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## **Chain Modeling for Life Cycle Systems Engineering**

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## Chain Modeling for Life Cycle Systems Engineering

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### Abstract

Throughout Sandia's history, products have been represented by drawings. Solid modeling systems have recently replaced drawings as the preferred means for representing product geometry. These systems are used for product visualization, engineering analysis and manufacturing planning.

Unfortunately, solid modeling technology is inadequate for life cycle systems engineering, which requires maintenance of technical history, efficient management of geometric and non-geometric data, and explicit representation of engineering and manufacturing characteristics. Such information is not part of the mathematical foundation of solid modeling. The current state-of-the-art in life cycle engineering is comprised of painstakingly created special purpose tools, which often are incompatible.

New research on "chain modeling" provides a method of chaining the functionality of a part to the geometric representation. Chain modeling extends classical solid modeling to include physical, manufacturing, and procedural information required for life cycle engineering. In addition, chain modeling promises to provide the missing theoretical basis for Sandia's parent/child product realization paradigm. In chain modeling, artifacts and systems are characterized in terms of their combinatorial properties: cell complexes, chains, and their operators. This approach is firmly rooted in algebraic topology and is a natural extension of current technology. The potential benefits of this approach include explicit hierarchical and combinatorial representation of physics, geometry, functionality, test, and legacy data in a common computational framework that supports a rational decision process and partial design automation. Chain modeling will have a significant impact on design preservation, system identification, parameterization, system reliability, and design simplification.

## **Acknowledgments**

The authors owe a special thanks to Steve Hudson for providing the current design, design history, engineering, and manufacturing information of a rolamite; and Yvonne Martinez for researching and identifying the life cycle considerations of a rolamite.

## Table of Contents

Introduction .....	1
Current Technical Approach .....	1
General Chain Model Approach.....	2
Solid Model.....	3
Elementary Chain Models.....	3
More General Chain Models.....	4
Rolamite .....	5
Chain Model of the Band.....	6
Representing Functionality .....	8
Functionality .....	9
Structural Refinement.....	10
Conclusions and Future Directions .....	13
Appendix A .....	15
Life Cycle Systems Requirements .....	15
References .....	20
Distribution .....	21

## Introduction

Geometry is used to describe mechanical devices and to support all life cycle system engineering disciplines, including conceptual design, detailed design, manufacturing, assembly, testing, inspection, and field maintenance. In the past this approach was justified because shape is relatively easy to describe using drawings. Until recently, engineering drawings also contained a variety of textual and symbolic information related to functionality and manufacture of the product. Information such as tolerances, material properties, process characteristics, standard components, and record of changes were shown on the drawing.

Engineering standards now specify that non-geometric information should not appear on the drawings[1]. This practice is consistent with the modern view that mechanical form is induced from function and fabrication follows form[2]. This allows for greater flexibility in manufacturing methods, but impacts design negatively in that:

- drafting, descriptive geometry, geometric modeling, and computer-aided design have become specialized engineering skills,
- omitted non-geometric information has led to poor communication between design and manufacturing, thus increasing lead-times, the number of iterations, and product cost, and
- it is easy to design geometric shapes that are difficult or impossible to manufacture.

The emergence of new disciplines such as “Design for X” is a result of the dominance of geometric information. Each of these disciplines are built on the premise that geometry provides the foundation for all engineering activities. The rise of solid modeling indicates that there are geometric features which correspond to physical mechanical parts. Features are specific to each discipline. With the emergence of feature-based solid modeling, the functionality of each discipline is tied to the form of the solid model.

## Current Technical Approach

Feature-based modeling captures and communicates more engineering information than just geometry. Features are not formally defined, but features capture and represent engineering knowledge and function and do not explicitly include any physical information. Features are named, typed geometric information structures that have to be combined with function and fabrication models. Geometric models created using features support a variety of spatial computations, but they do not explicitly represent general phenomena, such as heat, stresses, or magnetic flux. These quantities are not part of the assumed mathematical model.

Graph-based and schematic representations capture some interaction of mechanical form and function, but they do not deal with spatial representations of material or other

physical quantities. Heuristics are an important component of these representations for form-function interaction. It is difficult to model mechanical function in a manner that links function to form[2]. The process of inducing form from function is viewed as a creative activity that must be performed by humans.

In the area of form-fabrication, models of manufacturing processes represent mathematical problems of physics, but these models are quite complex and offer little insight into selection of a manufacturable form. Manufacturing process information is dominated by rules and tables that divide all manufacturing process plans into a finite number of categories, based on factors like tolerances, material types, gross dimensions, presence or absence of certain features, material thickness, and production volume. Traditional manufacturing methods are often dominated by tooling activities and small changes in design parameters can easily lead to large changes in tooling costs. Models of form-fabrication interaction can be incompatible with parametric definitions of functional form.

## **General Chain Model Approach**

We propose that the proper formal setting for systematic exploration, organization, and modeling of structural information is combinatorial or algebraic topology. General topology constructs called chains subsume the usual concepts of graphs, skeletons, sketches and meshes. Algebraic topological concepts, in particular chains, play an important role in the formulation of solid modeling[3][9]. Thus, it may be possible to formalize and unify many of the heuristic modeling methods mentioned above. For example, it may be possible to establish a direct relationship between schematic, bond-graph and skeletal descriptions of mechanical devices and to methodically transform them into spatially distributed solid models that preserve the same physical structure. Some transforms will be related to the functions of the device, e.g., plate thickness for a desired strength characteristic, and other to a particular manufacturing method, e.g., bend radii. A common structure must be preserved by all transformations.

Chains and structural operations may be used to capture and represent many physical laws of spatially distributed phenomena, such as conservation, equilibrium, and balance, in combinatorial terms that are independent of a particular choice of geometric representation. Initially a chain model does not represent the geometry of a given solution. It can be used to represent the functionality as a class of solutions. During the design process, this model can be systematically transformed into a geometry-embedded detailed chain model that corresponds to the final solid model. Each successive transformation involves refining cell decompositions and redistributing physical quantities, such as forces, displacement, and energy[3]. In Sandia parlance, an abstract chain model represents a “Parent” design and a geometrically-embedded detail chain model represents a “Child” design[4]. Each transformation must preserve the functionality of the part and take into account geometric characteristics of the manufacturing process. The geometric solution may not be unique, but production costs may help identify the most economical geometric design.

### Solid Model

Chain modeling works with the boundary representation (b-rep) of a solid model. The entities of a boundary representation are faces, edges, and vertices. Figure 1 shows the faces, edges, and vertices of a simple solid.

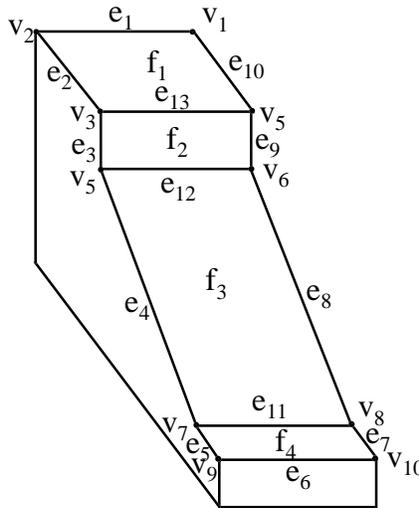


Figure 1. Simple solid model

Each of these entities are referred to as a cell. The combinatorial relationships of these cells are captured in a cell complex,  $K$  [5]. Geometric information is embedded for every cell to define the solid. A vertex is referred to as a 0-cell and can be oriented positively or negatively. Edges are referred to as 1-cells bounded by two vertices,  $m$  and  $n$ , and the orientation is determined by the order of  $m$  and  $n$ . 2-cells or faces can be oriented either clockwise or counterclockwise and are bounded by 1-cells or edges. Refer to Figure 2 for the cell map of four surfaces of the simple solid show in Figure 1.

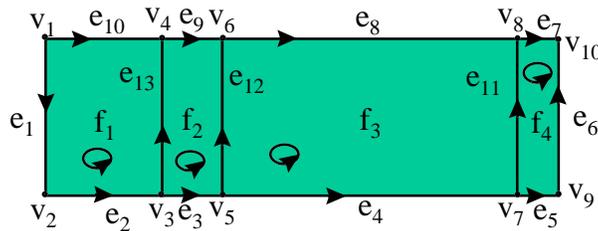


Figure 2. Cell map of simple solid model

### Elementary Chain Models

In classical solid modeling, chains are used to define an oriented boundary of a  $p$ -cell. Intuitively, it links the boundaries of the  $(p-1)$ -cells which make up the  $p$ -cell. For example, the boundaries of a face (2-cell) are the edges (1-cells) surrounding the face. More precisely, the boundary of a  $p$ -cell,  $\partial$ , is the  $(p-1)$ -chain consisting of all  $(p-1)$ -cells

that are faces of  $\partial$  with a +1 coefficient if the orientation of  $\partial$  is consistent with the orientation of the face and -1 otherwise.

For the following example, refer to Figure 2. The boundary of a 0-cell, vertex, is 0. An edge is a 1-cell, therefore the oriented boundary is a 0-chain:

$$\partial e_1 = v_2 - v_1$$

$$\partial e_2 = v_3 - v_2$$

This implies that  $e_1$  starts at vertex,  $v_1$  and ends at vertex,  $v_2$ . The oriented boundary of 2-cell,  $f_1$ , is 1-chain:

$$\partial f_1 = e_1 + e_2 + e_{13} - e_{10}$$

which tells us that the direction of the edges  $e_1$  and  $e_2$ , are consistent with the counterclockwise direction of  $f_1$ . The boundary of the surfaces is a 1-chain that is obtained by adding the 1-chains of the individual faces:

$$\begin{aligned} \partial F = \partial(f_1 + f_2 + f_3 + f_{10}) = & e_1 + e_2 + e_{13} - e_{10} + e_3 + e_{12} - e_9 - e_{13} + e_4 + e_{11} - e_8 - e_{10} + e_5 + e_6 \\ & - e_7 - e_{11} \end{aligned}$$

$$\partial F = e_1 + e_2 + e_3 + e_4 + e_5 + e_6 - e_7 - e_8 - e_9 - e_{10}$$

More generally, for an oriented cell complex,  $K$ , consisting of  $n$  oriented cells,  $\sigma_i$ :

$$C = a_1\sigma_1 + a_2\sigma_2 + a_3\sigma_3 + \dots + a_n\sigma_n$$

which is a  $p$ -chain, where  $a_i$  are integer coefficients. The boundary of  $C$  is a chain:

$$\partial C = a_1\partial(\sigma_1) + a_2\partial(\sigma_2) + a_3\partial(\sigma_3) + \dots + a_n\partial(\sigma_n)$$

This means that the oriented boundary of  $C$  is a chain which is equal to the summation of the oriented boundaries of each of the cells included in the cell complex.

### **More General Chain Models**

The general chain models are straightforward generalizations of the elementary chain models used in solid modeling and described above. The main difference is that coefficients of the general chain models include scalar, complex, vector and other values that can be added to or subtracted from each other. The coefficients can be attached to the  $p$ -cells of the same cell complex, forming a  $p$ -chain. General chains also allow coboundary operations. For formal definitions of chains, boundary and coboundary operations, as well as examples of their use in modeling, the reader is referred to [3]. Below we show how the concepts of chain modeling can be used to capture functional

and spatial decomposition of the rolamite, a precision roller-band mechanism with a wide range of applications, especially in instruments and miniature mechanical and electromechanical devices.

## Rolamite

The most common rolamite configuration consists of a flexible band and two rollers mounted between two fixed, parallel guide surfaces, as shown in Figure 3. As an acceleration switch, the rollers provide a sensing mass, and the flexible band provides a spring force. The cluster roller, comprised of the actuate roller and reset roller, can roll back and forth between the guides, along its “sensing axis.” As a force (acceleration) is applied to the rolamite, the rollers overcome the spring force of the band and roll or wrap along the band toward the actuate endcap. When the actuate roller reaches the actuate endcap, electrical contact is made between the contact-block subassembly and the pins. As deceleration occurs, the rollers roll back away from the actuate endcap toward the reset endcap and electrical circuit is opened. Damping fluid is contained within the case to filter out the effects of vibration and shock on the sensing mass.

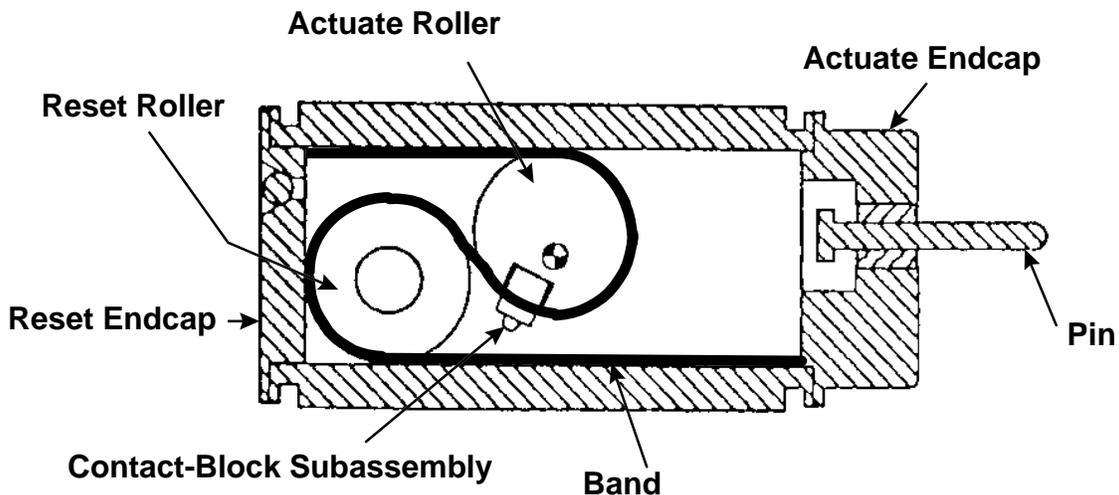


Figure 3. Rolamite

The band is attached to the top of the case near the reset endcap, wraps around the actuate and reset rollers, and is attached in the bottom actuate, endcap-to-case weld. The spring force of a rolamite band is determined by a balance of the strain energy at four locations: the two roller to case transitions of the band and the two transitions at the roller to roller interface. A force cutout in the band, at the top roller-to-case interface, is used to tailor the force deflection characteristics of the band. Because there are certain areas of the band where a break could cause the rolamite circuit to close, a band break is cut out in a location that will not allow the rolamite circuit to close if the band breaks. This ensures that if a break occurs, it will break at the designed location in a fail-safe mode. The contact-block subassembly will remain attached to the band, away from the pins. Also, the band will unwrap to prevent access to the pins of the rolamite’s electrical circuit in the event that the contact-block subassembly should become separated from the band[6].

Two of its most important advantages of the rolamite mechanics are:

- The coefficient of friction is low, even with low normal forces and preloads. The cluster has pure rolling motion with virtually no sliding. Lubrication is normally not required.
- The band can be designed to provide force generation. Cutouts in the band can be used to produce an unlimited variety of force-deflection characteristics.

### Chain Model of the Band

Geometrically, the rolamite band is probably the most complicated and delicate part of the rolamite. Its dimensions and shape of cutouts determine the required force profile of the rolamite, its breakage, fail-safe return, and the damping fluid flow. Yet, none of this information is represented explicitly in a computer. Figure 4 shows the geometry of a typical band. We will now show how chain models of the same band can be used to enhance the geometric representation with explicit links to the rolamite functionality and rational life-cycle considerations.

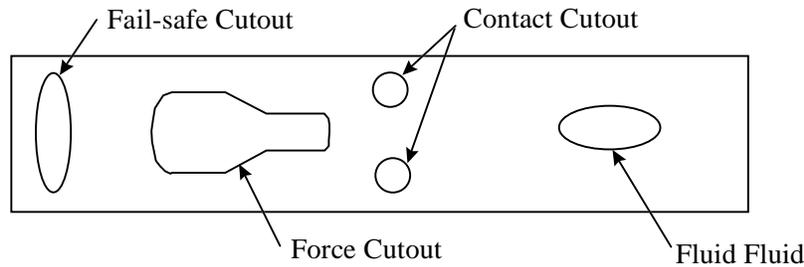


Figure 4. Rolamite band

At the first level of abstraction, the band is decomposed into four sections corresponding to its boundary representation. See Figure 5. Notice that we simplified all cutouts to be represented by a single edge (loop), indicating that its precise decomposition may not be important.

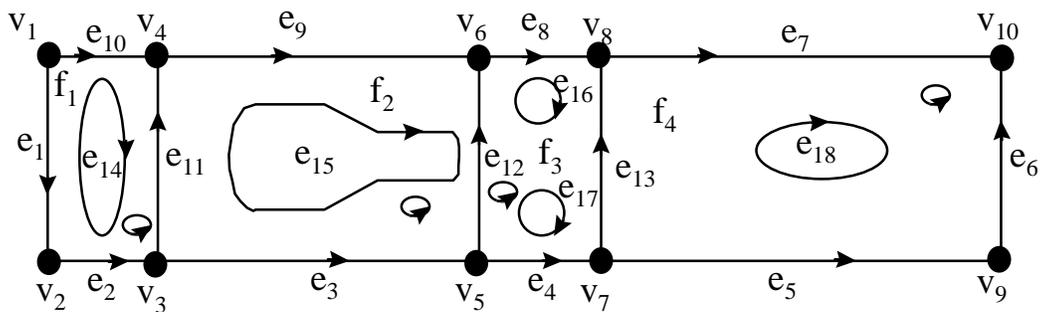


Figure 5. Decomposition of Rolamite band

The data structures corresponding to this decomposition are easily represented by chains that are consistent with the usual boundary representation. Specifically, vertices can be

used to construct corresponding 0-chains, edges used to construct corresponding 1-chains, and faces used to construct corresponding 2-chains.

0-chains:

$$\begin{aligned}
 \partial e_1 &= v_2 - v_1 \\
 \partial e_2 &= v_3 - v_2 \\
 \partial e_3 &= v_5 - v_3 \\
 \partial e_4 &= v_7 - v_5 \\
 \partial e_5 &= v_9 - v_7 \\
 \partial e_6 &= v_{10} - v_9 \\
 \partial e_7 &= v_8 - v_{10} \\
 \partial e_8 &= v_8 - v_6 \\
 \partial e_9 &= v_6 - v_4 \\
 \partial e_{10} &= v_1 - v_4 \\
 \partial e_{11} &= v_4 - v_3 \\
 \partial e_{12} &= v_6 - v_5 \\
 \partial e_{13} &= v_8 - v_7
 \end{aligned}$$

The important stored boundary relationships would include 1-chains:

$$\begin{aligned}
 \partial f_1 &= e_1 + e_2 + e_{11} - e_{10} + e_{14} \\
 \partial f_2 &= e_3 + e_{12} + e_9 - e_{11} + e_{15} \\
 \partial f_3 &= e_4 + e_{13} - e_8 - e_{12} + e_{16} + e_{17} \\
 \partial f_4 &= e_5 + e_6 - e_7 - e_{13} + e_{18}
 \end{aligned}$$

and for 2-chains:

$$\begin{aligned}
 \partial F = \partial(f_1 + f_2 + f_3 + f_4) &= e_1 + e_2 + e_{11} - e_{10} + e_{14} + e_3 + e_{12} - e_9 - e_{11} + e_{15} + e_4 + e_{13} - e_8 - \\
 &e_{12} + e_{16} + e_{17} + e_5 + e_6 - e_7 - e_{13} + e_{18}
 \end{aligned}$$

$$\partial F = e_1 + e_2 - e_{10} + e_{14} + e_3 - e_9 + e_{15} + e_4 - e_8 + e_{16} + e_{17} + e_5 + e_6 - e_7 + e_{18}$$

A more abstract version of the decomposed band is shown in Figure 6. At this level, the shape and location of the cutouts are generalized. Very little if any geometry exists in this version; on the other hand, the same structural relationships (for example, all boundary relationships) apply to the model even at this level. There is a clear relationship between the spatial model and the intended functionality of the band. The connection to functionality becomes explicit at the higher levels of abstraction as illustrated in Figures 7 and 8, where each portion of the band is related to the band's intended functions. In this particular case, the relationship between geometry and functionality is one-to-one, but in general this need not be the case.

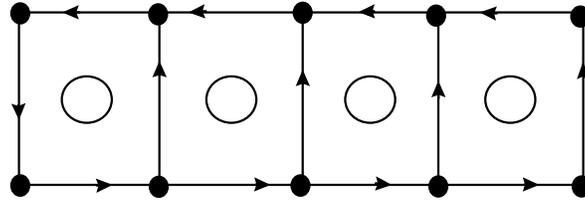


Figure 6. Abstract form of Rolamite band

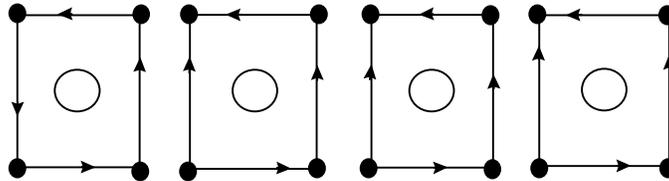


Figure 7. Separation of abstract Rolamite band

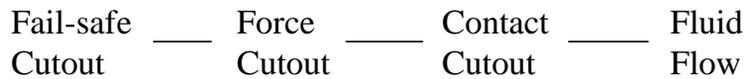


Figure 8. Functional diagram of a Rolamite band

Above, the chains were used for a hierarchical decomposition and representation of the existing design. Reversing the process, similar chain models could be used for a new design starting with specified functions, and systematically transforming them into final geometry through a series of chain mappings, including splitting, merging, collapsing, expanding, embedding and identification of various models. The associated boundary and coboundary constraints would remain valid at all levels of abstraction, enforcing integrity and validity of all the transformations throughout the life-cycle of the device and its computer model.

### ***Representing Functionality***

Neither parametric nor combinatorial models of a solid represent the functionality of the component. We need to find a way of representing the functionality in a computer in a manner that relates the functionality to the combinatorial representation of the geometry. General functionality requirements of the rolamite include [6][7][8]:

1. Closure must occur at a specific force and time lapse.
2. The system must re-open at a specific force and time lapse.
3. The system must remain closed at a specific force and time lapse.

4. Safe break is required.
5. The rolamite must be mountable.
6. Electrical contact is required.

Refer to Appendix A for specific rolamite requirements and additional considerations. We will focus on requirements 1, 2, and 3, which define the forces on the rolamite band with respect to time.

### Functionality

At the highest level of abstraction, the rolamite is a simple mass-spring device; an applied force on an object between time  $t_0$  and  $t_1$  will result on the object moving a distance  $X$  with respect to time. Refer to Figure 9.



Figure 9. Model of Motion

Therefore, at this level of abstraction, it is appropriate to follow the Laws of Physics for a particle in motion. Figure 10 shows the relationship between the usual physical quantities in such a model.

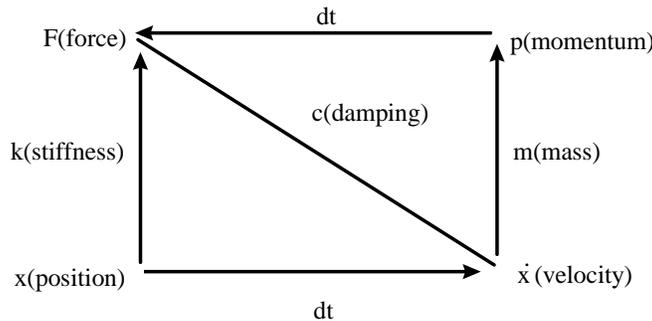


Figure 10. Particle in motion

The function of the rolamite is defined by the relationship of the applied force,  $F_A$ , and the resulting displacement,  $X$ . In Figure 10,  $F(t)$  is the summation of the forces: applied force ( $F_A$ ), band force ( $F_B$ ), fluid force ( $F_F$ ), and inertial force ( $F_i$ ). Dynamic equilibrium implies

$$F_A + F_i + F_F + F_B = 0$$

or

$$F_A(t) + m\ddot{x} + c\dot{x} + k(x)x = 0$$

This equation represents the relationship between  $F(t)$  and  $X(t)$  for the mass-spring system. The same equations can be easily described by general chains of forces attached to an abstract combinatorial graph representing the conceptual decomposition of the above device. In particular, the dynamic equilibrium equation can be simply restated as a coboundary equation of a 1-chain of forces for four coefficients.

This system consists of the rolamite case, the band ( $k(x)$ ), the cluster ( $m$ ), and the fluid. Once the band, the cluster, and the fluid are represented, the model of a rolamite would contain sufficient information to generate and simulate the physics model governing the functionality of a rolamite at this level of abstraction. This physics model, in turn, can be used as a formal specification for a rolamite, or as a virtual prototype for conducting what-if studies and comparison with experimental data.

### Structural Refinement

The cluster and band can be further refined. The cluster consists of two rollers, roller<sub>1</sub> and roller<sub>2</sub>, and is wrapped by the band. The mass model is shown in Figure 11, which indicates that a simple structural (chain) model together with a coboundary relationship can be used in place of the usual syntactic equation:

$$m_{\text{total}} = m_1 + m_2$$

Notice that if additional rollers are added in the cycle, a simple modification of this structural model will automatically update the resulting equation of mass conservation (represented by the corresponding coboundary equation).

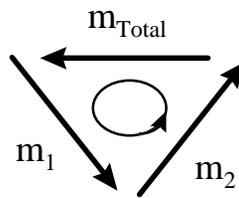


Figure 11. Mass model of rolamite cluster

The band can be modeled by a system of six beams, Figure 12, which are fixed at the end of Beams 1 and 6. The beam is divide into 6 beams because the tangency points of Beams 1 and 2 and Beams 5 and 6 represent the two roller to case transitions of the band and the tangency points of Beams 2 and 3 and Beams 4 and 5 represent the roller to roller interface. Beams 3 and 4 are separate because each of the beams has a slightly different radius of curvature to transition from one roller to the other. The beams can be chained to the functionality of the band developed earlier: Beam 1 contains the fail-safe cutout, Beam2 contains the force cutout, Beam 3 contains the contact cutout, and Beam 5 contains the fluid flow cutout.

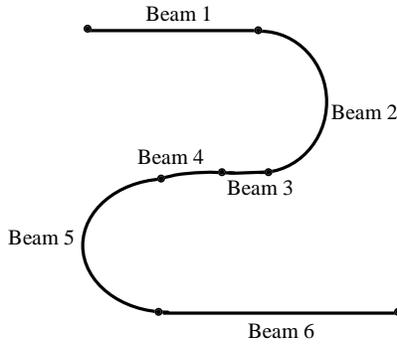


Figure 12. Six beams representing rolamite band

Figure 13 shows the force diagram for each beam which would be normally represented by the appropriate chains of forces and moments. Each beam must be in equilibrium and satisfy kinematic conditions: therefore, two coboundary relationships must be satisfied:  $\partial F = 0$  and  $\delta M = 0$ . Each of the force diagrams is a special case of the more general situation shown in Figure 14, which can be represented by a chain model of the behavior for all such beams and instantiated as any one of the beams shown in Figure 13.

The six-beam model of the band is sufficient to generate a useful model of the band at one time instant. To properly simulate the behavior of a rolamite over time, the equation

$$m\ddot{x} + c\dot{x} + k(x)x = F_A(t)$$

would need to be solved for a new  $x$  at every time step, with  $F_B(x) = k(x)x$  determined by the band configuration at a given time step. The positions of each of the beams can be mapped to the function of the band at a specific point in time of operation. This would be very useful in the simulation of a rolamite and mapping discrete events to geometric shape.

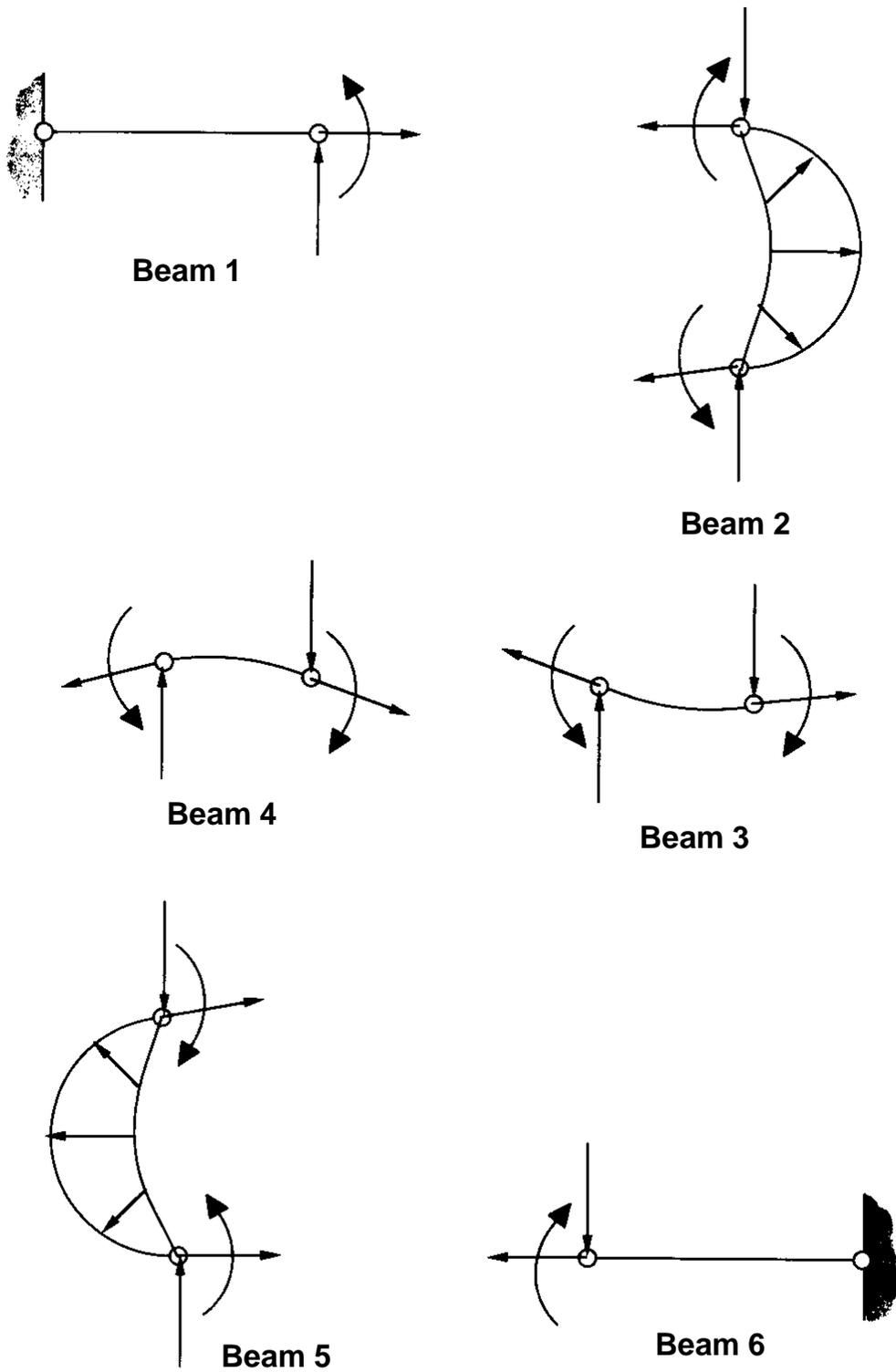


Figure 13. Force diagram of each beam of the rolamite band

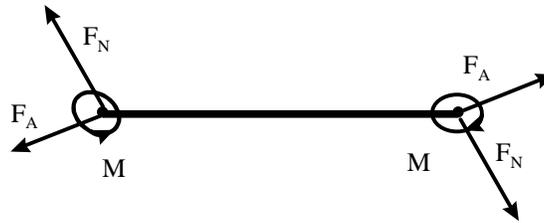


Figure 14. Force diagram of a beam

## Conclusions and Future Directions

The chain models of the rolamite indicate that very simple models using engineering equations can be developed to represent the functionality of the band. Currently, rolamite design decisions are based on experimental data. Chain models are well suited for representing such data as functional relationships between various chains of variables, e.g.,  $x$  and  $F(x)$ . From the existing documentation of the rolamite design, the force behavior of the band does not seem to be well understood. Different reports appear to give only partial and conflicting information. If chain models had been developed at the beginning of the rolamite design process, force behavior would be contained in the model. These models would be consistent and complete. The development of chain models of the rolamite in this phase productions, will clarify the conflicting information and provide documentation of the force behavior.

Transformations of the simple functional chain models to show the variable relationships within the cluster will begin to define the design. The beams of the band can be transformed into a chain representing the entire band. Now the rolamite chain models can be transformed to begin to add geometry, manufacturing, assembly and field use constraints.

The chain models developed for a rolamite are from an existing design for a product that has been in production for years. We need to start with a new design, develop the functional chain models for the parameter variables, and transform these models to add geometric constraints. Research into this work would prove very beneficial as to the usefulness of chain modeling.

Once the usefulness of chain modeling is established, simple computer-representable chain models could be developed to represent partial functionality of many products and devices. Such models would support automatic generation of physical simulation of the devices, as well as provide explicit links to geometric embodiment of the device. For example, the physical model of the rolamite as a mass-spring system would incorporate a functional description of the band as a moving system of beams. The integral properties of the beams could be mapped to functional portions of the band, as illustrated in this report. All of these models and transformations can be achieved within the same computational framework using chain modeling. Furthermore, since chains basically represent distribution of physical quantities in space, they are well suited to represent measured behavior and experimental data. Thus, it is also likely that analytic and

experimental data can be combined with analytic and geometric data within the same work. Similarly, chain models can be further enhanced to include geometry, manufacturing, assembly, and field use constraints.

Continued research in the area of chain modeling theory for life cycle engineering will encompass and extend solid modeling theory and Sandia's parent/child design theory. Chain models will have a major impact on the life cycle of a product:

- Design Preservation                      Collection of functionality and mapping through chains to final design
- System Identification                    Identification of variables that can change depending on the application
- Parameterization                        Identification of specific design parameters
- System Reliability                      Identification of possible failure modes and the weakest link in the design (physical properties)
- Simplification                            Identification of the features that are important and which ones are not necessary and can be removed for analysis

Chain modeling can become a vital part of a product design process.

## Appendix A

The following requirements were identified from rolamite documentation [6][7][8].

### *Life Cycle Systems Requirements*

- Effective roller radii and guide spacing measured to the neutral axis of the band for small deflection loose geometry are:

$$R1 = (d1 + t)/2$$

$$R2 = (d2+t)/2$$

$$S = sg+t$$

where R = effective roller radii  
d = roller diameter  
t = band thickness  
sg = guide surface spacing

- Curvature for slightly loose geometry is:

$$1/\rho = M/EI = d^2y/dx^2$$

- Bending moment is:

$$M(x,y) = Nx + Ty$$

- If none of the angles  $\theta_B$  (i.e.  $\theta_1, \theta_2, \theta_3,$  or  $\theta_4$ ) is greater than about 6 degrees the following equations give good results for rolamite geometry:

$$\delta = M_B L^2 / (3EI) \qquad \theta_B = M_B L / (2EI)$$

where  $\delta$  = maximum deflection  
 $\theta_B$  = slope angle  
L = length of band in a transition zone or beam  
E = elastic modulus for band material  
I = moment of inertia for band; for rectangular band,  $I = wt^3/12$

- From these equations, for small deflections:

$$\theta_B = L / (2R) \qquad \delta / L = 2 * \theta_B / 3$$

$$\delta C = 3/8 * \delta \qquad \delta H = \delta / 2$$

$$\delta_s = \delta/8$$

where  $\delta C$  = chord distance

$\delta H$  = vertical distance from root B to tip A

$\delta_s$  = net clearance or “air gap” between roller and guide

- For symmetric cases, equal diameters and a neutral band, the net clearance between rollers is approximately the beam deflection:

$$\delta_{RN} \approx \delta$$

- Clearance or “air gap” between rollers  $\delta_{RN}$  is about 8 times the clearance  $\delta_s$  between either roller and the guide:
- Net Clearance between rollers:

$$\delta_{RN} \approx 1/(3R^3)[EI/(T \cdot \tan \gamma)]^2$$

where  $\gamma$  = contact angle (also called force angle)

- For Large Deflection Nonsymmetric Geometry, curvature is:

$$1/\rho = M(x,y)/EI(s) = d\theta/ds$$

- Assume all external and inertia forces act through roller centers and parallel to the guide surfaces, and are in equilibrium with the band-generated force:

$$F_1 + F_2 = F$$

- Assume band is not precurved and has constant thickness (t) and modulus (E)
- Maximum bending strain in band is:

$$\epsilon_b = t/2R$$

- Strain due to band tension is:

$$\epsilon_t = T/Ewt$$

where T = band tension

w = band width

- Bending strain in band must be below the yield point  $\sigma_y$ :

$$\epsilon_b = t/2R \leq \sigma_y/E$$

- Guide spacing ratio for symmetric geometry is:

$$S/R = 2*(s_g - t)/(d+t)$$

- Tight geometry with high band tension.
- Roller diameters must be chosen so rollers fit properly between the guides.
- Band thickness must be limited to prevent yielding of band material when bent to roller curvature.
- Wrap angle  $\beta$  must be greater than 180 degrees.

- Guide spacing is:

$$S/R = 4*(\sin \gamma)^2$$

- To generate a force, the band stiffness or flexural rigidity,  $EI$ , may be varied by varying the elastic modulus  $E$ , the band width  $w$ , or the thickness  $t$ .
- The elastic strain energy stored per unit length, in a band subjected to pure bending is:

$$dU/ds = M^2/2EI$$

- To bend the band to the roller radius, a bending moment is required

$$M = EI(1/R \pm 1/\rho)$$

where  $\rho$  = precurvature radius measured to the center of the band

(+) sign for roller 1

(-) sign for roller 2

- Geometry is tight and band generates a force by a combination of precurvature and varying width.
- If cluster moves a small distance to right and static equilibrium is maintained by force:

$$F = -dU/dz$$

where  $-dU/dz =$  rate at which energy is absorbed by band

- Strain energy is:

$$U = \int_0^Q M^2(s)/(2EI(s))*ds$$

- To avoid discontinuities, strain energy is:

$$U = \int_{SO}^{SA(z)} M^2(s)/2EI(s)*ds + \int_{SA}^{SP} M^2(s)/2EI(s)*ds + \int_{SP}^{SD(z)} M^2(s)/2EI(s)*ds \\ + \int_{SD(z)}^{SG(z)} M^2(s)/2EI(s)*ds + \int_{SG(z)}^{SQ} M^2(s)/2EI(s)*ds$$

- If band is precurved, it may be forced to lie flat on the guides, thus

$$M(s) = EI1/2(1/R1^2 + 2/(R1*\rho1)) - EI2/2(1/R1^2+2/(R1*\rho2)) + \\ EI3/2(1/R2^2-2/(R2*\rho3)) - EI4/2(1/R2^2 - 2/(R2*\rho4))$$

- To maximize force generated for a given cross sectional area normal to sensing axis  $A_c$ , effective cross sectional area  $A_e$  should be close to  $A_c$ .
- Bending stress should be near yield point.
- Ratio  $\sigma_y^3/E^2$  is an index of the force generation potential of the band material.
- For tight geometry, if thickness ( $t$ ) and elastic modulus ( $E$ ) are constant:

$$I(SD) = I2 = I3 \\ \rho(SD) = \rho2 = \rho3$$

$$F = Et^3/24[(w1-w2)/R1^2+ (w3-w4)/R2^2]$$

- If initially flat, precurvature  $\rho$  is infinite and  $F$  is:

$$F = Et^3/24[(w1-w2)/R1^2 + (w3-w4)/R2^2 ]$$

- For  $\rho =$  constant, equal roller diameters and constant width, no force will be generated.

- For equal diameter rollers and  $w2 = w3$  :

$$F' = Et^3/(24R^2)*(w1-w4) < \sigma_y^3*A_e/(6*E^2)$$

where  $A_e = 2R(w_1 - w_4)$

$A_c = s_g * w_c$

$w_c =$  internal case width

- Typical Spring material is  $F/A_e = 2.5$  psi.
- Required to provide an open circuit after exposure to shocks of up to 10,000 g's and 1-ms in duration in any direction.
- Must close at spin rates of  $< 2.25$  rps ( $\approx 2.30$  g's).
- Once closed, must reopen before acceleration drops below 1.1 g's.
- Maximum and minimum storage and operation environments are 77 °C and -55 °C.
- Thermally shocked between 77 °C and -55 °C.
- Manufactured and tested at 1 atm.
- Exposed to relative humidities between 0 and 60% during normal processing and testing.
- Parallelism and perpendicularity of fixture rollers are critical.
- Band force trace should mirror shape of force cutout in band.
- Roller concentricity is critical.
- Roller spacer press fit.

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