadership Interactions.....	18
Figure 7: Isolated Economic Development.....	19
Figure 8: Economic Development with Security.....	20
Figure 9: Economic Development with Security and Societal Response.....	21

1. Introduction

This report documents a demonstration model of interacting insurgent leadership, military leadership, government leadership, and societal dynamics under a variety of interventions. The primary focus of the work is the portrayal of a token societal model that responds to leadership activities. The model also includes a linkage between leadership and society that implicitly represents the leadership subordinates as they directly interact with the population. The societal model is meant to demonstrate the efficacy and viability of using System Dynamics (SD) methods to simulate populations that these can then connect to cognitive models depicting individuals. SD models typically focus on average behavior and thus have limited applicability to describe small groups or individuals. On the other hand, cognitive models readily describe individual behavior but can become cumbersome when used to describe populations. Realistic security situations are invariably a mix of individual and population dynamics. Therefore, the ability to tie SD models to cognitive models provides a critical capability that would be otherwise unavailable.

To allow parallel development of SD and “real” cognitive models for this project, this work incorporates a “fake” cognitive model structure. The fake cognitive models are based on the same qualitative choice theory and cointegration (discussed below) used in the SD components, but the outcome is treated deterministically, even though these approaches only have probabilistic meaning at the individual level. Nonetheless, **for demonstrative purposes**, the dynamic response is a qualitatively consistent placeholder for a true cognitive model.

While Sandia National Laboratories is developing very sophisticated macroeconomic models to capture the dynamics of economies under stress or conflict, for the purposes here, the model uses a very simple representation that captures dynamic impacts solely associated with test interventions.

Good models focus on the key aspects needed for solving a particular problem. This proof-of-concept effort addresses oil bunkering. The premise is that military intervention will produce pressure on both the population and insurgency leadership to reduce bunkering. A further premise is that insurgency leadership and population are motivated to maintain their economic status quo. The model portrays the dynamic evolution of the bunkering activities as tension between these two premises interact under intervention scenarios.

The physical and economic behavioral implications are readily simulated using basic aspects of conventional simulation methods such as System Dynamics (Sterman 2000), engineering (Gershensfeld 1998), and economics (Hendry 1993). Societal and economic realities are the consequence of behavioral decisions. The simulation and understanding of these processes is only recently possible. Decisions are the process of making choices. All behaviors are the consequence of choices made. Daniel McFadden pioneered the use of (psychologically framed) qualitative choice theory (QCT). QCT (McFadden 1982, 1986) is actually very quantitative and determines the importance people place on information, tastes, beliefs, and preferences when making decisions. The robust parameterization of QCT is often based on data readily obtainable in the field. Other techniques can further determine the correct functional representation of QCT formulation for the problem at hand (Keeney 1976).

A key part of the decision process is the filtering of information and the extent to which longer term experience biases the decision process. Our personalities reflect this semi-permanent characterization of the person that consistently (albeit possibly irrationally) acts upon the information he/she perceives. The final outcome is certainly probabilistic due to variations in peripheral conditions (e.g., health/stress) and the fact that the human brain is always internally in transient. At a group level, the probabilistic nature leads to a mean-value response because random variation in one direction by one person is balanced by the reverse variation of another person. The enduring aspects of the population (society) dominate the group behaviors. The identification of the transient and stable components of the decision process uses cointegration (also Granger Causality) methods pioneered by Clive Granger. These same methods also ascertain the filtering and delayed-response processes associated with information perception and behavior (Granger 1987, 1991). These methods and others are summarized in Backus (2005, 2006) and Boslough (2004).

2. Model Equations

The model is divided into five interacting components. Feedback interactions are what produce the model's distinctive dynamics. The five components are: Societal Behavior, Economic Development, Cognitive-SD Interface, Cognitive Insurgent Leader, Cognitive Government Leader, and Cognitive Military Leader. A real cognitive model would connect with the Cognitive-SD interface and replace the "fake" cognitive components. The model's term for pressure applied to the population is called "coercion" and can be thought of as having physical and psychological aspects that are implemented by insurgent leaders' subordinates. Similarly, military and governmental initiatives by "leadership" are also implicitly assumed to occur through the agency of subordinates. Further, note that this construct recognizes that even US military interventions are limited by counter activities of other stakeholder leadership and the societies involved.

Essentially all behaviors are part of feedback processes. Therefore, most phenomena flow via loops of interactions where most causes are then due to previous effects and vice versa.

2.1 Societal Behavior

Social behaviors start as a response to recognized insurgent coercion (RIC – as defined later). This coercion finally occurs after subordinates act on behalf of leadership demands, and the eventual recognition of the activities' extent as quantity, direct experience, and information about them increases within the target population.

Society responds directly to perceived coercion (PC) that is a filtered delay for initiation of coercion activities. This filtering delay is the societal perception time (SPT). The functional form shown below represents a first order delay in SD terms (generating a first order Erlang distribution in statistical terms) as captured using cointegration. Note that this equation is actually an integration process ("t" is the current time, "t-1" is the previous time period, and "dt" is the delta time (notionally in months here) solved using simply Euler integration in this instance)¹. SD argues that all dynamics systems operate in feedback. All feedback is driven by integration. Humans (and nature) cannot causally act on (mathematical) derivatives.

$$PC(t)=PC(t-1)+dt*(RIC(t)-PC(t-1))/SPT$$

¹ Variables are defined at time "t" unless otherwise noted.

The PC, Military Pressure on Insurgents (MPI), economic activity (EA), and governmental pressure on the population (GPP) compared to reference coercion (RC) and reference military pressure (RM) determine the ordinal utility (UR) of participating in bunkering. Behavioral and normalization parameters (Ceteris Paribus Utility (CPU), Societal Coercion-Response Behavior (SCRB), Societal Economic-Response Behavior (SERB), Societal Government Response (SGR), Societal Military-Response Behavior (SMRB)) denote each factor's importance and others not modeled (i.e., in CPU) on the decision process.

$$UR = CPU + SCR B * \ln(PC/RC) + SMRB * \ln(MPI/RM) + SERB * \ln(1 + EA) + SGR * \ln(GPP/RC)$$

The use of the logarithm (ln) use denotes the idea that the impact on utility is proportional (relative) rather than measured in absolute terms.

The fraction of the population that then participates in bunkering (FPB) is the integral of the QCT probability distribution over all choices and, in this instance, has the form noted below.

$$FPB = 1 / (1 + \exp(-UR))$$

In this exercise, UR is defined to produce a value of 20% at initial levels of coercion. Without coercion (ceteris paribus), the economic advantages of bunkering would, by assumption, still cause 5% of the population to participate in some form of bunkering.

For a few extreme scenarios, the model simulates coercion rising to levels where insurgent control breaks down when societal counter responses (CCR) “eliminate” the insurgent leadership – or they eliminate infrastructure (subordinates) oppressing the population. Such (successful) processes have abundant local precedence in both Afghanistan and Iraq. This term should be probabilistic and more “complex” but for (generalizable) demonstration purposes, its purpose is to simply produce a response when coercion (PC) reaches a specified threshold (SCT).

$$CCR = \text{IF}(PC > SCT), \text{ THEN } 1, \text{ ELSE } 0$$

2.2 Economic Development

Exogenous economic development (EDM) is an exogenously specified intervention, define as an index referenced to current conditions (e.g., 1.0 = 100% increase). This entire SD model, in general, uses unitary and index-based constructs for simplicity and generalizability. Actual parameterization is possible, but it is not part of the demonstration effort.

Economic development response is the “exogenous” reference response (MEI) increased by EDM.

$$EDR=MEI*EDM$$

Simulated adjustments to economic development attempt to capture the impact of security (law and order) required for economic activities to transpire. The Beneficial Economic Impacts (BEI) of security are simply noted as exponential asymptotic benefits of added security presence (Military pressure on Insurgent – MPI) It assumes a logic that after you put several soldiers or police on every corner, adding more will have minimal security benefits. PFER is just a parameter relating MPI to the marginal benefit.

$$BEI=1-\exp(-PFER*(MPI-1))$$

Conversely, a large military presence can ”scare” economic activity and interfere with its flow. The Detrimental Economic Impact (DEI) simply simulates an exponential penalty that in the extreme could stop all activity. PDER simply relates MPI to the marginal detrimental impact.

$$DEI= \exp(-PDER*(MPI-1))$$

The net policy impact on the economy (PIE) of military security forces is the combination of BEI and DEI relative to what is already present (Reference Law & Order Index – RLOI).

$$PIE=(BEI+RLOI)*DEI$$

The change in economic activity (EA) is then definitionally net growth integrated with current activity and with the addition of any insurgent acts to disrupt activity (IRE – discussed later).

$$EA(t)=EA(t-1)+dt*EA(t-1)*(EDR(t)*PIE(t)*IRE(t))$$

2.3 Cognitive-SD Interface

The insurgent coercion is implemented by subordinates. If orders from the leader (Coercion Directives – CD) declines, subordinates will eventually attempt to minimize their efforts over time (Organizational perception time – OPT). The Indicated Coercion (IC) is the maximum of CD and the atrophying RIC.

$$IC(t)=MAX(CD(t),RIC(t-1)*(1-1/OPT))$$

The RIC is a cointegrated delay of IC over the time it takes leadership to mobilize subordinates to complete new directives (Leadership response Rate – LRR).

$$RIC(t)=RIC(t-1)*dt*(IC(t)-RIC(t-1))/LRR$$

Typically insurgency leadership needs to provide monetary incentives in addition to "physical" ones. Further, subordinates expect part of the “Take” from bunkering sales. These Coercion Costs (CC) are defined as a Normal Cost (CCN), and they proportionally increase as IC exceeds the initialized level of coercion (DCC).

$$CC=CCN*IC/DCC$$

The “Take” depends on how much of the population is helping oil bunkering (FPB) and the Take per unit of population (TC):

$$Take=TC*PFR$$

2.4 Cognitive Insurgent Leader

The remaining discussion describes “fake” cognitive structures used to demonstrate the linkage and feedback among model components. The insurgent leader reduces his visibility and risk (Insurgent Response from Military – IRM) as security interventions (MPI) increase. The model uses a very simplistic (implicit) QCT formulation, comparable to a log-log econometric approach that indexes the response to a reference security presence (Reference Military – RM) with a diminishing-returns response (Military Activity Impact on Insurgent Leader – MAIL).

$$IRM=(MPI/RM)^MAIL$$

Conversely, data indicates that monetary benefit drives much of the bunkering activities. It takes some time before the initial reduced activity of leadership spills over to reduced subordinate activity and reduced actual bunkering (for the insurgency) by the population. However, once Perceived Take (PT) declines noticeably relative to Expected Take (ET), the insurgent responds by commanding subordinates to increase coercion and do all necessary (Insurgent Response from Take – IRT) to again restore the Take to expected levels. As noted previously, it takes time (LRR) for the leader to recognize the situation, formulate a response, and execute. The insurgent leader also recognizes that added coercion efforts have a cost (CC) relative to normal costs (CCN) experienced prior to security intervention. These costs produce diminishing benefits (returns) to excessive coercion efforts. IRT is a slightly modified cointegration process.

$$IRT(t)=IRT(t-1)+dt*IRT(t-1)*((ETake(t)-CC(t)+CCN)/PTake(t)-1)/LRR$$

The orders (Coercion Directives – CD) from leadership are combined impacts of IRM, IRT, and (to be discussed later) societal counter responses (IRO). A normalization term is defined to convert directives to implied coercion efforts (Directive Coercion Conversion – DCC), as based on assumed initial conditions.

$$CD=DCC*IRM*IRT*IRO$$

The PT is simply the filtering of information on actual Take over the Leadership Perception Time (LPT).

$$PT(t)=PT(t-1)+dt*(Take(t)-PT(t-1))/LPT$$

Expected Take (ET) assumes that leadership fixates on (acclimates to) the highest sustained levels of Averaged Take (AT).

$$ET=MAX(ET,AT)$$

Average Take is just the filtering of the Take (net of daily variations).

$$AT(t)=AT(t)+dt*(PT(t-1)-AT(t-1))/LRR$$

The model allows for the possibility of a societal counter response (CCR – discussed previously) to excess coercion that “overthrows” insurgent leadership (IRO). As noted above, this formulation should be probabilistic and contain more terms, but it does produce the key dynamics of primary interest here.

$$IRO=MIN(IRO,1-CCR)$$

When the population has lower-risk, higher pay-off alternatives to bunkering activities, it can reduce both the Take and prestige of the insurgency. Experience in Iraq, Afghanistan, and elsewhere indicates that economic infrastructure can quickly become the target of insurgent attacks if it negatively affects the insurgency reward-structure. The Insurgent Response to Economic Development (IRE) is based on a QCT formulation that implicitly reduces the probability that populations can act (behavior) to maintain maximum economic impact (MEI). The reduced probability comes from assumed insurgent activities driven by recognition of changes in Perceived Take (PT) relative to Expected Take (ET), where the perceived relation to economic development activities (EDM) is increased as EDM becomes more extensive.

$$IRE=2*(1+MEI)/(1+(PT/AT)^(EDM-1))-MEI$$

(The “2” in the equation normalizes it to 1.0 for purposes here, because a multiplier of unity acts as a numaire meaning “no impact” in this model design. QCT naturally normalizes to .50, i.e., a 50/50 chance in absence of information.)

2.5 Cognitive Government Leader

Historical interventions reveal that local (or national) government leadership can perceive U.S. intervention in a competitive manner. Governments can perceive U.S. interventions as highlighting local government inadequacy, as improving perceived relevancy of insurgent leadership, and as diluting political support by generating support for intervention forces. The pressure that the local government places on the (U.S.) military (GPM) is simply assumed to be proportional to an exogenous intervention intensity (MPI) relative to the “norm” (RM), with a response term (GMR).

$$GPM = GMR * (MPI / RM)$$

As the insurgency increases its presence (measured as RIC), the government may attempt to persuade (Pressure on Population - GPP) the population to remain committed to the government's position. Realized pressure (GPP) is a filtered version of indicated pressures (IGPP) over the government response time, due to delays in implementation and realization by the populations at large.

$$GPP(t) = GPP(t-1) + dt * (IGPP(t) - GPP(t-1)) / GRT$$

The IGPP uses the same logic as GPM but is based on RIC, the “norm” level of insurgent activity (DCC), and a government's response to insurgency term (GSR).

$$IGPP = GSR * (RIC / DCC)$$

2.6 Cognitive Military Leader

The U.S. response to local government is assumed to be sub-linear. Executed interventions (MPI) are multiplicatively reduced from intended levels (IMPI) based on perceived local government pressure (PGP) relative to the norm (RM), in diminishing returns as specified by U.S. responsiveness to local government pressures (MGR).

$$MPI = IMPI * (PGP / RM)^{MGR}$$

The perceived government pressure (PGP) is simply a cointegrated filtering of government pressure (GPM) over the operational response time (OPT).

$$PGP(t) = PGP(t-1) + dt * (GPM(t) - PGP(t-1)) / OPT$$

All the above equations sequence through time to produce the feedback dynamics discussed in the next section.

3. Demonstrative Examples of Model Implications

The first test shown below only includes the interaction of cognitive leadership, his subordinates, and the population. All other parts of the model are inactive. The primary response is to show tension between the insurgency’s “caution” due to security interventions and by its subsequent efforts to still maintain the “Take.”

Shortly after the security intervention, insurgent leadership limits their newly perceived risks (blue line). After a delay, the subordinates and population reduce “Take” efforts. Once leadership realizes this impact on “Take” (green line), they initiate directives to “coerce” subordinates and population into restoring “Take” cash flow (red line). They continue to increase pressure until they achieve their goals. Due to delays in physical realization and perceptions, there are overshoot and undershoot dynamics that eventually bring the Take to former levels – but now with increased intervention and insurgency violence. Note that the dynamics are probably valid, but the level of response (until modified by actual field data and validation testing) is based on input assumptions that the insurgency will act to maintain its status quo conditions – the same as organized crime works in most countries.

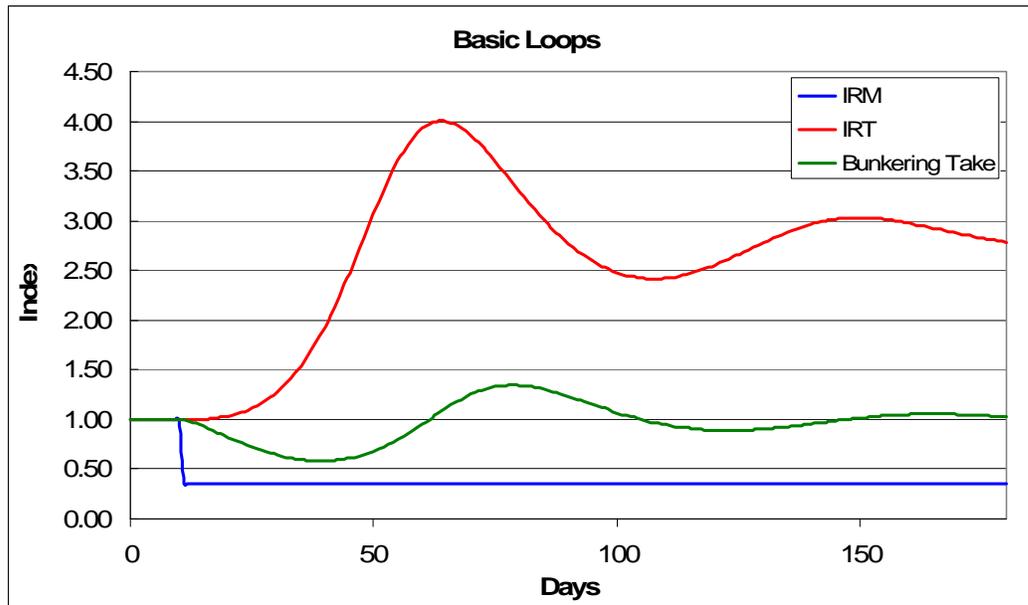


Figure 1: Basic Response.

In the next figure, the model not only includes costs of added coercion but also acclimation to increased “Take” during the overshoot dynamics. The net affect is slightly reduced maximum coercion but with increased net final “Take.” The security intervention (as modeled here for demonstrative purposes) would thus be counterproductive.

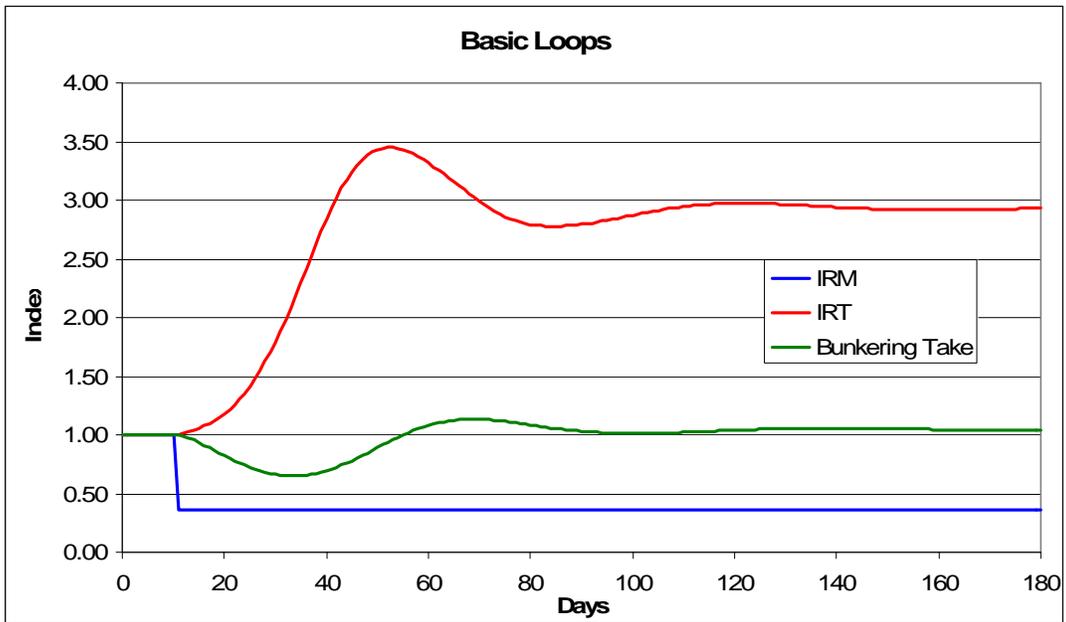


Figure 2: Base Run with Added Insurgency Costs and “Wake” Expectations.

The next test simply increases security (military) intervention to illustrate “assumed” behavior where the insurgency will simply counter respond to security interventions. Despite the “extremism” of the simulated responses, it does appear consistent with early Iraq experience (albeit not associated with bunkering).

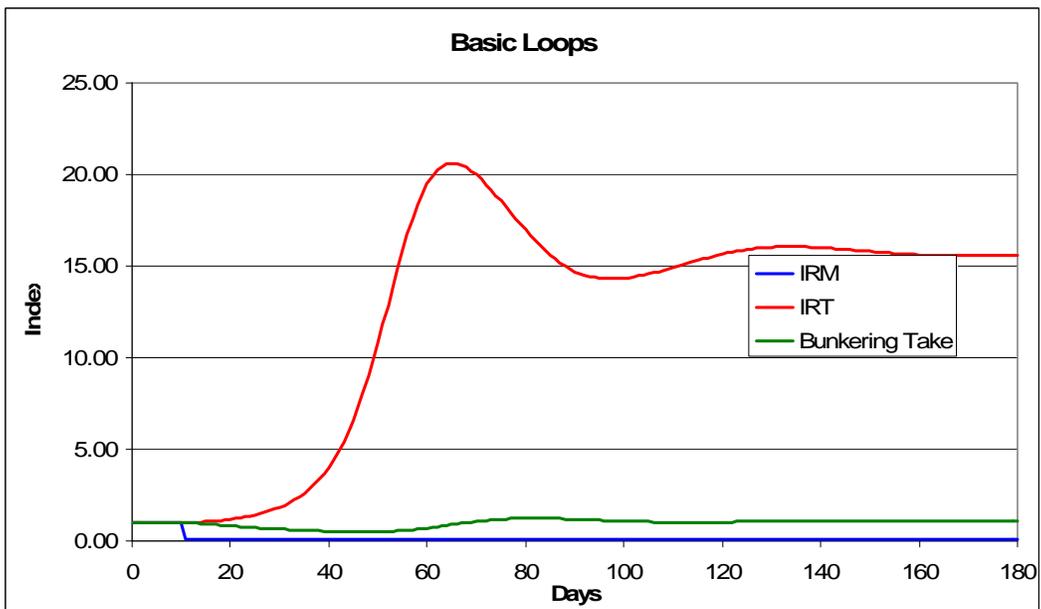


Figure 3: Base response with Intensified Security Intervention.

In Figure 4, the societal counter response is active in the model. An intense security response causes an intense insurgency counter response. The population eventually “mutinies” and eliminates insurgent leadership. The model assumes that there is NO insurgent leadership replacement to fill the power void. More advanced models could include the exploration of this (excluded) dynamic. The bunkering level drops dramatically to the levels the population might pursue on its own without added influence.

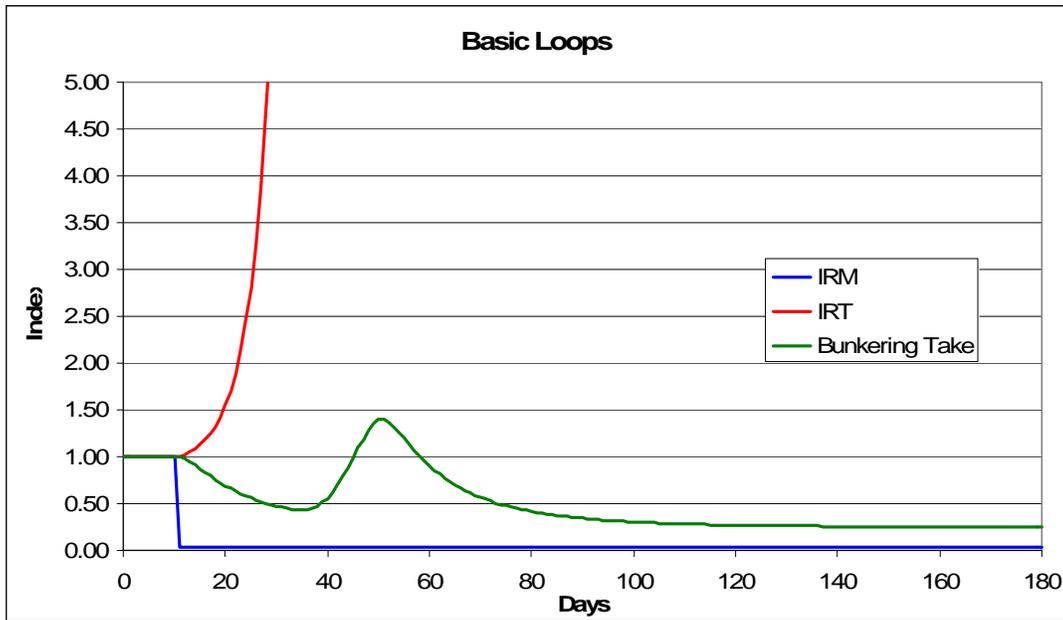


Figure 4: Societal Counter Response to Excessive Coercion.

Figure 5 shows the impact of adding security pressure (risk) to the general population for supporting insurgency bunkering. This tension reduces the efficiency of insurgency coercion, with a steady-state response slightly below its initial value. The intervention causes marginal improvement, but at the expense of more violence.

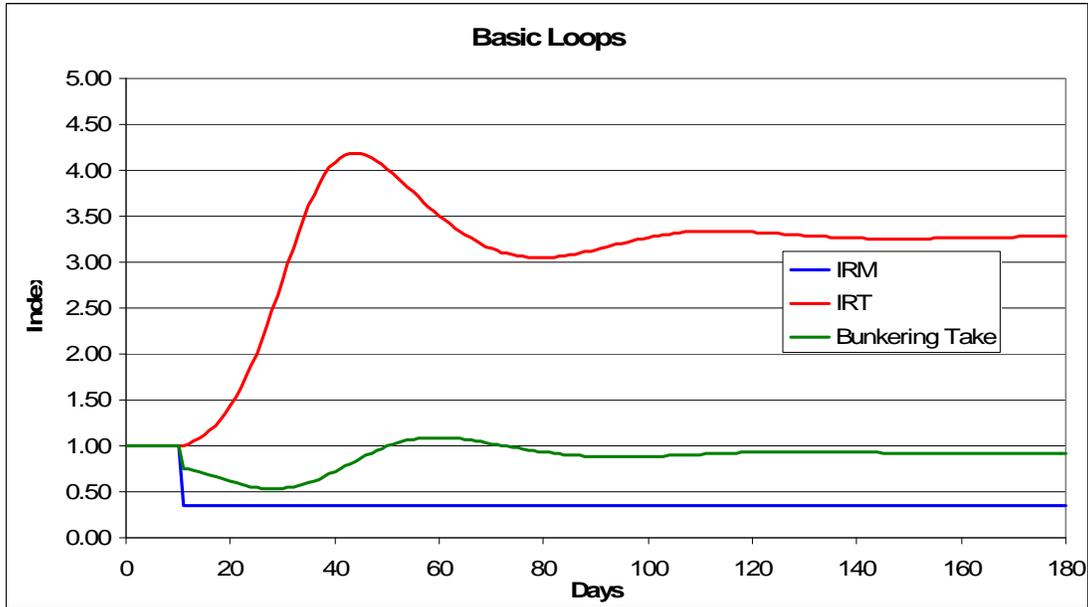


Figure 5: Added Security impacts on the Population.

The run in Figure 6 adds interacting dynamics among the three leadership levels. The initial intervention is eventually reduced. The insurgency recognizes the lack of resolve (via sustained increase “take”) and thus the worst outcome occurs with concomitant significantly increased “take”, violence, and local government/U.S government efforts that only dilute net mitigation efforts.

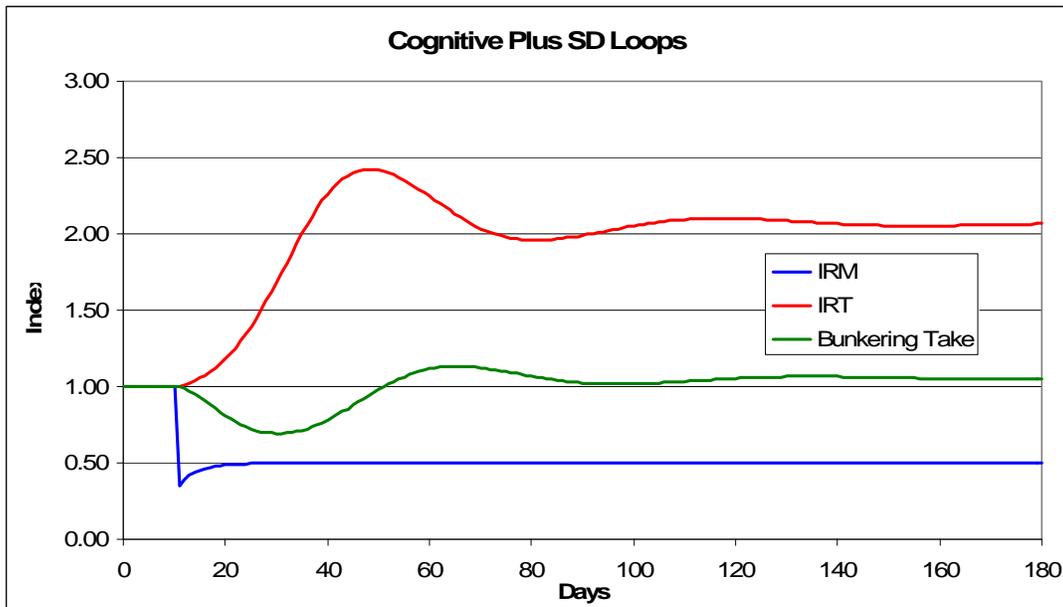


Figure 6: Added Leadership Interactions.

Figure 7 removes cognitive leadership interactions but adds broader economic interactions that allow testing of economic development efforts. The implicit assumption used for the model is the addition of appropriate (regionally compatible) economic development activities. In this case, an assumed, modernized (covered) marketplace with access to roads and electricity (refrigeration/heat) allows improved trade in indigenous goods. Added development alone does produce some longer term affects as long as it is below insurgents' radar. Lack of security, however, limits the dramatic increase in economic activity.

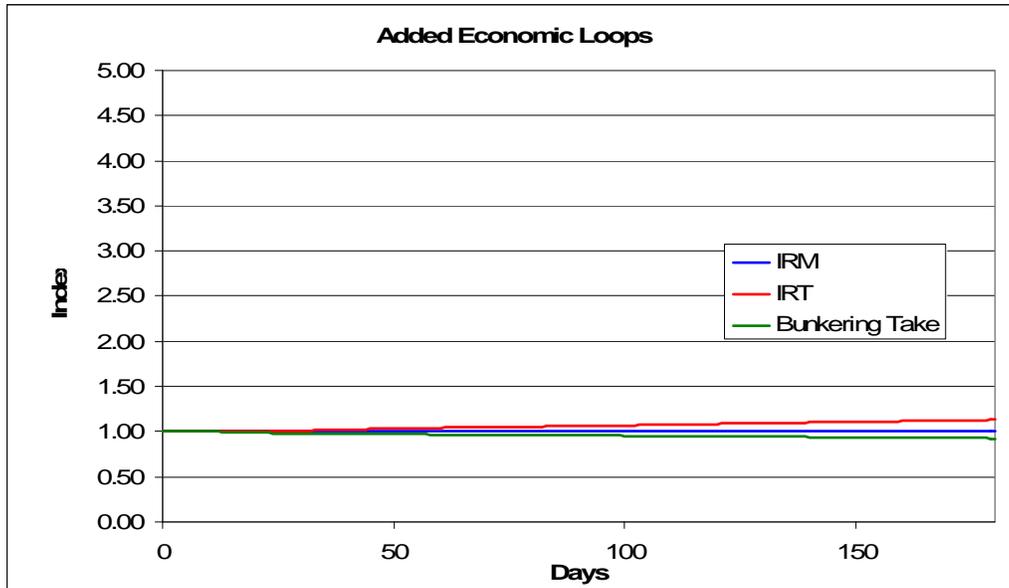


Figure 7: Isolated Economic Development.

Figure 8 adds stabilizing security support to economic development. There is unavoidable added violence due to insurgent counter responses, but added development is achieved; and a large segment of the population selects legitimate economic activity over bunkering in the long term.

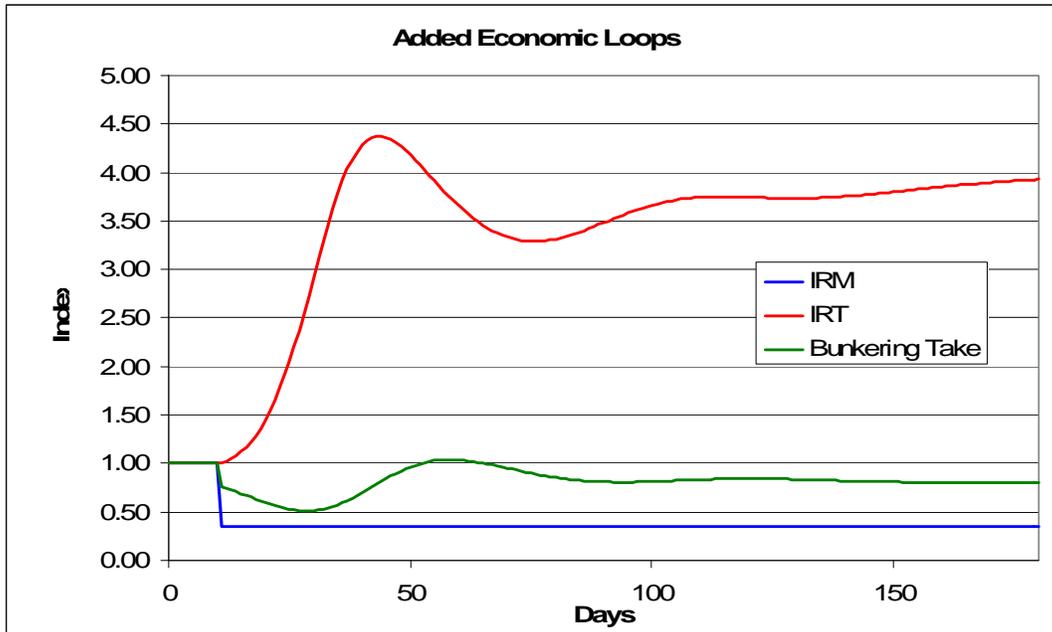


Figure 8: Economic Development with Security.

Finally, Figure 9 again adds the societal counter response to model dynamics and includes added economic development with security. The insurgent leadership is eliminated, and intervention can cease because the system is self-sustaining – where legitimate economic activity essentially eliminates bunkering. Historically, such situations limit the ability for insurgency to again take root. (There is an implicit and limiting assumption here that local government remains legitimate and corruption levels remain tolerable.)

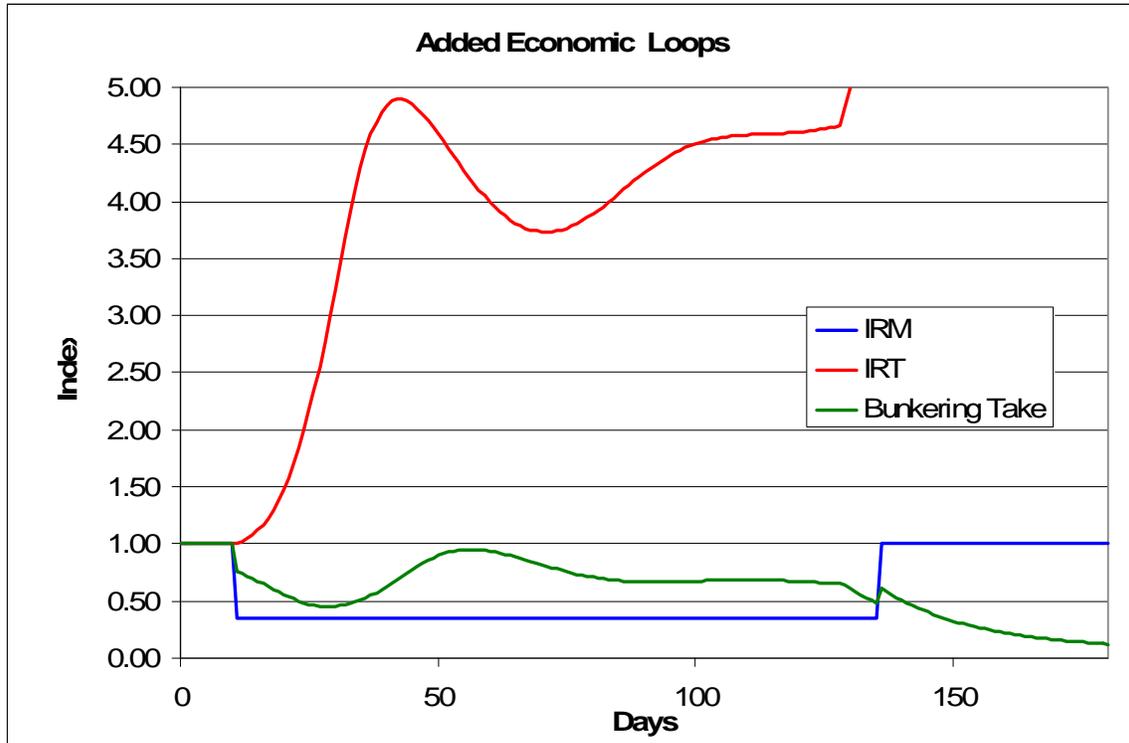


Figure 9: Economic Development with Security and Societal Response.

4. Conclusion

These explorations indicate the value and viability of combining cognitive models to represent individual leadership with System Dynamics models to simulate groups and societal interactions. The demonstration model also shows it is possible to design a model that does allow field data for parameterization (and thereby allows validation testing/modification) of the model. While the qualitative results shown here are possibly intriguing and plausible, the use of normalized parameters and unsubstantiated assumptions means that there is no legitimacy to quantitative results.² Confidence in model results/recommendations would require client supported data efforts, SME review, and formal model validation & verification.

² Next stage efforts should certainly include ensuring (making) units-of-measure consistent within the model, along with other basic testing (such as extremum tests).

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Appendix 1: Listing on Model Variable and Parameters

Variables

Societal Behavior

Perceived Coercion (PC)

Utility of Response (UR)

Population Fraction Bunkering (PFB)

Coercion Counter-Response (CCR)

Cognitive-SD Interface

Indicated Coercion (IC)

Realized Coercion (RIC)

Coercion Costs (CC)

Take (Take)

Economic Development

Economic Development Multiplier (EDM)

Economic Activity (EA)

Policing Benefit Economic Impact (BEI)

Policing Detrimental Economy Impact (DEI)

Policing Impact on the Economy (PIE)

Economic Development Response (EDR)

Cognitive (Insurgent Leader)

Insurgent Response from Military (IRM)

Insurgent Response from Take (IRT)

Coercion Directives (CD)

Perceived Take (PTake)

Expected Take (ETake)

Average Take (ATake)

Insurgent Response from Overthrow (IRO)

Insurgent Response to Econ Development (IRE)

Cognitive (Local Government Leader)

Pressure on Military (GPM)

Pressure on Population (GPP)

Indicated Pressure on Population (IGPP)

Cognitive (US Military Leader)

Pressure on Insurgents (MPI)

Perceived Government Pressure (PGP)

Indicated Pressure on Insurgents (IMPI)

Parameters

Delta Time (dt)

Base PFB (BPFB)

Ceteris Paribus Utility (CPU)

Coercion Costs Normal (CCN)

Directive Coercion Conversion (DCC)
Economic Development on Day 10 (ED10)
Government Military Response (GMR)
Government Response Time (GRT)
Government Societal Response (GSR)
Initial PFB (PFBI)
Leader Perception Time (LPT)
Leader Response Rate (LRR)
Maximum Economic Impact (MEI)
Military Activity Impact on Insurgent Leader (MAIL)
Military Government Response (MGR)
Military Increase on Day 10 (MI10)
Military Response Time (MRT)
Organizational Perception Time (OPT)
Policing Beneficial Economic Response (PFER)
Policing Detrimental Economic Response (PDER)
Reference Coercion (RC)
Reference Economy (RE)
Reference Law & Order Index (RLOI)
Reference Military (RM)
Reference Take (RTake)
Societal Coercion Threshold (SCT)
Societal Coercion-Response Behavior (SCRB)
Societal Economic-Response Behavior (SERB)
Societal Government Response (SGR)
Societal Military-Response Behavior (SMRB)
Societal Perception Time (SPT)
Take Conversion (TC)

Appendix 2: Initial and Demonstrative Model Variable/Parameter Values

Cognitive (Insurgent Leader)

Insurgent Response From Military	IRM	1.00
Insurgent Response from Take	IRT	1.00
Coercion Directives	CD	2.18
Perceived Take	PTake	1.00
Expected Take	ETake	1.00
Average Take	ATake	1.00
Insurgent from Overthrow	IRO	1.00
Insurgent Response to Econ Development	IRE	1.00

Cognitive (Local Government Leader)

Pressure on Military	GPM	1.00
Pressure on Population	GPP	1.00
Indicated Pressure on Population	IGPP	1.00

Cognitive (US Military Leader)

Pressure on Insurgents	MPI	1.00
Perceived Government Pressure	PGP	1.00
Indicated Pressure on Insurgents	IMPI	1.00

Economic Development

Economic Development Multiplier	EDM	0.00
Economic Activity	EA	1.00
Policing Benefit Economic Impact	BEI	0.00
Policing Detrimental Economy Impact	DEI	1.00
Policing Impact on the Economy	PIE	0.20
Economic Development Response	EDR	0.00

Cog-SD Interface

Indicated Coercion	IC	2.18
Realized Coercion	RIC	2.18
Coercion Costs	CC	0.50
Take	Take	1.00

Societal Behavior

Perceived Coercion	PC	2.18
Utility of Response	UR	-1.39
Population Fraction Bunkering	PFB	0.20
Coercion Counter-Response	CCR	0.00

Delta Time	DT	1.00
Parameters		
Military Activity Impact on Insurgent Leader	MAIL	-1.50
Base PFB	BPFB	0.05
Initial PFB	PFBI	0.20
Reference Take	RTake	1.00
Reference Coercion	RC	1.00
Reference Economy	RE	1.00
Reference Military	RM	1.00
Societal Coercion-Response Behavior	SCRB	2.00
Societal Economic-Response Behavior	SERB	-1.00
Societal Military-Response Behavior	SMRB	-0.50
Societal Perception Time	SPT	3.00
Organizational Perception Time	OPT	3.00
Leader Perception Time	LPT	3.00
Leader Response Rate	LRR	14.00
Take Conversion	TC	5.00
Directive Coercion Conversion	DCC	2.18
Societal Coercion Threshold	SCT	5.00
Coercion Costs Normal	CCN	0.50
Government Military Response	GMR	1.00
Government Societal Response	GSR	1.00
Government Response Time	GRT	30.00
Military Response Time	MRT	7.00
Military Government Response	MGR	-0.50
Societal Government Response	SGR	-0.50
Economic Development on Day 10	ED10	0.00
Military Increase on Day 10	MI10	0.00
Maximum Economic Impact	MEI	0.01
Policing Beneficial Economic Response	PFER	0.50
Policing Detrimental Economic Response	PDER	0.50
Reference Law & Order Index	RLOI	0.20
Ceteris Paribus Utility	CPU	-2.94

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