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Modeling, Simulation & Optimization of the Landing Craft Air Cushion Fleet Readiness

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Modeling, Simulation & Optimization of the Landing Craft Air Cushion Fleet Readiness

An Exploration of Customer Needs and the connection to Sandia's ProOpta Capabilities

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Abstract

The Landing Craft Air Cushion is a high-speed, over-the-beach, fully amphibious landing craft capable of carrying a 60-75 ton payload. The LCAC fleet can serve to transport weapons systems, equipment, cargo and personnel from ship to shore and across the beach. This transport system is an integral part of our military arsenal and, as such, its *readiness* is an important consideration for our national security. Further, the best way to expend financial resources that have been allocated to maintain this fleet is a critical issue.

There is a clear coupling between the measure of Fleet Readiness as defined by the customer for this project and the information that is provided by Sandia's ProOpta methodology. Further, there is a richness in the data that provides even more value to the analyst. This report provides an analytic framework for understanding the connection between Fleet Readiness and the output provided by Sandia's ProOpta software. Further, this report highlights valuable information that can also be made available using the ProOpta output and concepts from basic probability theory. Finally, enabling assumptions along with areas that warrant consideration for further study are identified.

Executive Summary

The Landing Craft Air Cushion is a high-speed, over-the-beach, fully amphibious landing craft capable of carrying a 60-75 ton payload. The LCAC fleet can serve to transport weapons systems, equipment, cargo and personnel from ship to shore and across the beach. This transport system is an integral part of our military arsenal and, as such, its *readiness* is an important consideration for our national security. Further, the best way to expend financial resources that have been allocated to maintain this fleet is a critical issue.

Sandia has been heavily involved in modeling, simulation, and optimization of system reliability, maintainability and availability for many years. A key question is the connection between *Fleet Readiness* and the concept of LCAC craft *availability*. This report provides the mathematical framework for that connection. It is shown that not only can Sandia's ProOpta provide valuable information in the estimation of Fleet readiness using extant failure and repair data, but also this data can be mined for additional information that is of significant value. Further, it is likely that ProOpta can provide guidance regarding investment strategies that result in fleet readiness levels that are superior to levels that are achievable without the supporting analysis.

There is a clear coupling between the measure of Fleet Readiness as defined by the customer for this project and the information that is provided by Sandia's ProOpta methodology. Further, there is a richness in the data that provides even more value to the analyst. In particular, the probability density function(s) that characterizes the number of LCAC craft that are available can be used to explore various important sensitivities of overall fleet readiness to variables such as the...

- availability of the hypothetical representative craft,
- relative degrees of site readiness, and
- differences in (equipment) class readiness.

This report provides an analytic framework for understanding the connection between Fleet Readiness and the output provided by Sandia's ProOpta software. Further, this report highlights valuable information that can also be made available using the ProOpta output and concepts from basic probability theory. Finally, enabling assumptions along with areas that warrant consideration for further study are identified.

Introduction

Sandia National Laboratories undertook a project in support of the United States Navy's Amphibious Warfare Program Office focused on delivering a capability to help determine system upgrades, repair, spare parts, and maintenance strategies that cost-effectively improve the *readiness* of the Landing Craft Air Cushion (LCAC) fleet. In addition to the Amphibious Warfare Program Office, there are other offices within the Navy's Program Execution Office that will find value in using this capability

The LCAC (see figure 1) is a high-speed, over-the-beach, fully amphibious landing craft capable of carrying a 60-75 ton payload. The LCAC can serve to transport weapons systems, equipment, cargo and personnel from ship to shore and across the beach.

Figure 1: The Landing Craft Air Cushion Transport System

LCACs can carry heavy payloads, such as an M-1 tank, at high speeds. Their payload and speed mean more forces reach the shore in a shorter time, with shorter intervals between trips. Determining the optimal allocation of fiscal resources that will maximize the readiness of our country's LCAC Fleet is important to our national security.

The connection between *Fleet Readiness* and the concept of craft *availability* is described in this report. The concepts of availability, and its closely allied area of reliability, are well established in the technical community[1]. Many organizations, including Sandia National Laboratories, have been actively engaged in the modeling, analysis, and optimization of system reliability and availability for decades (see: references [1] through [10]. The Sandia capability that is being adapted and applied to the LCAC Readiness project is ProOpta; a next generation version of SyOp (SyOp evolved from WinR [11]). ProOpta consists of a modeling and analysis framework and a collection of tools that provide reliability, fault-tree, uncertainty, sensitivity and optimization analyses.

This report focuses on five key questions; namely…

(1) What does the customer want?
(2) What can ProOpta deliver?
(3) What are the connections between what ProOpta can deliver and what the customer needs?
(4) What are the associated gaps?
(5) What are the recommendations?

The remainder of this report is organized to present the answers, including supporting rationale, to these questions.

[1] See, for example, http://www.reliabilityweb.com/ , http://www.reliability-magazine.com/ , and http://en.wikipedia.org/wiki/Reliability_engineering.

What does the Customer Want?

The *project scope* articulated within the Statement of Work was developed in collaboration with the customer. As such it sets the context for what the customer is expecting. The project scope is…

> "Sandia National Laboratories (SNL) is requested to generate a Statement of Work (SOW) to develop a model (or system of models) to perform analyses of current and future LCAC maintenance and support operations. The effort should consider the related current and historical databases to determine the effect of fiscal changes to the Assault Craft Units' (ACU's) operational funds and the larger LCAC program funding (Service Life Extension Program, System Upgrade, Phased Maintenance Plan, and Fleet Modernization Plan) on the overall craft readiness of the ACU. In other words, SNL is to perform modeling and simulation of the LCAC enterprise to determine the best use of funds that will optimize the number of each ACU's Mission Capable craft. Once developed, the model(s) will be maintained, updated, and used by the LCAC commcrafty to develop "what if" cost and material scenarios to optimize the organization and accomplishment of budgeting for, operating, supporting, and maintaining all LCAC."

Further, in the Objectives & Constraints of Proposed Effort section of the Statement of Work, the *primary objective* of this project is articulated as follows…

> "Primary objective is to answer the question as to the best repair/sparing strategy and what maintenance procedures should or should not be enacted in order to maximize fleet readiness given an increase or decrease of the requested budgets. (In short, how much will readiness be affected by a defined budget increase or decrease.)"

Clearly, the focus for this project is to provide a capability to *maximize Fleet Readiness* by determining the best allocation of fiscal resources among feasible (1) repair strategies, (2) spare parts strategies, and (3) maintenance procedures.

An important consideration is the mathematical definition of the objective function; namely, Fleet Readiness. The central theme for this report is the *readiness* of the fleet of LCAC. The customer has clearly defined *fleet readiness* as:

The number of craft that are Mission Capable[2]
divided by
the Total Number of craft.

This report presents a metric for the Fleet Readiness that is, in essence, the measure defined above and that can be computed from ProOpta output. This report also expands upon this measure and illustrates the value of the inherent richness of the information beyond a simple expected value analysis. The next section in this report focuses on the mathematical definition of the estimates of *measures related to Fleet Readiness* that are computed by the ProOpta software.

2 Note: The number of craft that are Mission Capable is the number of craft that are either Fully Mission Capable or Partially Mission Capable.

What ProOpta Can Deliver[3]*?*

In order to describe the ProOpta output, concepts from basic probability theory will be relied upon. The following notation[4] will be used...

> "i" is the *index* of the craft. This index is used for algorithmic purposes and has no other significance.
>
> "j" is the *class*[5] of crafts of which the craft is a member.
>
> "k" is the *geographic region*[6] to which the craft is assigned.
>
> ${}^{f,n}T_{i,j,k}$ is the time at which $LCAC_{i,j,k}$ transitions from the Mission Capable State to the Non-Mission Capable State for the n^{th} time.
>
> ${}^{r,m}T_{i,j,k}$ is the time at which $LCAC_{i,j,k}$ experienced the m^{th} repair[7].

It is noteworthy that the *tail number* of each craft is associated with one, and only one, triplet[8] *i,j,k.*

A concept that is needed to understand the ProOpta measures is that of a random variable.

> A *Random Variable* is a function that, to each sample point in the sample space, assigns a (usually real) number.

The two random variables that are fundamental to the ProOpta calculation are:

$${}^{f,n}\Delta_{i,j,k} = {}^{f,n}T_{i,j,k} - {}^{r,n-1}T_{i,j,k}$$

and

---

[3] It is noteworthy that ProOpta can be used to provide information related to optimization of an objective function. The discussion in this section focuses on the calculation of Fleet Readiness and not on optimization *per se*. A discussion of the connection to optimization is deferred to the *Gaps and Recommendations* section of this report.

[4] The nomenclature used in the following is a general case that, in principle, allows the analyst to have the resolution of both "class" and "location" (AKA ACU Unit). It is noteworthy that the analyst may choose to not have either the "class" or the "location" resolution.

[5] The set of LCAC craft that have similar types of equipment fall within the same *class*.

[6] For purposes of this study there are two geographic regions; i.e., ACU-4 and ACU-5.

[7] Note that *repair* as used in this context is not necessarily a repair in the purest sense. For purposes of this study, ${}^{r,m}T_{i,j,k}$ is simply the m^{th} time at which the craft $LCAC_{i,j,k}$ transitions from a non-mission-capable state to a mission-capable status.

[8] This association is, indeed, unique. However, it is possible that an LCAC can be assigned to a different geographic region at different times; therefore, the index "k" may possibly change for a given LCAC.

$$^{r,m}\Delta_{i,j,k} = {}^{r,m}T_{i,j,k} - {}^{f,m-1}T_{i,j,k}$$

Two important estimators that are produced by ProOpta are the Mean Time Between Failures and the Mean Down Time. These estimators are computed as the arithmetic means of the samples[9] for Time Between Failures and Time to Repair, respectively. Hence,

$MTBF_{i,j,k}$ is an estimator for $E[{}^{f,n}\Delta_{i,j,k}]$

and

$MDT_{i,j,k}$ is an estimator for $E[{}^{r,n}\Delta_{i,j,k}]$

where E[y] is the *expected value*[10] of the random variable "y".

Availability has been defined[11] as:

1. The degree to which a system, subsystem, or equipment is operable and in a committable state at the start of a mission, when the mission is called for at an unknown, *i.e.,* a random, time. Simply put, availability is the proportion of time a system is in a functioning condition. (Note: the conditions determining operability and committability must be specified.)
2. The ratio of (a) the total time a functional unit is capable of being used during a given interval to (b) the length of the interval.

ProOpta computes the *availability* ($A_{i,j,k}$) of an LCAC craft with the MTBF and MDT estimators for that craft using the following relationship...

$$A_{i,j,k} = \frac{MTBF_{i,j,k}}{\left(MTBF_{i,j,k} + MDT_{i,j,k}\right)}$$

The notion of failure and repair *rates* are also important to understanding the ProOpta methodology. These rates are denoted as $\lambda_{i,j,k}$ and $\mu_{i,j,k}$, respectively. The relationships between the MTBF and MDT estimators and the failure and repair rates are:

$$E[{}^{f,n}\Delta_{i,j,k}] = (\lambda_{i,j,k})^{-1} \quad \text{and} \quad E[{}^{r,m}\Delta_{i,j,k}] = (\mu_{i,j,k})^{-1}$$

The transition rate model is a useful construct in understanding the relationship between transition rates and the state of the craft. In figure 2, the craft is in state "1" if it is

---

[9] The failure and repair data for this study was contained within {**Need a set of reference data bases here. For example: 2-KILOs**}

[10] See: http://mathworld.wolfram.com/ExpectationValue.html

[11] See: http://en.wikipedia.org/wiki/Availability

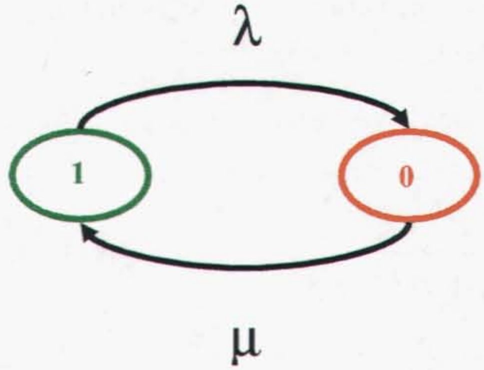

Figure 2: Transition Rate Model

available and "0" if it is unavailable. If the craft is in the available state then it transitions to the unavailable state at a rate of λ. Similarly, if the craft is in the unavailable state then it transitions to the available state at a rate of μ. An underlying assumption is that these transition rates are constant; i.e., they do not vary with time.

Another useful construct is the transition probability model shown in figure 3. In this

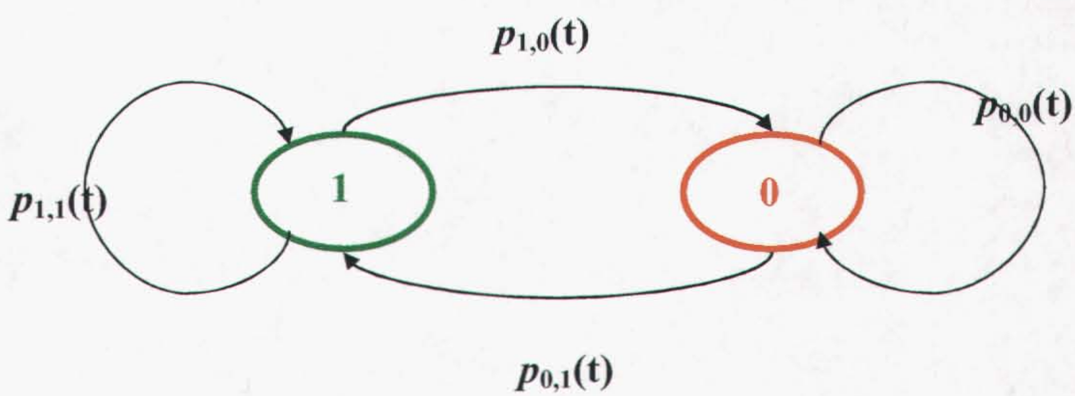

Figure 3: Transition Probability Model

representation, the transitions from one state to the other or from a state to itself are depicted as arrows flowing out of each of the two states. The labels on the arrows are the probabilities that the transitions will be made at a particular time "t".

The relationships between the transition rates and the transition probabilities can be derived from the continuous-time, Chapman-Kolmogorov equations[12]. The basic assumption in the ProOpta calculation is that the underlying stochastic process[13] is Markovian[14]. This assumption gives rise to the following transition probabilities[15]...

$$p_{0,0}(t) = \frac{\lambda + \mu e^{-(\lambda+\mu)t}}{\lambda + \mu} \qquad p_{0,1}(t) = \frac{\mu + \mu e^{-(\lambda+\mu)t}}{\lambda + \mu}$$

$$p_{1,0}(t) = \frac{\lambda + \lambda e^{-(\lambda+\mu)t}}{\lambda + \mu} \qquad p_{1,1}(t) = \frac{\mu + \lambda e^{-(\lambda+\mu)t}}{\lambda + \mu}$$

From these equations, the *steady-state* probabilities can be determined. *The steady-state probabilities are important in that they provide the fundamental linkage between the ProOpta computation and the Fleet Readiness that was requested by the customer.*

Define ${}^{\langle 1\rangle}\Pi$ and ${}^{\langle 0\rangle}\Pi$ as the steady state probabilities that the craft is in the Mission-Capable and Non-Mission-Capable state, respectively. Then we have

$$
\begin{aligned}
{}^{\langle 1\rangle}\Pi &= \lim_{t\to\infty}\left[\omega p_{0,1}(t) + (1-\omega)p_{1,1}(t)\right] \\
&= \lim_{t\to\infty}\left\{\omega\left[\frac{\mu - \mu e^{-(\lambda+\mu)t}}{\lambda+\mu}\right] + (1-\omega)\left[\frac{\mu + \lambda e^{-(\lambda+\mu)t}}{\lambda+\mu}\right]\right\} \\
&= \frac{\omega\mu + (1-\omega)\mu}{\lambda+\mu} \\
&= \frac{\mu}{\lambda+\mu}
\end{aligned}
$$

---

[12] See: http://eom.springer.de/K/k055680.htm

[13] Doob (6) defines a stochastic process as a family of random variables $\{x\,(t, \bullet), t \in \mathcal{T}\}$ from some probability space $(S, \mathbb{S}, P)$ into a state space $(S', \mathbb{S}')$. Here, $\mathcal{T}$ is the index set of the process. Typically, "t" represents time.

[14] In probability theory, a stochastic process is said to be Markovian if it has the Markov property; i.e., the conditional probability distribution of future states of the process, given the present state, depends only upon the current state. Simply stated, the present state is conditionally independent of the past states or the path of the process.

[15] See: http://eom.springer.de/t/t093760.htm

Here, ω is the probability that the initial state is "0"; hence, the probability that the initial state is "1" must be (1-ω). Therefore,

$$^{\langle 1 \rangle}\Pi = \frac{\mu}{\lambda + \mu} \qquad \text{and} \qquad ^{\langle 0 \rangle}\Pi = \frac{\lambda}{\lambda + \mu}$$

These steady-state probabilities can be used to determine the availability of each of the individual LCAC craft using the following relationships…

$$^{\langle 1 \rangle}\Pi_{i,j,k} = \frac{\mu_{i,j,k}}{\lambda_{i,j,k} + \mu_{i,j,k}} \qquad \text{and} \qquad ^{\langle 0 \rangle}\Pi_{i,j,k} = \frac{\lambda_{i,j,k}}{\lambda_{i,j,k} + \mu_{i,j,k}}$$

The interpretation of these two probabilities is that if one randomly selects an LCAC craft – say the craft with designators *i,j,k* – within the fleet and randomly selects a time to check the status of that craft, then the probability that that craft is mission capable is $^{\langle 1 \rangle}\Pi_{i,j,k}$ and the probability that it is not mission capable is $^{\langle 0 \rangle}\Pi_{i,j,k}$.

What is the Connection between the ProOpta Computation and the Customer Need?

With the preceding sections as prolog, the connection between the overall objective *Fleet Readiness*, as defined by the customer, and the output that is provided by ProOpta can be readily drawn. Basic concepts from probability theory will be used to draw this connection.

Recall that a Random Variable is a function that to each sample point in the sample space *S* assigns a (usually real) number. The following notation is needed to understand the connecting concepts...

$L_{j,k}$ = the total number of LCACs of class "j" that are assigned to region "k".

$N_{j,k}$ = the minimum number of LCACs of class "j" that are required for geographic region "k" to be *ready*[16]. This number can be set by the analyst.

$X_{i,j,k}$ is a random variable that defines the status of craft $i \in \{1,2, \ldots, L_{j,k}\}$ of class $j \in \{1,2,\ldots,6\}$ located within geographic region $k \in \{1,2\}$.

$X_{i,j,k} = 1$ if LCAC craft *i,j,k* is mission capable, 0 otherwise

$O_{j,k}$ is a random variable that defines the number of *mission capable* LCAC crafts of class "j" that are located within geographic region "k". That is,

$$O_{j,k} = \sum_i X_{i,j,k}$$

Using the terminology of classical probability theory;

- The random variable $X_{i,j,k}$ has a *Bernoulli* probability density function[17], and
- The random variable $O_{j,k}$ has a *Binomial* probability density function.

16 For the purposes of this study, as per guidance from the customer, the default value for Nj,k is simply 60% of the total number of LCACs of class "j" that are assigned to geographic region "k" (hence: Nj,k is the smallest integer that is greater than or equal to 60% of Tj,k ; i.e., Nj,k = .6* Tj,k if 0.6* Tj,k = INT(.6* Tj,k) otherwise Nj,k = [INT(0.6* Tj,k) + 1]

17 See: http://mathworld.wolfram.com/ProbabilityFunction.html

The mathematical expressions for these probability density functions are;

$$\Pr[X_{i,j,k} = 1] = p$$
$$= \{1 - \Pr[X_{i,j,k} = 0]\},$$

and

$$\Pr[O_{j,k} = l] = \binom{L_{j,k}}{l} p^{l}(1-p)^{(T_{j,k}-l)}$$

respectively.

The expected value of $O_{j,k}$ is then simply $L_{j,k}\ \Xi\ p$

Recall that the customer defined LCAC Fleet Readiness as…

The number of craft that are Mission Capable
divided by
the Total Number of craft.

One measure of the Fleet Readiness, R, that is supportive of this definition is the *expected value* of the number of craft in the fleet that are Mission Capable divided by the total number of craft in the fleet. Define the following fleet-wide variables; O is the number of craft in the fleet that are Mission Capable, T is the total number of craft in the fleet, and p is the probability that the hypothetical representative LCAC craft within the fleet is mission capable. Then, using the *expected val*ue as a measure, and the results shown above for the expected value of a Binomial random variable,

$$R = \frac{E(O)}{T} = \frac{T \times p}{T} = p$$

The conclusion is that Fleet Readiness is equivalent to the availability of a hypothetical, representative craft within the fleet.

It is noteworthy that the customer for this project has specified that the fleet is ready if at least 60% of the LCAC are mission capable. The simple expected value measure defined above does not account for the minimum number of LCAC needed. Fortunately, the information that is available can provide for an even richer understanding of Fleet Readiness than that provided by the expected value of O alone and, in particular, it can provide valuable insights into the relationship between the minimum number of LCAC required and their availability. To expand upon this richness, the following are key definitions specific to the LCAC modeling and analysis activity.

One particularly valuable parameter is the *minimum number of LCAC craft that must be mission capable if the fleet is to be ready.* This parameter, $N_{j,k}$, was defined above in the context of class *j* and site *k* but it can readily be generalized beyond site and class. This parameter can set by the analyst.

If p_k is defined as the probability that an LCAC within region *k* is mission capable then *Site Readiness* (R_k) can be defined as:

$$R_k \;=\; Pr\{\, O_k \geq N_k \,\} \;=\; \sum_{j=N_K}^{T_K} \binom{T_K}{j} p_k^{\,j} (1-p_k)^{T_K-j}$$

This is the readiness of *geographic region k* and it is simply the probability that the number of mission capable LCAC craft within region *k* exceeds the minimum number specified by the analyst. For purposes of this study, the geographic regions correspond to ACU-4 and ACU-5. However, additional or alternative regions can be readily included.

Fleet Readiness can be defined in the context of this definition of *Site Readiness*. If Fleet Readiness is defined as the probability that all of the sites are ready then, assuming independence between sites, Fleet Readiness is given by:

$$R = \prod_{l=1}^{K} R_l \text{, where K is the total number of geographic regions of interest.}$$

Similarly, the question of readiness can be asked in the context of the classes of LCAC. First, recall that the set of LCAC craft that have similar types of equipment fall within the same class. If p_c is defined as the probability that an LCAC within class *c* is mission capable, T_c is the total number of LCAC within class *c*, and N_c is the number of LCAC required to be mission capable if the class is to be *ready*; then *Class Readiness* (R_c) can be defined as:

$$R_c \;=\; Pr\{\, O_c \geq N_c \,\} \;=\; \sum_{j=Nc}^{T_c} \binom{T_c}{j} p_c^{\,j} (1-p_c)^{T_c-j}$$

For purposes of this study, there are six classes of LCAC craft. However, additional or alternative classes can be readily included.

Fleet Readiness can be then defined in the context of *Class Readiness*. If Fleet Readiness is defined as the probability that all of the classes are ready then, assuming independence between classes, Fleet Readiness is given by:

$$R = \prod_{l=1}^{N_c} R_l$$

The notions of Class Readiness and Site Readiness may be important if the analyst needs resolutions in either of these dimensions. For example, it may be important to determine the Site Readiness to understand asymmetry in the mission capable status of different sites. Similarly, if the distinction in types of equipment on board the individual craft is important in understanding the mission capable status in different mission scenarios then the Class Readiness may provide valuable information.

At a more aggregate level, the analyst may choose to compute the fleet readiness independent of both Site Readiness and Class Readiness. Define *T*, *N*, and *O* as the total number of LCAC crafts that are in the fleet, needed for readiness, and mission capable; respectively. Then, if *p* is defined as the probability that a hypothetical representative LCAC within the fleet (independent of location or class) is mission capable, then Fleet Readiness is given by:

$$R = \Pr\{O \geq N\} = \sum_{l=N}^{T} \binom{T}{l} p^l (1-p)^{T-l}$$

For purposes of this exercise T = 82. Note that 60% of 82 is 49.2, so N = 50. Therefore, fleet readiness can be computed as:

$$R = \Pr\{O \geq 50\} = \sum_{l=50}^{82} \binom{82}{l} p^l (1-p)^{82-l}$$

As an illustration of the information richness associated with using the probability density function, and not simply the expected value of the number of mission capable craft, figure 4 displays the probability that at least 60% of the Craft within the Fleet are Mission

Pr {at least 60% of the Craft within the Fleet are Mission Capable}
Pr {at least 60% of the Craft within the Fleet are Mission Capable}
Pr {at least 60% of the Craft within the Fleet are Mission Capable}

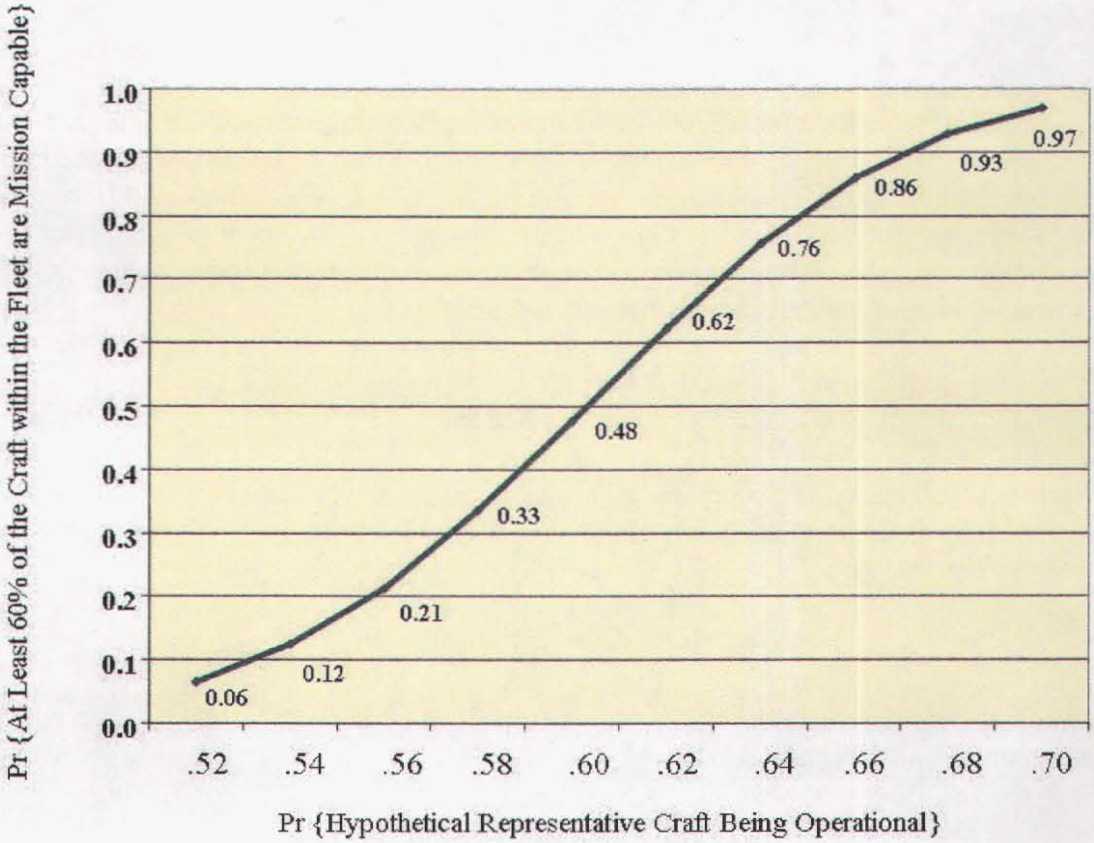

Figure 4: **Fleet Readiness**

Capable as a function of the probability that the hypothetical representative craft within the fleet is mission capable. This segment of the curve was chosen because it displays the region that the Fleet Readiness is most sensitive to changes in the mission capability of individual craft. From the figure one can see that in order to have at least a 90% probability of at least 60% of the LCAC craft within the fleet to be mission capable; then the hypothetical representative craft within the fleet must have at least a 66% probability of being mission capable.

At the other extreme, suppose that the analyst wishes to use a different mission capable probability for each of the 82 craft. This creates a computational burden that renders the task intractable if the desired outcome is to be a closed-form solution[18]. The recommendation is to employ either Monte-Carlo sampling or clustering[19] techniques to

18 In mathematics, an equation or system of equations is said to have a closed-form solution if, and only if, at least one solution can be expressed analytically in terms of a bounded number of certain "well-known" functions . Typically, these well-known functions are defined to be elementary functions; infinite series, limits, and continued fractions are not permitted (See: http://en.wikipedia.org/wiki/Closed_form_solution)

19 Here, clustering is meant grouping of the of LCAC craft into categories of crafts that have mission capable probabilities that are approximately equivalent. The definition of these clusters is left to the analyst.

address the curse of dimensionality through approximation. Either of these techniques has the potential to produce valuable approximations of the estimators.

Gaps and Recommendations

This Report has answered the questions (1), (2), and (3) below using basic concepts from probability theory.

(1) What does the customer want?
(2) What can ProOpta deliver?
(3) What are the connections between what ProOpta can deliver and what the customer needs?
(4) What are the associated gaps?
(5) What are the recommendations?

The answers to questions (4) and (5) fall more in the realm of the mathematical optimization problem that is articulated within the Statement of Work as…

> "Primary objective is to answer the question as to the best repair/sparing strategy and what maintenance procedures should or should not be enacted in order to maximize fleet readiness given an increase or decrease of the requested budgets. (In short, how much will readiness be affected by a defined budget increase or decrease.)"

The Standard Form[20] of a *mathematical program* is an optimization problem of the form:

$$\text{Maximize } f(x)\text{: } x \text{ in } X,\ g(x) <= 0,\ h(x) = 0,$$

where X is a subset of R^n and is in the domain of the real-valued functions, f, g and h. The relations, g(x) <= 0 and h(x) = 0 are called *constraints*, and f is called the *objective function*. To place the LCAC optimization problem – as stated within the Statement of Work – in Standard Form, the following notation will be used:

R = Fleet readiness

U = {All Feasible System Upgrades "u"}

S = {All Feasible Spare Parts Strategies "s"}

M = {All Feasible Maintenance and Repair Procedures "m"}

B = Budget

The Standard Form for this problem is:
Maximize R = R(u,s,m)

[20] **See: http://glossary.computing.society.informs.org/index.php?page=nature.html** . It is well known that the standard form for the constrained optimization formulation can readily be transformed into a standard form for the unconstrained formulation by appropriately moving the constraint(s) into the objective function. This is the procedure used in ProOpta.

$\ni$: $u \in U$, $s \in S$, $m \in M$, and $c_{u,s,m} \leq B$,

where

$c_{u,s,m}$ is the cost to implement scenario (u,s,m)

The ProOpta software uses a Genetic Algorithm[21] to estimate the allocation of financial resources among the feasible upgrades, spare parts strategies, and maintenance procedures in order to maximize fleet readiness. The application of Genetic Algorithms to optimize system reliability has been documented in (5), (7), (8), and (9).

Three of the more important assumptions needed for the ProOpta computation to provide valuable results are...

(1) The past is prologue (i.e., the availability of the LCAC craft in the future can be approximated by the availability of the LCAC craft in the past),
(2) Subject Matter Experts can predict – with reasonable accuracy – the impacts that investments in upgrades, spare parts strategies, and changes to maintenance procedures have on the availability of the craft within the LCAC fleet, and
(3) The fitness function used by the Genetic Algorithm within the ProOpta software provides a good approximation to the constrained, non-linear optimization problem articulated above.

Each of these assumptions warrants exploration in the context of the *validity* of the ProOpta calculation. The validity of calculations provided by computer models is a topic that has been explored for many years. A valuable paper that provides a sound overview of the topical area of model validation is [12].

The notions of class readiness and site readiness were introduced earlier in this paper. Recall that these measures may be important if the analyst needs resolutions in either of these dimensions. To better understand the potential value of these measures consider two scenarios, say σ_1 and σ_2, for which the decision maker is planning. It is conceivable that σ_1 and σ_2 may require a different minimum number of LCAC to be provided from each site. For example, scenario σ_1 may occur nearer to the geographic location of ACU-4 than to ACU-5 thereby requiring more LCAC to be available from ACU-4 than from ACU-5. A similar, albeit inverse, argument may be made for σ_2 necessitating more LCAC be available from ACU-5 than from ACU-4. Using the notation:

$$ {}_{\sigma_i}N_{j,k} $$

[21] A genetic algorithm is a search technique used to find approximate solutions to optimization and search problems. Specifically it falls into the category of local search techniques and is therefore generally an incomplete search. (See: http://en.wikipedia.org/wiki/Genetic_algorithm). A good on-line introduction to Genetic Algorithms can be accessed at: http://www.rennard.org/alife/english/gavintrgb.html

for the minimum number of LCAC of class “j” that are required if site “k” is to be ready to respond to scenario σ_i then we have…

$${}_{\sigma_1}N_{j,ACU-4} > {}_{\sigma_1}N_{j,ACU-5}$$

while

$${}_{\sigma_2}N_{j,ACU-4} < {}_{\sigma_2}N_{j,ACU-5}$$

Similarly, if the distinction in types of equipment on board the individual craft is important in understanding the mission capable status in different mission scenarios then the Class Readiness may provide valuable information.

The important point is that if the decision maker chooses to understand the sensitivity of fleet readiness in the context of site or class readiness, the information needed to conduct these analyses is available within the LCAC data sets. The only additional information that is needed to conduct this analysis is the minimum number of LCAC that is required per class, per site, per scenario; i.e., ${}_{\sigma_i}N_{j,k}$.

Commentary

Summary

The Landing Craft Air Cushion is a high-speed, over-the-beach, fully amphibious landing craft capable of carrying a 60-75 ton payload. The LCAC fleet can serve to transport weapons systems, equipment, cargo and personnel from ship to shore and across the beach. This transport system is an integral part of our military arsenal and, as such, its *readiness* is an important question for our national security. Further, the best way to expend financial resources that have been allocated to maintain this fleet is a critical consideration.

Sandia has been heavily involved in modeling, simulation, and optimization of system reliability, maintainability and availability for many years. A key question is the connection between *Fleet Readiness* and the concept of LCAC craft *availability*. This report provides the mathematical framework for that connection. It is shown that not only can Sandia's ProOpta provide valuable information in the estimation of Fleet readiness using extant failure and repair data, but also this data can be mined for additional information that is of significant value. Further, it is likely that ProOpta can provide guidance regarding investment strategies that result in fleet readiness levels that are superior to levels that are achievable without the supporting analysis.

Conclusions

There is a clear coupling between the measure of Fleet Readiness as defined by the customer for this project and the information that is provided by Sandia's ProOpta methodology. This coupling has been documented in this report. Further, there is a richness in the data that provides even more value to the analyst. In particular, the probability density function(s) that characterizes the number of LCAC craft that are available can be used to explore various important sensitivities of overall fleet readiness to variables such as the…

- availability of the hypothetical representative craft,
- relative degrees of site readiness, and
- differences in (equipment) class readiness.

Key issues that warrant further exploration include…

- fidelity of failure rates,
- fidelity of "repair" rates,
- accuracy of estimates provided by subject-matter-experts that investments in improved upgrade, repairs, spare parts, and maintenance strategies have on availability on LCAC craft,

- the validity of fitness function used by the Genetic Algorithm within the ProOpta software as a good approximation for the constrained, non-linear optimization problem articulated above, and
- scenario-based analysis that provides the analyst with the capability to explore the effects that different investment portfolios have on fleet readiness as a function of various hypothetical, yet plausible, threats.

References

[1] Wright, Donald L., *Nuclear Weapon Reliability Evaluation Methodology*, SAND93-0704, June 1993

[2] Campbell, James E., Dennis E., Longsine, and Joel Atkins, *Algorithms for treating redundancy in reparable and non-reparable systems*, SAND93-1822.

[3] Painton, Laura, A., and James Campbell, *Identification of components to optimize improvement in system reliability*, SAND93-2420C.

[4] Cutler, Robert M., *Spacecraft Availability Enhancement by In-Flight Testing of Spare Parts*, AIAA Paper 95-0938, January 1995.

[5] Painton, Laura A., and James E. Campbell, Genetic Algorithms in Optimization of System Reliability, SAND95-0381J.

[6] Lalli, Vincent R., and Michael H. Packard, *Designing for Maintainability and System Availability*, NASA-TM-107398, January 1997.

[7] D.W. Coit and Smith, A.E, *Reliability optimization of series-parallel systems using a genetic algorithm*, IEEE Transactions on Reliability, June 1996, Volume: 45, Issue: 2

[8] Painton, Laura A., and James E. Campbell, *Optimization of reliability allocation strategies through use of genetic algorithms*, SAND96-1998C.

[9] Chen R.-S., C.C. Chiu, and Y.S. Yeh, *A Genetic Algorithm for the Reliability Optimization of a Distributed System*, *dexa*, p. 484, 9th International Workshop on Database and Expert Systems Applications (DEXA'98), 1998.

[10] Wyss, Gregory Dane, and Felicia Angelica Duran, *Object-based approaches to risk and reliability analysis: OBEST methodology and application*, SAND2002-0670C, March 2002

[11] Painton, Laura A., WinR: A system reliability modeling and analysis tool, SAND96-1999C, August 1996.

[12] Sargent, Robert G., Verification and Validation of Simulation Models, Proceedings of the 2003 Winter Simulation Conference.

[13] Database Name: OARS (Open Architectural Retrieval System)
Database Owner: NAVSEA (Naval Sea Systems Command)
URL: https://mechctx.nslc.navy.mil/OARS/Citrix/auth/login.aspx (requires authetication certificate to view)

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