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## The Role of Noise in Analog-to-Digital Converters

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# The Role of Noise in Analog-to-Digital Converters

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## Abstract

Because of the widespread use of digital systems in radars, instrumentation, and communication systems, an understanding of the role played by noise at the input to the analog-to-digital (A/D) converter is important. When digital signal processing is performed on the output of the A/D, it is crucial that the A/D respond linearly to the signal. The noise level at the input of the A/D is a determining factor for the linearity of the system. Many texts discuss the operation and performance of analog-to-digital converters and, although the understanding of the role of noise is not new, it seems that few, if any, discuss noise from the point of view presented here. This omission appears to lead to a misunderstanding of the importance of noise in these analog-to-digital systems. Single-bit and multiple-bit analog-to-digital converters will be analyzed, and it will be shown, that with the appropriate noise level at the input, even the single-bit converter can behave as a linear device. An example will be described whereby a "feature" of a particular commercial instrumentation system was based on a misunderstanding of the role of noise, and the use of this "feature" caused serious degradation of the system linearity and performance.

### Acknowledgment

Understanding of a subject is always enhanced, and misconceptions are often corrected through discussions with learned colleagues. I wish to thank G. K. Froehlich, J. T. Cordaro, and B. L. Burns for the many fruitful discussions I have had with them regarding this and similar topics. Inspiration is also an important contributor to understanding, and I wish to thank Scientific Atlanta for inspiring me to understand how noise affects the analog-to-digital converter.

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## Introduction

Because of the widespread use of digital systems in radars, instrumentation, and communication systems, an understanding of the role played by noise at the input to the analog-to-digital (A/D) converter is important. Naturally, one wishes to achieve a good signal-to-noise ratio (SNR). For example, in a radar receiver, the signal-to-noise ratio of the final signal (after all processing) is the quantity to be maximized. Signal strength is only important to the extent that it improves SNR. Gain is only useful when the signal-to-noise ratio (SNR) is already adequate because gain alone cannot improve the signal-to-noise ratio. When digital signal processing is performed on the output of the A/D, it is crucial that the A/D respond linearly to the input signal. The noise level at the input of the A/D is a determining factor for the linearity of the system. Many texts discuss the operation and performance of analog-to-digital converters and, although the understanding of the role of noise is not new, it seems that few texts, if any, discuss noise from the point of view presented here. This omission appears to lead to a misunderstanding of the importance of noise in these analog-to-digital systems.

Single-bit and multiple-bit analog-to-digital converters will be analyzed, and it will be shown, that with the appropriate noise level at the input, even the single-bit converter can behave as a linear device. The number of bits required in multiple-bit A/D will also be discussed. An example will be described whereby a “feature” used to extend the dynamic range of a particular commercial instrumentation system was based on a misunderstanding of the role of noise, and the use of this “feature” caused serious degradation of the system linearity and performance.

To achieve best performance, the total analog gain for a digital receiver should be determined by the input noise power. Specifically, the gain should be chosen so that the rms noise voltage is about the voltage associated with the least significant bit of the analog-to-digital converter (A/D); the specific recommended range for the noise voltage will be detailed below. Although system linearity is determined by the actual level of the input noise voltage, nothing is gained by placing the noise voltage much higher than the least significant bit of the A/D. However, if the noise voltage is significantly lower than the least-significant-bit voltage, the system performance will be limited by quantization (at the level of the least significant bit) and nonlinearity errors. The statistics (mean, standard deviation) of the output of the A/D will be nonlinear functions of the statistics of the input signal *unless the noise level is sufficient* to toggle the least-significant bit of the A/D.

Intuitively, one would suspect that averaging (integration or other signal processing) of the output of the A/D to detect weak signals buried in the noise would be successful only if the output changed from sample to sample. It is also true, but not intuitively obvious, that if the noise level is insufficient to allow integration to improve the signal-to-noise ratio, the response of the A/D to signal levels much higher than the least-significant bit (signals for which the signal-to-noise ratio is high) will be nonlinear. This fact does not seem to be well known. For example [1] states that the rms noise voltage input to the A/D should be less than half the least significant bit (more than  $-6$  dB from the least

significant bit). While acceptable performance may be obtained in many circumstances with this recommendation, the errors in the A/D output will be dominated by quantization error, and the statistics of the A/D output will exhibit nonlinear behavior when this recommendation is followed.

In order to examine the role of noise in analog-to-digital converters, a one-bit A/D is examined first. Expressions for the mean and standard deviation of the output of the A/D are derived in terms of the mean and standard deviation of the input signal. It will be shown that the one-bit A/D is only linear when used to measure signal levels significantly *below* the noise level. A similar analysis is next applied to a multiple-bit analog-to-digital converter. In this case, it will be shown that linear performance is obtained when the signal standard deviation (noise) is about equal to the least-significant bit voltage. The choice of the number of bits in the A/D is then addressed. Finally, the futility of using overlapping A/D channels to extend the instantaneous dynamic range is demonstrated.

### Single-bit quantizer:

Consider a two-state (one bit) quantizer

$$q(v) = \begin{cases} -1 & \text{for } v \leq 0 \\ 1 & \text{for } v > 0 \end{cases} \quad (1)$$

Let  $v$  be a Gaussian random process with mean  $v_0$ , and standard deviation  $\sigma_v$ . The expected value (mean) of a large number of measurements of  $v$  with the quantizer will be

$$\bar{q} = \frac{1}{\sigma_v \sqrt{2\pi}} \int_{-\infty}^{\infty} q(v) e^{-\frac{(v-v_0)^2}{2\sigma_v^2}} dv = \frac{1}{\sigma_v \sqrt{2\pi}} \left( - \int_{-\infty}^0 e^{-\frac{(v-v_0)^2}{2\sigma_v^2}} dv + \int_0^{\infty} e^{-\frac{(v-v_0)^2}{2\sigma_v^2}} dv \right). \quad (2)$$

The Gaussian probability integral  $P(x)$  with mean  $m$  and standard deviation  $\sigma$  is defined

$$P(x) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{(t-m)^2}{2\sigma^2}} dt, \quad (3)$$

and has the properties

$$\begin{aligned} \lim_{x \rightarrow \infty} P(x) &= 1 \\ P(m-x) &= 1 - P(m+x) \end{aligned} \quad (4)$$

Also,

$$\frac{1}{\sigma \sqrt{2\pi}} \int_x^{\infty} e^{-\frac{(t-m)^2}{2\sigma^2}} dt = 1 - P(x). \quad (5)$$

The Gaussian probability integral (3) can be written in terms of the error function [2]

$$P(x) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{x-m}{\sigma \sqrt{2}} \right) \right], \quad (6)$$

where [2]

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt, \quad (7)$$

and

$$\lim_{x \rightarrow \infty} \operatorname{erf}(x) = 1. \quad (8)$$

The power series expansion for the error function (good for small argument) is [2]

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{n!(2n+1)}. \quad (9)$$

Since  $\operatorname{erf}(-x) = -\operatorname{erf}(x)$

$$P(-x) = \frac{1}{2} \left[ 1 - \operatorname{erf} \left( \frac{x+m}{\sigma\sqrt{2}} \right) \right], \quad (10)$$

and since  $P(\infty) = 1$ , the mean of the one-bit quantizer measurements is

$$\bar{q} = (1 - 2P(0)) = \operatorname{erf} \left( \frac{v_0}{\sigma_v \sqrt{2}} \right), \quad (11)$$

or

$$\begin{aligned} \bar{q}(v_0) &= \frac{v_0}{\sigma_v} \sqrt{\frac{2}{\pi}} \left[ 1 - \frac{v_0^2}{6\sigma_v^2} + \frac{v_0^4}{40\sigma_v^4} - \frac{v_0^6}{336\sigma_v^6} + \dots \right] \\ &\cong v_0 \sqrt{\frac{2}{\pi\sigma_v^2}} \end{aligned} \quad (12)$$

Thus, the mean (averaged) output of the quantizer will be linearly related to  $v_0$ , when  $v_0 \ll \sigma_v$ . In other words, the averaged output of the quantizer can be modeled as a linear function of the signal  $v_0$ , *as long as the signal-to-noise ratio is small* (much less than unity). Note, though, that the gain of the quantizer depends on the standard deviation,  $\sigma_v$ .

Unfortunately, the single-bit quantizer is not useful for measuring strong signals, because the averaged output becomes very nonlinear for high signal-to-noise ratios. The deviation from linear performance for large signals is clearly indicated in Fig. 1, where the mean output,  $\bar{q}(v_0)$ , of the quantizer has been normalized,

$$\text{normalized quantizer output} = \frac{\bar{q}(v_0)}{v_0 \sqrt{\frac{2}{\pi\sigma_v^2}}},$$

so that zero dB indicates zero linearity error.

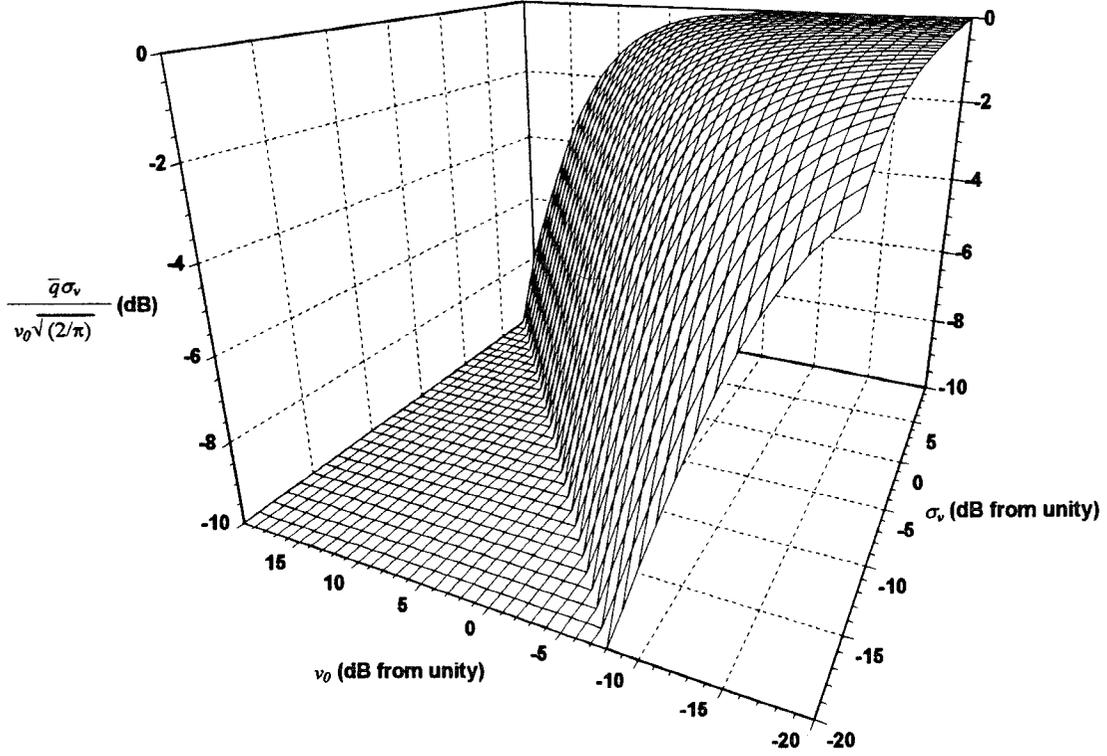


Fig. 1 Normalized output of quantizer as a function of signal,  $v_0$ , and noise,  $\sigma_v$ . Zero dB indicates no error in the mean,  $\bar{q}$ .

In addition, the standard deviation of the output of the quantizer matches that of the signal as long as the signal-to-noise ratio is small enough. The standard deviation,  $\sigma_q$  of the output of the quantizer is obtained from

$$\sigma_q^2 = \frac{1}{\sigma_v \sqrt{2\pi}} \int_{-\infty}^{\infty} [q(v) - \bar{q}]^2 e^{-\frac{(v-v_0)^2}{2\sigma_v^2}} dv = \frac{1}{\sigma_v \sqrt{2\pi}} \left( (1 + \bar{q})^2 \int_{-\infty}^0 e^{-\frac{(v-v_0)^2}{2\sigma_v^2}} dv + (1 - \bar{q})^2 \int_0^{\infty} e^{-\frac{(v-v_0)^2}{2\sigma_v^2}} dv \right) \quad (13)$$

$$= (1 + \bar{q})^2 \operatorname{erf}\left(\frac{v_0}{\sigma_v \sqrt{2}}\right) + (1 - \bar{q})^2 \left(1 - \operatorname{erf}\left(\frac{v_0}{\sigma_v \sqrt{2}}\right)\right)$$

Fig. 2 shows the normalized standard deviation of the output of the quantizer as a function of signal and noise. Only when the signal-to-noise ratio is *small* is the standard deviation of the output the same as that of the input ( $\sigma_q/\sigma_v = 0$  dB).

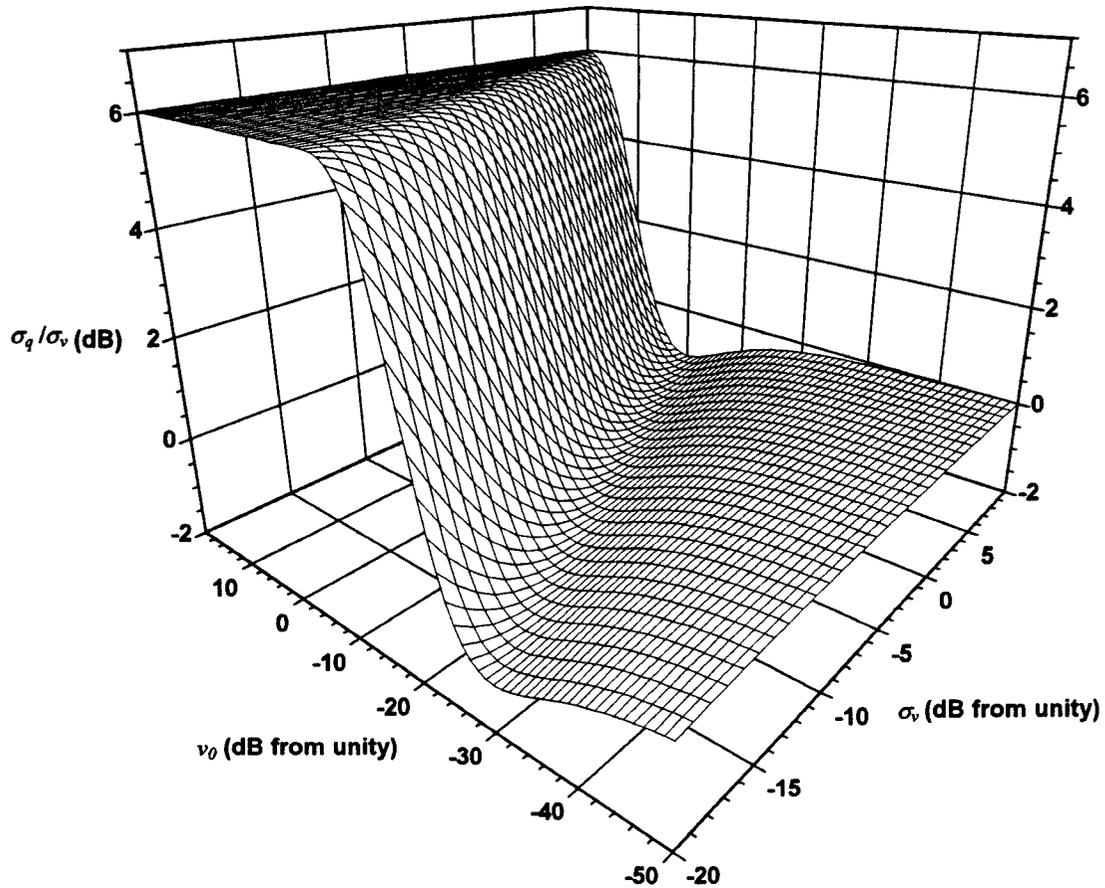


Fig. 2 Standard deviation of the output of quantizer,  $\sigma_q$ , normalized by the signal standard deviation,  $\sigma_v$ , as a function of signal,  $v_0$ , and noise,  $\sigma_v$ .

The one-bit quantizer behaves as a linear device, preserving the mean and the standard deviation of the input *when the signal-to-noise ratio is very small*, but behaves as a *nonlinear* device for large signal-to-noise ratio. Note, to determine  $v_0$  accurately, the gain of the quantizer must be known. Since the gain depends on  $\sigma_v$ , the standard deviation must also be computed from the measurements in order to determine the gain factor. Thus, the output of the one-bit quantizer can be averaged over many samples to provide an accurate estimate of the signal, but only when the signal is much smaller than the noise. This implies that the number of measurements contained in the average must be very large. The standard deviation of the average of  $N$  measurements,  $\sigma_N$  is [3] is

$$\sigma_N = \frac{\sigma_v}{\sqrt{N}}. \quad (14)$$

For each doubling of  $N$ , a 3-dB improvement in the output signal-to-noise ratio is achieved.

### Multiple-bit analog-to-digital converter

Now, consider an  $m$ -bit A/D with a least-significant bit corresponding to  $\Delta v$  volts. With one bit giving the sign, the converter can measure voltages,  $v$ , in the range

$$-(2^{m-1} - 1)\Delta v \leq v \leq (2^{m-1} - 1)\Delta v.$$

The output,  $S$ , of the A/D for an input voltage,  $v$ , is (in units of  $\Delta v$ )

$$S = \text{sgn}(v) \text{int}\left(\frac{|v|}{\Delta v} + \frac{1}{2}\right), \quad (15)$$

where  $\text{int}(x)$  is the nearest integer to  $x$ . The transfer function would appear as indicated in Fig. 3.

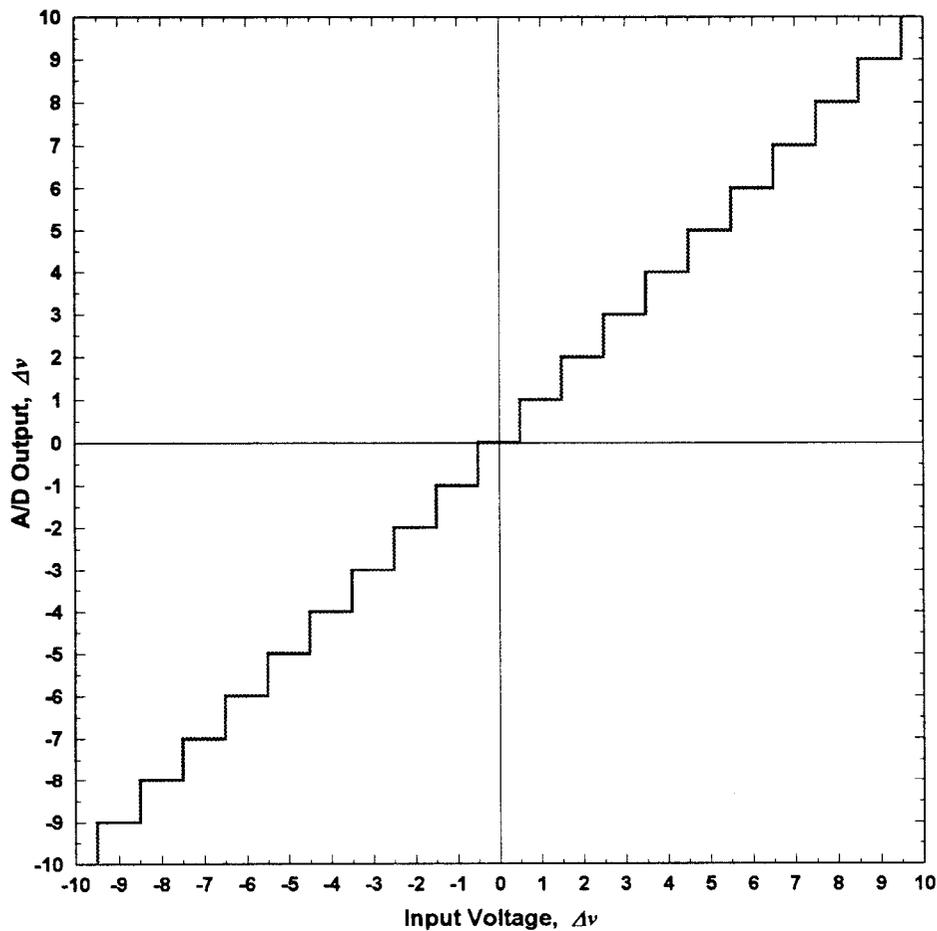


Fig. 3 Transfer function for A/D converter.

Suppose the input voltage is

$$v = v_0 + v_{noise},$$

where  $v_{noise}$  is zero-mean Gaussian with standard deviation  $\sigma_n$ . The probability of the output count of the A/D being  $k$  is the probability that the input lies between  $(k - \frac{1}{2})\Delta v$  and  $(k + \frac{1}{2})\Delta v$ . This probability is

$$\begin{aligned} P\left((k - \frac{1}{2})\Delta v \leq v < (k + \frac{1}{2})\Delta v\right) &= \int_{-\infty}^{(k + \frac{1}{2})\Delta v} \frac{1}{\sigma_{noise} \sqrt{2\pi}} e^{-\frac{(v-v_0)^2}{2\sigma_{noise}^2}} dv - \int_{-\infty}^{(k - \frac{1}{2})\Delta v} \frac{1}{\sigma_{noise} \sqrt{2\pi}} e^{-\frac{(v-v_0)^2}{2\sigma_{noise}^2}} dv \\ &= P\left(\frac{(k + \frac{1}{2})\Delta v - v_0}{\sigma_{noise}}\right) - P\left(\frac{(k - \frac{1}{2})\Delta v - v_0}{\sigma_{noise}}\right) \\ &= \frac{1}{2} \left[ \operatorname{erf}\left(\frac{(k + \frac{1}{2})\Delta v - v_0}{\sigma_{noise} \sqrt{2}}\right) - \operatorname{erf}\left(\frac{(k - \frac{1}{2})\Delta v - v_0}{\sigma_{noise} \sqrt{2}}\right) \right] \end{aligned} \quad (16)$$

The expected value of a measurement with the A/D would be

$$\begin{aligned} \bar{S} &= \sum_{k=-(2^{m-1}-1)}^{(2^{m-1}-1)} k\Delta v P\left((k - \frac{1}{2})\Delta v \leq v < (k + \frac{1}{2})\Delta v\right) \\ &\quad - (2^{m-1} - 1)\Delta v P\left(v < -(2^{m-1} - \frac{1}{2})\Delta v\right) \\ &\quad + (2^{m-1} - 1)\Delta v P\left(v \geq (2^{m-1} - \frac{1}{2})\Delta v\right) \end{aligned} \quad (17)$$

or, in terms of the error function

$$\begin{aligned} \bar{S} &= \sum_{k=-(2^{m-1}-1)}^{(2^{m-1}-1)} k\Delta v \frac{1}{2} \left[ \operatorname{erf}\left(\frac{(k + \frac{1}{2})\Delta v - v_0}{\sigma_{noise} \sqrt{2}}\right) - \operatorname{erf}\left(\frac{(k - \frac{1}{2})\Delta v - v_0}{\sigma_{noise} \sqrt{2}}\right) \right] \\ &\quad + (2^{m-1} - 1)\Delta v \frac{1}{2} \left[ \operatorname{erf}\left(\frac{(2^{m-1} - 1)\Delta v + v_0}{\sigma_{noise} \sqrt{2}}\right) - \operatorname{erf}\left(\frac{(2^{m-1} - 1)\Delta v - v_0}{\sigma_{noise} \sqrt{2}}\right) \right], \end{aligned} \quad (18)$$

The terms outside the summation represent the contribution due to voltages which lie outside the nominal range of the A/D,  $v < -(2^{m-1} - \frac{1}{2})\Delta v$  and  $v > (2^{m-1} - \frac{1}{2})\Delta v$ . Fig. 4 shows the normalized mean for a six-bit A/D as a function of input voltage and noise. A more quantitative view is shown in Fig. 5. Low signal values are severely under-estimated when the noise is small with respect to the least significant bit ( $\sigma_v \ll \Delta v$ ). When the signal is substantial ( $v_0 \geq \Delta v$ ), considerable distortion is present in the low-noise region. However, even for very small signals, the mean of the output matches the signal if the noise level is *great enough* ( $\sigma_v/\Delta v \geq -4$  dB). For example, at  $\sigma_v/\Delta v = -4$  dB, the error in the mean is less than 0.01 dB. Clearly, a certain amount of noise is *necessary* in order for the A/D to behave linearly over the full range of signal levels.

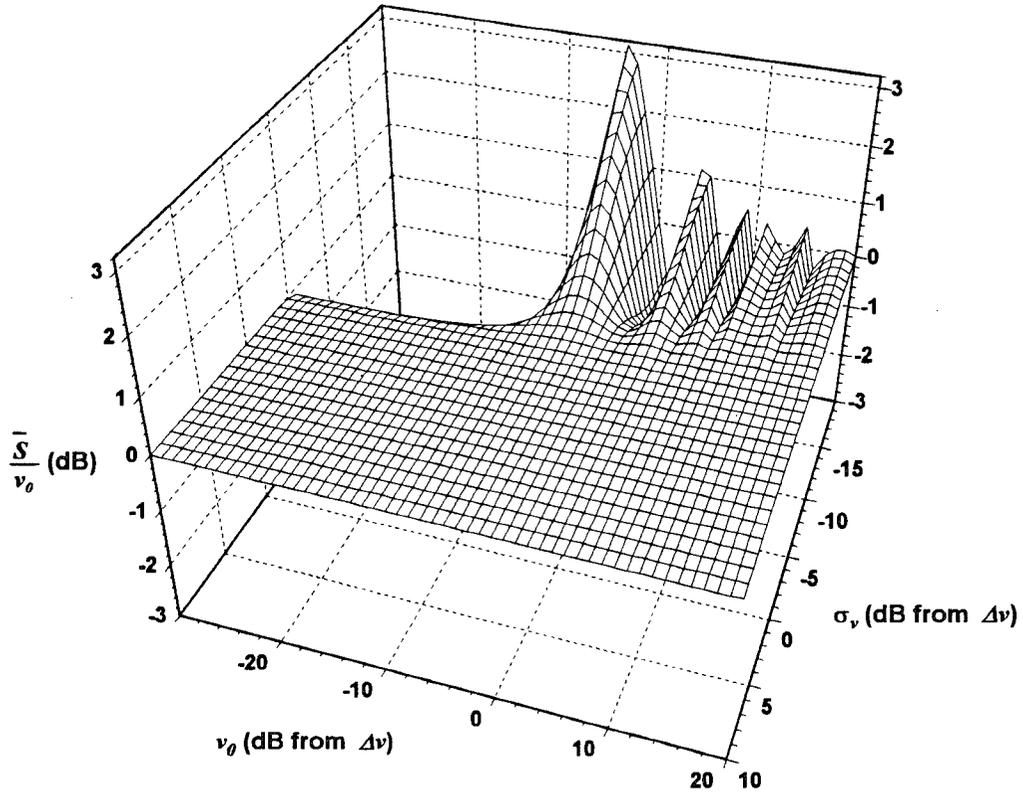


Fig. 4 Normalized mean output of a six-bit A/D as a function of signal,  $v_0$ , and noise,  $\sigma_v$ , levels.

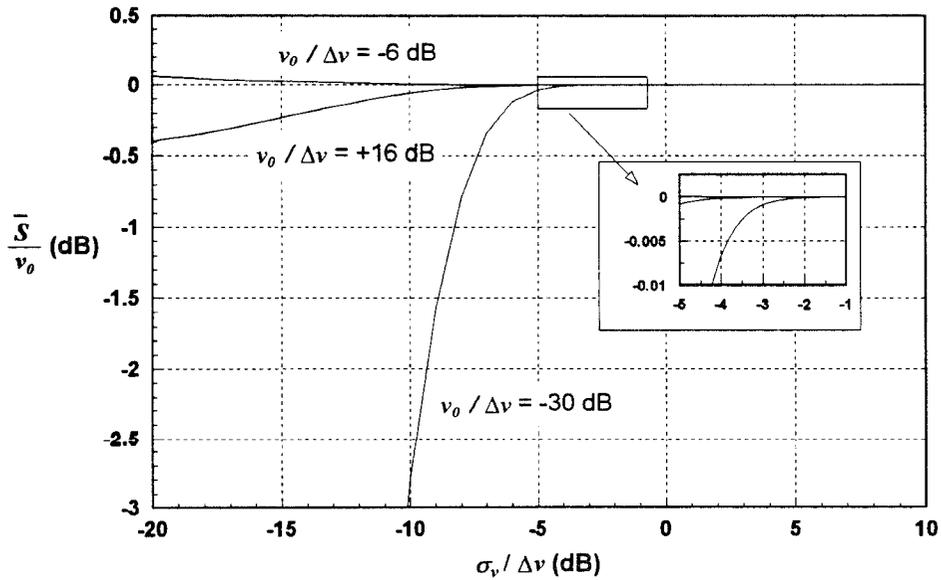


Fig. 5 Normalized mean output of the A/D as a function of noise,  $\sigma_v$ , for three signal levels.

Similarly, the variance of the output will be

$$\begin{aligned}\sigma_s^2 = & \sum_{k=-(2^{m-1}-1)}^{(2^{m-1}-1)} (k\Delta v - \bar{S})^2 P((k - \frac{1}{2})\Delta v \leq v < (k + \frac{1}{2})\Delta v) \\ & + \left( -(2^{m-1} - 1)\Delta v - \bar{S} \right)^2 P(v < -(2^{m-1} - \frac{1}{2})\Delta v) \\ & + \left( (2^{m-1} - 1)\Delta v - \bar{S} \right)^2 P(v \geq (2^{m-1} - \frac{1}{2})\Delta v)\end{aligned}\quad (19)$$

so that

$$\begin{aligned}\sigma_s^2 = & \sum_{k=-(2^{m-1}-1)}^{(2^{m-1}-1)} (k\Delta v - \bar{S})^2 \frac{1}{2} \left[ \operatorname{erf}\left(\frac{(k + \frac{1}{2})\Delta v - v_0}{\sigma_{noise} \sqrt{2}}\right) - \operatorname{erf}\left(\frac{(k - \frac{1}{2})\Delta v - v_0}{\sigma_{noise} \sqrt{2}}\right) \right] \\ & - \left( (2^{m-1} - 1)\Delta v + \bar{S} \right)^2 \frac{1}{2} \left[ 1 - \operatorname{erf}\left(\frac{(2^{m-1} - 1)\Delta v + v_0}{\sigma_{noise} \sqrt{2}}\right) \right] \\ & + \left( (2^{m-1} - 1)\Delta v - \bar{S} \right)^2 \frac{1}{2} \left[ 1 - \operatorname{erf}\left(\frac{(2^{m-1} - 1)\Delta v - v_0}{\sigma_{noise} \sqrt{2}}\right) \right]\end{aligned}\quad (20)$$

Fig. 6 illustrates the normalized standard deviation of the output of the A/D as a function of input voltage and noise. Severe nonlinear behavior is exhibited for small noise levels ( $\sigma_v \ll \Delta v$ ). A more quantitative representation is shown in Fig. 7. The standard deviation of the output always exceeds that of the input when the noise is greater than -10 dB from the least significant bit. However, it approaches the input standard deviation as the noise increases. When the noise,  $\sigma_v$ , is -5 dB below the least significant bit,  $\Delta v$ , the standard deviation of the output is only about 1 dB above the input standard deviation, dropping to about 1/3 dB above at 0 dB from the least significant bit and less than 0.1 dB above the input when the noise is 6 dB above the least significant bit.

If a new random variable,  $V$ , is defined by averaging  $N$  measurements of  $S$ ,

$$V \equiv \frac{1}{N} \sum_{n=1}^N S_n, \quad (21)$$

so that the mean value of  $V$  is  $\bar{V} = \bar{S}$ , then the variance of  $V$  is [3]

$$\sigma_N^2 = \frac{\sigma_s^2}{N}. \quad (22)$$

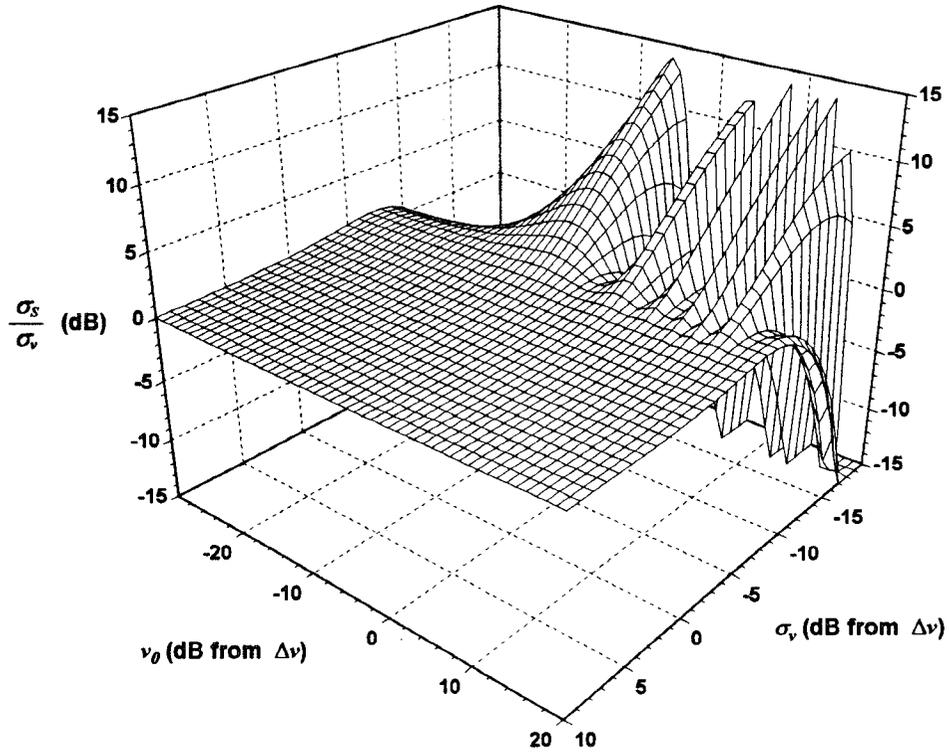


Fig. 6 Normalized standard deviation of the output of the A/D as a function of signal,  $v_0$ , and noise,  $\sigma_v$ .

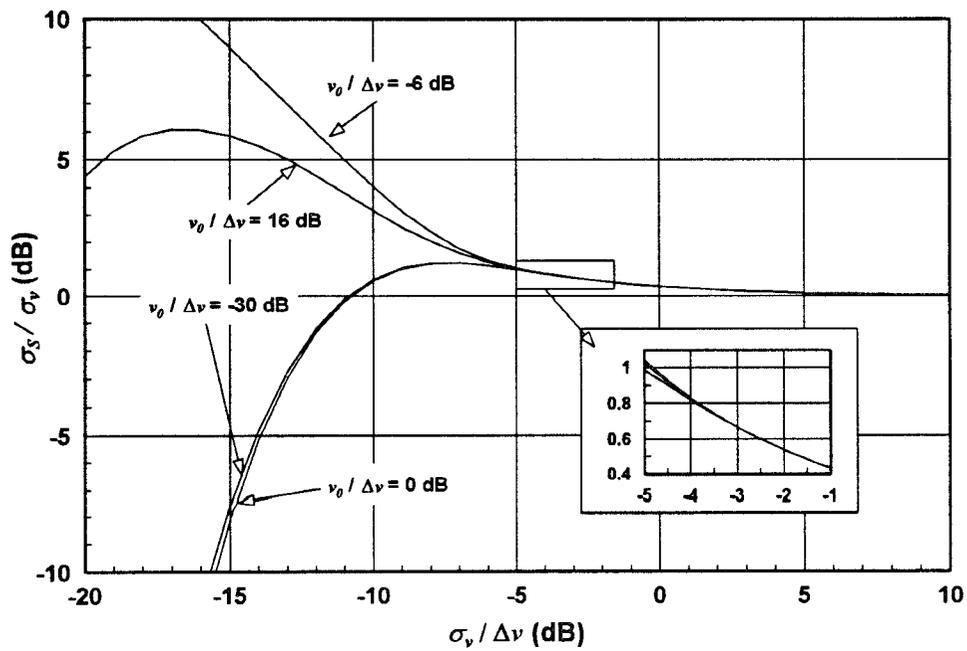


Fig. 7 Normalized standard deviation,  $\sigma_s$ , of the output of the A/D as a function of noise level,  $\sigma_v$ , for several signal levels,  $v_0$ .

Fig. 8 shows the output signal-to-noise ratio,  $\bar{V}/\sigma_N$ , of a system which averages  $N = 1024$  measurements from a 6-bit A/D. Note the nonlinear behavior of the signal-to-noise ratio when the signal reaches the least significant bit,  $\Delta v$ , while the noise level is low ( $\sigma_v/\Delta v < -5$  dB). To avoid the nonlinearity and to obtain an accurate measurement of the signal, the noise level must be at least  $\sigma_v/\Delta v \geq -4$  dB (see Fig. 5). The actual value used must be determined by the degree of linearity required by the system. Operating with the noise lower than this level will subject the output to nonlinearities. Operating with a significantly higher noise level will degrade the signal-to-noise ratio of the averaged output. With reference to Fig. 8, it is seen that an input noise level in the range of  $-4$  dB  $\leq \sigma_v/\Delta v \leq 0$  dB will allow a very linear measurement even when the input signal-to-noise ratio is greater than 0 dB. Since the output is linear, signals below the input noise level can be detected when an adequate number of measurements are averaged. For example, averaging with  $N = 1024$  allows signals as low as 30 dB below the noise to be detected, *but the noise must be present* for the averaging to work.

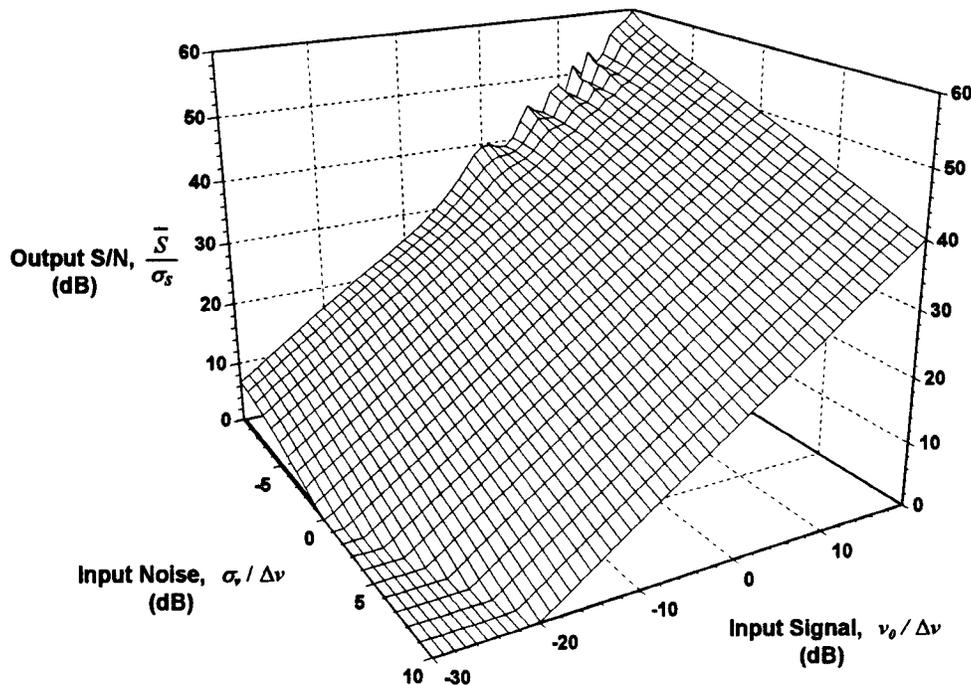


Fig. 8 Signal-to-noise ratio for average of 1024 measurements from 6-bit A/D.

Quantization error will be determined by the size of the least significant bit of the A/D when no coherent integration is used, or when the rms noise voltage is significantly below the least significant bit, and in this case, nonlinear performance will seriously degrade the measurement. The quantization step size will drop 6 dB for each bit of coherent integration, while the effective noise level will drop by 3 dB per bit. By applying coherent integration, the system can become noise-limited rather than quantization-error limited, and signals can be detected below the noise floor of the receiver, but *only* if the proper amount of noise is present.

### Number of bits in A/D

The required instantaneous dynamic range (largest operating signal-to-noise ratio) determines the number of bits in the analog-to-digital converter. To achieve greater dynamic range than allowed by the number of bits in the A/D will require the use of coherent integration to detect signals below the noise floor. The use of attenuation at the front end of the receiver will allow larger signals to be measured, but will not extend the dynamic range. Further, if the system noise floor is set by atmospheric noise rather than internal thermal noise (as in low-frequency receivers), then applying attenuation at the front end of the receiver may reduce the noise voltage below the least significant bit, limiting the receiver performance by quantization and nonlinearity errors. Thus, given a fixed number of bits, the dynamic range can only be extended beyond that of the A/D through coherent integration, and extended only in the direction of weak signals. Extension in the direction of larger signal strength requires additional bits in the A/D.

### Over-lapping A/D converters to extend dynamic range

It might appear that a useful way to extend the range of an A/D system would be to provide two A/D converters, one for weak signals and one for strong signals. Such an arrangement has been proposed and implemented by Scientific Atlanta, Inc. in their model 2090 RCS measurement system [4]. In this example, the signal is split into two channels. One channel, designated the high-gain channel, has 30 dB more gain than the other. Each channel is followed by a 14-bit (13-bits plus sign) A/D. The arrangement is diagrammed in Fig. 9. Weak input signals would be measured from the high-gain channel, while strong input signals would be measured from the low-gain channel. In a system utilizing this block diagram, the digitized voltage used by the system software, corresponding to the input voltage, will be

$$V_{System} = \begin{cases} V_H / G; & V_L < V_{threshold} \\ V_L; & V_L \geq V_{threshold} \end{cases}, \quad (23)$$

where  $V_{threshold}$  is the threshold voltage for switching between the two channels.

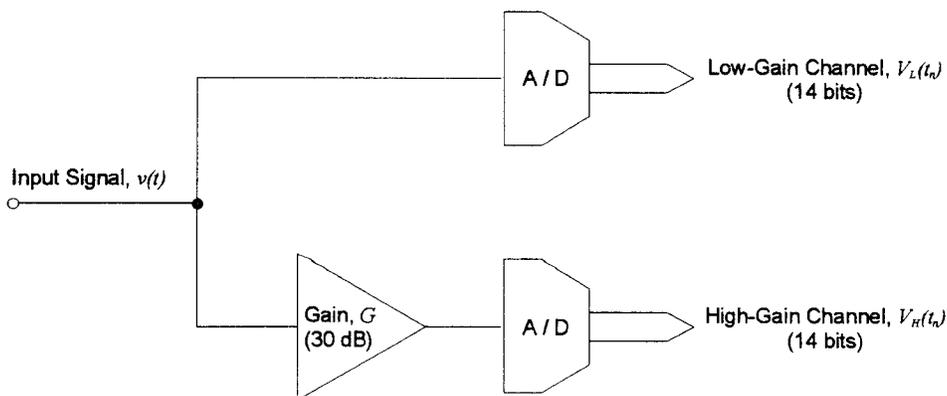


Fig. 9 Configuration of overlapping A/D converters used in the Scientific Atlanta, Inc. 2090 radar system in attempt to extend digital dynamic range.

As will be seen, this approach *does not* usefully extend the dynamic range of the system. If there is adequate noise in the low-gain channel (standard deviation at the level of the least-significant bit), then the noise in the high-gain channel will be a factor of  $G$  above its least-significant bit. Under this condition, there is no advantage to using the high-gain channel, since integration is required to see below the noise, and this integration can be performed with data from only the low-gain channel. On the other hand, if the noise is just adequate in the high-gain channel, it will be low by a factor of  $1/G$  in the low-gain channel, causing severe nonlinear performance in the low-gain channel, even for strong signals.

To illustrate the problem with this approach, Fig. 10 shows the difference between the system's mean digitized voltage,  $V_{system}$ , and the input signal,  $v(t)$  for the example of the Scientific Atlanta, Inc. model 2090 radar system. The difference is plotted in units of the least-significant bit of the low-gain channel, corresponding to signals greater by a factor of  $G$  than the least-significant bit of the high-gain channel. The system uses 14-bit A/D converters, the gain of the high-gain channel is 30 dB, and the noise level is set to the level of the least-significant bit in the high-gain channel. The threshold for switching between the two channels is set to 40 dB above the least significant bit of the low-gain channel. This arrangement uses only about half of the bits in the high-gain channel. Note that the input voltage corresponding to the least-significant bit of the low-gain channel is greater by a factor of  $\sim 32.6$  than the input voltage corresponding to the least-significant bit of the high-gain channel.

The error in the mean of a measurement is essentially zero until the system begins to use the data from the low-gain channel (larger signals). Then the error ranges between about  $\pm 1/2$  least-significant bit of the low-gain channel, which corresponds to about 24 dB above the least significant bit of the high-gain channel. The magnitude of this error is thus much larger than the noise in the input signal.

In addition, the standard deviation of the measurement (Fig. 11) also increases dramatically when the data is taken from the low-gain channel. The standard deviation peaks at about 16 times the input standard deviation (about 24 dB above the input noise) for data taken from the low-gain channel, after remaining at about the input noise level while the data is taken from the high-gain channel. (The seemingly erratic behavior in Fig. 11 for large values of the input voltage is an artifact of the sample spacing in the plot.) The standard deviation does become zero for certain input values, but the error in the mean of the measurement is different from zero for these inputs. Note that the error in the mean is also different from zero when the standard deviation is maximum.

Clearly, the statistics of the input voltage have been modified in a detrimental and nonlinear fashion as a result of insufficient noise in the low-gain channel. Integration (averaging) of multiple measurements to reduce the noise and obtain a better estimate of the mean will not work when the standard deviation is zero, so the system will suffer quantization error at the level of the low-gain-channel least-significant bit for these

conditions. When the standard deviation is not zero, integration will work, but the noise will be as much as 24 dB higher than the input noise, drastically reducing the effectiveness of the integration. Even if enough integration is applied to obtain a good estimate of the mean, it will be a *biased* estimate of the input voltage, since the error in the mean is nonzero for most of the range. The use of over-lapping A/D converters to extend the dynamic range does not work. The solution is to use a single A/D channel with the gain properly adjusted so that the standard deviation of the input signal (noise) is about the level of the least-significant bit of the A/D.

## Conclusion

The discussion and analysis contained here demonstrate that noise is an important contributor to the linear operation of an analog-to-digital converter. The correct analog gain for a system is determined by the noise level and the size of the least-significant bit of the A/D; the noise level must be at least  $\sigma_v/\Delta v \geq -4$  dB, where  $\Delta v$  is the voltage corresponding to the least-significant bit. The specific noise-level requirement depends on the degree of linearity required. The number of bits in the A/D is determined by the largest signal that the system will see. For example, if the analog gain places  $\sigma_v/\Delta v = 0$  dB (higher than necessary, in many cases), then the number of bits will be determined by the maximum signal-to-noise ratio expected. If the analog gain is increased beyond that required to bring the noise to an adequate level, all that is accomplished is a reduction in the strong-signal capability of the system. The only way to increase dynamic range while maintaining linearity, given a specified number of bits in the A/D, is to take advantage of signal-processing gain through coherent integration (through simple summation of the A/D output, or some other more complex signal-processing algorithm). However, this only increases the dynamic range in the direction of low-level signals. Addition of multiple channels with different analog gain levels, in an attempt to extend the dynamic range upward, destroys the linearity of the system. Increasing the dynamic range upward requires more bits in the A/D.

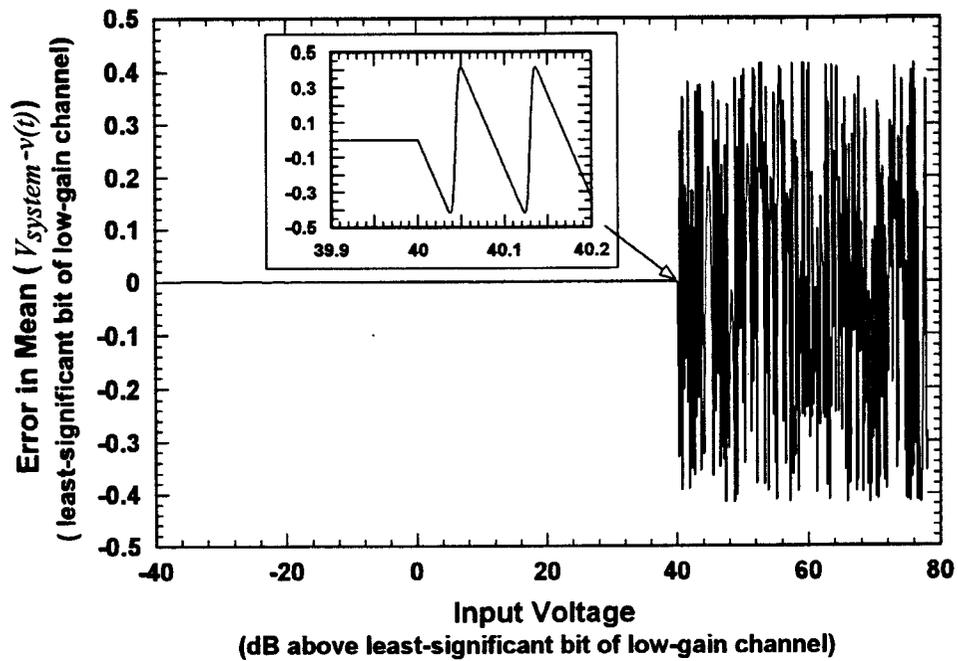


Fig. 10 Error in the mean of the system measurement relative to the least-significant bit of the low-gain channel as a function of input voltage.

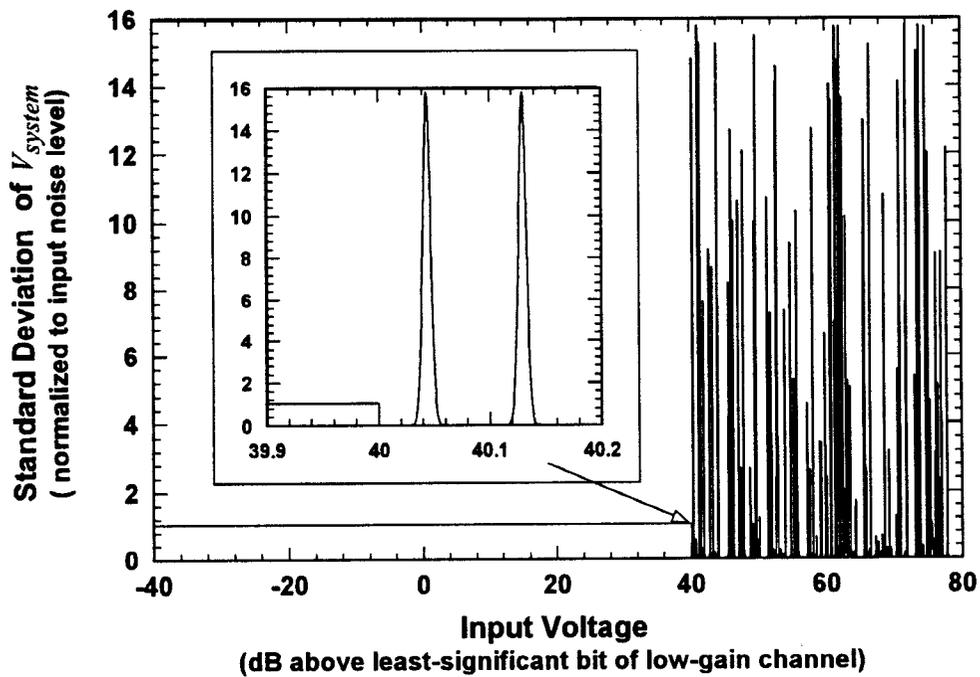


Fig. 11 Standard deviation of the measurement, normalized to the input standard deviation (noise level), as a function of input voltage.

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