Sizing Handbook for Stand-Alone Photovoltaic/Storage Systems

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Sizing Handbook for Stand-Alone Photovoltaic/Storage Systems

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Abstract
This report details the use and development of a simplified sizing technique for stand-alone photovoltaic/storage systems. The array size and storage capacity are chosen to achieve the required loss-of-load probability (LOLP). The LOLP represents the level of confidence at which the system will satisfy the load. An LOLP of 0 means that the load will always be satisfied, and an LOLP of 1 means that the load will never be satisfied. Array sizes are read from array-sizing nomograms as a function of latitude, tilt angle, and average horizontal insolation in December (in the northern hemisphere) or June (in the southern hemisphere). Storage capacities are read from storage-sizing nomograms as a function of the required LOLP. The technique is valid for systems with a fixed tilt array, product or energy storage, and for any hourly load profile or daily load profile within a given month. The only constraint is that the average monthly load must not vary more than ± 10% from month to month. The designer can choose from tilt angles from latitude minus 20° to latitude plus 20° and can choose from a range of array size/storage capacity combinations for any given LOLP.
A stand-alone photovoltaic/storage system can be defined as a power source dedicated to some load such as a water pump, refrigerator, or radio transmitter. The phrase "stand-alone" implies that the photovoltaic/storage system is the only source from which the load may draw power. The power generated by the photovoltaic (PV) array is equal to the insolation incident on the plane of the array's surface (plane-of-array insolation, POA) multiplied by the array's area and efficiency. The power generated by the array varies not only with the daily and seasonal cycles of the sun, but also with changing weather conditions. Because the power required by the load does not usually track the magnitude of the insolation, a portion of the energy produced by the array must be stored to be able to satisfy the load when there is insufficient insolation. The system designer must somehow quantify the variability in the insolation and from that choose a combination of array size and storage capacity that will satisfy the load at the confidence level required by the user.

This confidence level is mathematically described by the loss-of-load probability (LOLP), which represents how often the PV/storage system will not be able to satisfy the load. An LOLP of 0 means that the load will always be satisfied, and an LOLP of 1 means that the load will never be satisfied. The LOLP is characterized by an average or long-term value and the distribution about that average value. For conventional power sources, like a diesel generator, the distribution about the long-term value is narrow and smooth. For PV/storage systems, the distribution can be wide and erratic. The importance of this type of distribution is that if the variability in the insolation is not accurately quantified, the system may be over- or undersized.

A PV/storage system fails to satisfy the load when the power generated by the array is insufficient and the storage is depleted. Energy is stored when the power generated by the array is greater than the load. Energy is taken from storage when the power generated by the array is less than the load. The amount of stored energy at any given time depends on the sequence of instantaneous insolation and load levels from the last time that the storage was either depleted or at full capacity. An infinite number of sequences can lead to depletion of storage. Consequently, continuous insolation and demand profiles for the expected life of the system are required to determine the long-term LOLP and its distribution.

Because we cannot predict the weather, we are forced to rely on long-term historical data to define the insolation profile. There are long-term records of hourly direct normal and horizontal insolations from which the POA insolation can be calculated. These data, however, are available for only a handful of sites worldwide. Even if these data are available, their direct use requires extensive computer support, which is both expensive and time-consuming.

The demand profile is a function of the application and can have many forms. For example, a telecommunication system would have a nearly constant demand profile. An irrigation system would have a well-defined seasonal profile. A vaccine refrigerator system might have a highly erratic, unpredictable profile. In fact, the demand profile for most stand-alone applications is not well defined. We
will show that short-term variations in the demand have little effect on the long-term LOLP and that monthly average values of the demand are adequate for defining the load profile. Furthermore, if the monthly average demands do not vary by more than ± 10%, the average yearly demand is adequate. This sizing handbook describes an accurate noncomputerized sizing technique that allows the designer to size for the required LOLP given the average yearly demand and readily available insolation data.

The technique was derived using a loss-of-load simulation model and 23.5 years of hourly insolation data from the 20 U. S. weather stations listed in Table 1. These data were obtained from the SOLMET data base (1), which is a compilation of hourly solar and meteorological observations. Correlations were derived that relate the variability in the POA insolation to the average December horizontal insolation (in the northern hemisphere) or the average June horizontal insolation (in the southern hemisphere). These correlations were then used to generate sizing nomograms that give the storage capacity as a function of the loss-of-load-probability (LOLP), and the array size as a function of the array tilt angle, the average horizontal insolation, and latitude of the site.

Table 1. Solmet Sites Used to Develop Sizing Technique.

<table>
<thead>
<tr>
<th>Site</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albuquerque, New Mexico</td>
<td></td>
</tr>
<tr>
<td>Miami, Florida</td>
<td></td>
</tr>
<tr>
<td>Apalachicola, Florida</td>
<td></td>
</tr>
<tr>
<td>Dodge City, Kansas</td>
<td></td>
</tr>
<tr>
<td>Brownsville, Texas</td>
<td></td>
</tr>
<tr>
<td>Cape Hatteras, North Carolina</td>
<td></td>
</tr>
<tr>
<td>Lake Charles, Louisiana</td>
<td></td>
</tr>
<tr>
<td>Great Falls, Montana</td>
<td></td>
</tr>
<tr>
<td>Bismarck, North Dakota</td>
<td></td>
</tr>
<tr>
<td>Madison, Wisconsin</td>
<td></td>
</tr>
<tr>
<td>Medford, Oregon</td>
<td></td>
</tr>
<tr>
<td>Nashville, Tennesse</td>
<td></td>
</tr>
<tr>
<td>New York, New York</td>
<td></td>
</tr>
<tr>
<td>Phoenix, Arizona</td>
<td></td>
</tr>
<tr>
<td>Ely, Nevada</td>
<td></td>
</tr>
<tr>
<td>Charleston, South Carolina</td>
<td></td>
</tr>
<tr>
<td>Caribou, Maine</td>
<td></td>
</tr>
<tr>
<td>Seattle, Washington</td>
<td></td>
</tr>
<tr>
<td>Fresno, California</td>
<td></td>
</tr>
<tr>
<td>Columbia, Missouri</td>
<td></td>
</tr>
</tbody>
</table>

The sizing nomograms give the array size in terms of design insolations and the storage capacity in terms of days of storage. The design insolation and days of storage are translated into the array area and storage capacity based on the magnitude of the demand, the system configuration, and the performance characteristics of the system components.

Four sets of array and storage-sizing nomograms were derived that define a curve of array size versus storage capacity from which the designer can choose equivalent array/storage size combinations ranging from small array/large storage capacity to large array/small storage capacity. The array/storage combinations are equivalent in that all result in the same LOLP. Each set of sizing nomograms contains five array-sizing nomograms, one each for tilt angles of latitude minus 20 and 10°, latitude, latitude plus 10 and 20°, and one storage-sizing nomogram.

The 20 sites represent most climates found worldwide. The fact that these correlations were derived from data representing such a wide range of climates indicates that the variability in insolation
does not depend on the climate. The work of Liu and Jordan (2) and
Klein (3) support this observation.

Liu and Jordan noted that the distribution of daily horizontal
insolations for a given month is dependent only on the average
monthly clearness index and not on the type of climate. Klein has
shown that persistence of weather, that is the dependence of today's
insolation on yesterday's insolation, is weak and fairly constant
from site to site regardless of the climate. The implication of
these observations is that there is no correlation between climate
and variability in insolation. Because this technique is based on
the variability in the insolation and not on the magnitude of the
insolation, it can be applied to sites outside the U. S. even if the
cclimate is not represented by one of the 20 sites.

The technique is applicable to any system with a fixed tilt
array. The sizing nomograms are valid for any hourly demand profile
or daily demand profile within a given month and for any type of
storage (i.e., energy or product storage). The only limitation of
this technique is that the average monthly demand must not vary by
more than ±10%. We recently developed sizing nomograms that are
based on monthly values of the long-term LOLP and can be used for
applications in which the average monthly demand varies by more than
±10%.

The sizing technique is presented in the next section and is
followed by detailed sizing examples for general power load and
direct coupled water pumping applications. The sizing nomograms can
be found in Appendices A1 through A4. The derivation of the
loss-of-load simulation model is given in Appendix B along with model
verification and an overview of how the technique was developed.

SIZING PROCEDURE

This section describes the four steps in applying the sizing
technique. (Detailed sizing examples are given in the following
section.) The first step involves defining site- and
application-specific parameters. The second and third steps involve
using the sizing nomograms found in Appendices A1, A2, A3, and A4 to
determine the required array size and storage capacity. This
technique gives the designer four sets of array size/storage capacity
combinations. These combinations define a curve-of-array size versus
storage capacity that satisfies the required LOLP defined in step
one. The sizes vary from large array area/small storage capacity to
small array area/large storage capacity.

After performing these three steps, the designer will have array
sizes in terms of design insolations and storage capacities in terms
of days of storage. The translation of the design insolations and
days of storage into array size/storage capacity combinations is the
next step in the design sequence. The procedure for this step
depends on the system's configuration and it cannot be generalized.
A comprehensive discussion of this phase of design is beyond the
scope of this document. We do, however, present specific examples of
this design effort in step four of the sizing procedure.
Step One. Define Site- and Application-Specific Parameters

The following sections define the site and application parameters and give guidelines on how they are specified.

Site-specific parameters - These parameters consist of the latitude and the average horizontal insolation in December (in the northern hemisphere) or June (in the southern hemisphere). (Some readers who are more familiar with calculations of insolation incident of flat surfaces may wonder why the ground reflectance is not included as a site-specific parameter. The degree of uncertainty inherent in insolation data and this sizing technique do not warrant such detail in the description of the site. We found that the ground reflectance could be preset at an average value without reducing accuracy.)

Monthly average horizontal insolutions have been tabulated for many sites worldwide (4,5). If a tabulated value for the horizontal insolation cannot be found for the particular region, an estimate can be obtained from worldwide insolation maps (5,6).

Application-specific parameters - These parameters include the daily energy demand and the desired LOLP. The sizing nomograms were derived assuming a "constant" demand. Figure 1 shows, however, that the hourly demand has very little effect on the sizing nomograms. Figure 1 was generated by deriving the four storage-sizing nomograms for hourly demand profiles ranging from

![Figure 1. Effect of hourly demand profile on storage-sizing nomogram.](image-url)
an 8-hour nighttime demand to an 8-hour daytime demand without altering the array-sizing nomograms. It shows that the array size and storage capacity required for a given LOLP are nearly independent of the hourly demand profile. The technique is valid for any hourly demand profile.

Figure 2 shows that the daily demand profile within a given month has little effect on the sizing nomograms. Figure 2 was generated by randomly varying the total daily demand within each month as shown in Figure 3. The daily demand values range from 0 to 200% of the average daily demand and have a standard deviation of approximately 40%. The technique is also valid for any daily demand profile within a given month provided that the average daily demand does not vary by more than ± 10% from month to month.

(A more difficult issue is how to deal with seasonally varying demands, especially summer peaking demands. We have recently developed sizing nomograms that are based on long-term monthly LOLP values and can be used to size systems with seasonally dependent loads. The use of these nomograms is essentially the same as the use of the nomograms presented in this report. The difference is that these recent nomograms are used to size a system that will achieve the required LOLP in a given month as opposed to a system that will achieve the required overall LOLP. As a result, the designer must size a system for each month, determine which system size satisfies the monthly LOLP criterion for all months, and then calculate the overall LOLP. These monthly sizing nomograms will be published in a separate document.)

The LOLP represents the fraction of the demand that will probably not be satisfied over the life of the PV/storage system. This long-term LOLP is the summation of 20 plus yearly LOLPs. Unlike conventional energy sources that have a narrow distribution of yearly LOLPs about the long-term LOLP, photovoltaic energy sources have a very wide distribution of yearly LOLPs as shown in Figure 4. This type of distribution is to be expected given the nature of solar energy sources: their performance changes as the weather changes. This characteristic of PV/storage systems must be considered when a long-term LOLP is chosen, because even if the long-term LOLP is very low, there can still be years with relatively high LOLPs. To eliminate all periods of high LOLP, the designer would have to oversize a PV/storage system to the point that it would probably not be cost-effective or practical. If no periods of high LOLP can be tolerated, a PV/storage/diesel hybrid system should be considered.

**Step Two.** Determine Four Sets of Array Sizes in terms of Design Insolations

The design insolation (POA₀) is read as a function of the latitude, the tilt angle of the array, and the average horizontal design month insolation from array-sizing nomograms found in Appendices A1 - A4. Each of these appendices contains one set of array size and storage capacity nomograms. The design insolation
Figure 2. Effect of daily demand profile on storage-sizing nomograms.

Figure 3. Example of random daily demand profile within a 31-day month.
is a strong function of the tilt angle. This dependency is handled by providing a series of array-sizing nomograms, one each for tilt angles of latitude minus 20 and 10°, latitude, and latitude plus 10 and 20°, in each set of sizing nomograms. At some point in the design process, the designer must select a tilt angle. He must then use the POA₀ values corresponding to that tilt angle for sizing the array. The minimum theoretical array size is achieved by selecting the tilt angle that gives the largest value of POA₀. In practice, however, this tilt angle may not be the best choice. For example, if the ground is normally snow covered during the winter, it may be best to tilt the array at a larger angle to help keep the array surface free of snow.

Step Three. Determine Four Storage Capacities in terms of Days of Storage

The days of storage required (S) are a function of the long-term LOLP and are read off the storage capacity nomograms found in Appendices A1 - A4. The storage capacity required to achieve a given LOLP is not a function of the tilt angle. Therefore, each set of sizing nomograms contains a single storage-sizing nomogram. At the end of step 3, the designer has four sets of design insolations (POA₀)/days of storage (S) combinations, all for the chosen LOLP (see Tables 2 and 5). Each set consists of one S value and five POA₀ values. Each POA₀ value corresponds to one of the five tilt angle options.
Step Four. Determine Array Area and Storage Capacity

The objective of this step is to specify an array area and storage capacity for the components chosen by the designer. To do this, the designer must define two parameters; the insolation to storage path efficiency ($\eta_{in}$) and the storage to demand path efficiency ($\eta_{out}$).

The path efficiencies are the product of the efficiencies of the components that comprise the energy flow streams from insolation to storage and storage to demand. For example, examine the schematic shown in Figure 5. The components in the insolation to storage path are the array, the maximum power tracker, the charge controller, and the batteries. The components in the storage to demand path are the batteries, the charge controller, and the inverter.

The correct values for the path efficiencies are the average daily efficiencies in the design month defined as

$\eta_{in} = \frac{\text{daily POA} \cdot \text{A}}{\text{daily energy added to storage}}$

$\eta_{out} = \frac{\text{daily energy taken from storage}}{\text{demand}}$

where daily POA is the total average insolation in the design month, A is the array area, and demand is the average total daily demand.

The determination of the path efficiencies is complicated when the instantaneous component efficiencies are interrelated. In such cases, it is necessary to perform a single-day hourly simulation representing an average day in the design month.

The relationship between the array area, the path efficiencies, the daily demand, and the design insolation is defined as

$\text{demand} / \eta_{out} = \text{POA} \cdot \text{A} \cdot \eta_{in}.$

The relationship between the storage capacity (CAP), the days of storage (S), the path efficiencies, and the total daily demand is defined as

$\text{CAP} = (S \cdot \text{demand}) / \eta_{out}.$

Assume that we are sizing the system shown in Figure 5 and that the average daily efficiencies of the components are

- array - 0.10
- maximum power tracker - 0.95
- charge controller - 0.95
- battery charge - 0.75
- battery discharge - 1.00
- inverter - 0.85.

For these components, $\eta_{in}$ is $(0.10)(0.95)(0.95)(0.75)$ or 0.068 and $\eta_{out}$ is $(1.00)(0.85)$ or 0.85. If the daily demand is
10 kWh/day and POA0 is 5 kWh/m²-day, the array area is \((10)/(5)(0.068)(0.85)\) or 35 m². If the value for S is 3, the required storage capacity is \((3)(10)/(0.85)\) or 35 kWh. If the maximum allowable depth of discharge for the battery is 0.8, the required battery rating is \((35)/(0.8)\) or 44 kWh.

It is important to re-emphasize that this technique gives the designer a curve of array size versus storage capacity from which the designer can choose array/storage size combinations that satisfy the LOLP. An economic analysis should be performed to determine which system is the most cost effective. Keep in mind, however, that there may be other criteria that have a higher priority than costs. For example, the system might power a concealed security sensor, in which case a minimally exposed array would have a high priority. (We do intend to develop an economics handbook for stand-alone systems, which will be published as a separate document.)

SIZING EXAMPLES

This section steps the reader through specific applications of the sizing technique for a general load electrification system with battery storage and a direct coupled water pumping system with water storage. Sizing worksheets are provided to help organize and clarify the sizing procedure. Table 2 shows a completed worksheet for the electrification system and Table 5 shows a completed worksheet for the water pumping system.

Table 2. Completed worksheet for electrification system.

<table>
<thead>
<tr>
<th>SIZING WORKSHEET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
</tr>
<tr>
<td>Design month</td>
</tr>
<tr>
<td>Average horizontal insolation in design month</td>
</tr>
<tr>
<td>Loss-of-load probability</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Insolation, POA0 (kWh/m²-day)</th>
<th>Days of Storage, S</th>
</tr>
</thead>
<tbody>
<tr>
<td>tilt angle, latitude plus</td>
<td>0</td>
</tr>
<tr>
<td>Size Set</td>
<td>-20</td>
</tr>
<tr>
<td>1</td>
<td>3.53</td>
</tr>
<tr>
<td>2</td>
<td>3.73</td>
</tr>
<tr>
<td>3</td>
<td>3.85</td>
</tr>
</tbody>
</table>
Electrification

Step One.
Assume that the system is at latitude 30° north, the average horizontal insolation in December is 3 kWh/m²-day, and that the daily demand is 5 kWh ac. Assume that the electrical load is primarily vaccine refrigeration. For this critical load we will require an LOLP of 0.001.

Step Two.
The design insolation values listed in the worksheet for this system (Table 2) were read from Figures A1a-e, A2a-e, A3a-e, and A4a-e for a latitude of 30° north and a horizontal insolation of 3 kWh/m²-day. We will select the tilt angle that gives the largest POA values. For this example, that tilt angle is latitude plus 10°.

Step Three.
The days of storage values listed in Table 2 were read from Figures A1f, A2f, A3f, and A4f for an LOLP of 0.001.

Step Four.
Figure 5 shows the schematic for this electrification system. The components in the insolation to storage path are the array, a maximum power tracker, a battery charge controller, and the batteries. The components in the storage to demand path are the batteries and an inverter.

![Figure 5. Schematic for electrification system.](image)

The operating voltage in this system is regulated. The component efficiencies are not interrelated and can be evaluated independently. Therefore, it is not necessary to perform a single-day simulation. Once the designer selects the components and specifies their performance, he can translate the design insolation and days of storage values directly into array configurations and storage capacities.

Component specifications
The inverter rating is matched to the anticipated peak demand. For this example, we will assume the demand profile given in Table 3, which shows that the peak demand is 1000 Wac. We will select a
1200 Wac/12 Vdc inverter. The inverter's efficiency is a function of its load factor, which is the ratio of the actual demand to its rating. Table 3 shows the hourly values for the load factor and efficiency along with the resulting daily efficiency.

**Table 3. Hourly values for demand, inverter load factor, and inverter efficiency.**

<table>
<thead>
<tr>
<th>hour</th>
<th>demand (Wac)</th>
<th>load factor</th>
<th>efficiency</th>
<th>dc input (Wdc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>200</td>
<td>.167</td>
<td>.40</td>
<td>500</td>
</tr>
<tr>
<td>16</td>
<td>600</td>
<td>.500</td>
<td>.84</td>
<td>714</td>
</tr>
<tr>
<td>17</td>
<td>1000</td>
<td>.833</td>
<td>.90</td>
<td>1111</td>
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<td>200</td>
<td>.167</td>
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<td>500</td>
</tr>
<tr>
<td>totals</td>
<td>5000</td>
<td></td>
<td></td>
<td>6592</td>
</tr>
</tbody>
</table>

average daily efficiency = 5000/6592 = 0.758

The performance of batteries is extremely complicated and cannot be predicted -- especially when subjected to the uncontrolled charge/discharge cycles that occur in stand-alone photovoltaic systems. Furthermore, it is difficult to measure separate charge and discharge efficiencies of batteries. Consequently, battery efficiencies are usually given in terms of a constant round-trip efficiency. The choice of battery depends on the environment of the site, how often the battery can be serviced, and on how the battery is cycled. (The economics handbook will contain information on how the battery is cycled as a function of the required LOLP and the days of storage.) Given this information, a battery manufacturer can suggest an appropriate battery. For this example, we will select a 12 Vdc deep-discharge battery with a maximum depth-of-discharge of 0.8 and a round-trip efficiency of 0.75.

The battery capacity is usually defined in terms of the amount of energy that can be extracted, not the amount of energy that is actually stored. To be consistent with these conventions, the battery charge efficiency is set equal to the round trip efficiency and the battery discharge efficiency is set equal to 1.

The maximum power tracker adjusts the array voltage to the maximum power point. The array efficiency at the maximum power point ($\eta_{mp}$) is a function of the cell temperature ($T_c$) and the array's efficiency at some reference temperature ($\eta_r$, $T_r$) and is given by

$$\eta_{mp} = \eta_r [1 - C_T(T_c - T_r)], \quad (5)$$

where $C_T$ is the maximum power temperature coefficient. We will
select a 0.36 m² module with an efficiency of 0.10 at 48°C and a temperature coefficient of 0.005/°C. In general, the average daily array temperature is from 5 to 15 °C above the average daylight ambient temperature. We will assume that the the average ambient daylight temperature in the design month at this site is 10°C and that the average array temperature is 20°C, which results in an average daily array efficiency of 0.114. The maximum error introduced by this assumption is about 2%. The maximum power voltage of the module is 16.5 V and the maximum power current is 2.45 A.

Both the maximum power tracker and the battery charge controller have small parasitic power draws. We will assume that both have constant efficiencies of 0.95.

**Select array configuration/storage capacity configurations**

The insolation to storage path efficiency ($\eta_{in}$) is the product of the array, maximum power tracker, battery charge controller, and the battery round trip average daily efficiencies and is equal to 0.077. The storage to demand path efficiency ($\eta_{out}$) is the product of the charge controller and the inverter efficiencies which is 0.720. Using equation (3) and the four POA₀ values (tilt angle of latitude plus 10°) obtained from the array-sizing nomograms, we find that applicable array areas range from 17.2 m², corresponding to 48 modules, to 24.9 m², corresponding to 69 modules. Since the nominal battery and inverter voltage is 12 V, the modules would be grouped in parallel.

The storage capacities are calculated using equation (4), the days of storage, and $\eta_{out}$. Because the maximum depth-of-discharge for the selected battery is 0.8, the storage capacities are increased by (1)/(0.8). Table 4 lists several array configuration/storage capacity combinations that will satisfy an LOLP of 0.001. The days of storage corresponding to each array configuration were determined by interpolating between the four POA₀ values.

![Table 4](image)

**Water Pumping**

**Step One.**

For this example, we will assume that the site latitude is 10° south, the average horizontal insolation in June is 4 kWh/m²-day, and that the daily water demand is 10 m³. Assume that the demand...
is primarily for potable water and that an LOLP of 0.01 is acceptable.

The power required to pump water is the mass flow rate multiplied by the total dynamic head (H). Assume that the total dynamic head for this application is 5 m. The density of water is 1000 kg/m$^3$. The mass flow rate is the volume of water pumped (V) multiplied by the density ($\rho$) divided by the time (t) during which the water was pumped. Since the total energy requirement (E) is the power multiplied by the time, we have

$$E = V \rho H.$$  \hspace{1cm} (6)

The daily demand for this application is then $10(1000)(5) = 75,000$ kg-m/day or 0.204 kWh/day.

**Step Two.**

The design insolation values read from the array-sizing nomograms (Figures A1a-e, A2a-e, A2a-e, and A4a-e) are shown on the worksheet (Table 5). The values correspond to a latitude 10° south and a horizontal June insolation of 4 kWh/m$^2$-day. We will select the tilt angle that gives the largest POA$_0$ values. For this example, that tilt angle is latitude plus 10°.

**Step Three.**

The values of days of storage listed on the worksheet (Table 5) were read from the storage-sizing nomograms (Figures Alf - A4f). These values correspond to an LOLP of 0.01.

Table 5. Completed sizing worksheet for direct coupled water pumping system.

<table>
<thead>
<tr>
<th>SIZING WORKSHEET</th>
</tr>
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<tbody>
<tr>
<td>Latitude</td>
</tr>
<tr>
<td>Design month</td>
</tr>
<tr>
<td>Average horizontal insolation in design month</td>
</tr>
<tr>
<td>Loss-of-load probability</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Insolation, POA$_0$ (kWh/m$^2$-day)</th>
<th>Days of Storage, S</th>
</tr>
</thead>
<tbody>
<tr>
<td>tilt angle, latitude plus</td>
<td></td>
</tr>
<tr>
<td>-20</td>
<td></td>
</tr>
<tr>
<td>-10</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.03</td>
</tr>
<tr>
<td>2</td>
<td>3.55</td>
</tr>
<tr>
<td>3</td>
<td>3.74</td>
</tr>
<tr>
<td>4</td>
<td>3.87</td>
</tr>
</tbody>
</table>

13
Step Four.

Figure 6 is a schematic for this direct coupled water pumping system with water storage. The components in the insolation to storage path are the array and the motor/pump set. Since the water distribution is gravity fed, there are no components in the storage to demand path. The storage to demand path's efficiency ($\eta_{\text{out}}$) is therefore 1.

![Schematic diagram of direct coupled water pumping system]

Figure 6. Schematic for direct coupled water pumping system.

There are no components in this system to control the operating voltage. Because the array and motor/pump efficiencies vary with the operating voltage, the designer must perform a single-day simulation to determine the insolation to storage path efficiency $\eta_{\text{in}}$. To perform the single-day simulation, the designer must first select the building block module and the motor/pump set and specify their performance characteristics. He must also specify the array configuration. The array configuration defines the array area. In cases for which the designer must perform a single-day simulation to determine the path efficiencies, he must first select an array configuration, determine the path efficiencies, and then use equation (3) to calculate $POA_0$. The $S$ value is determined by interpolating between the four $POA_0/S$ combinations that correspond to the selected tilt angle, obtained from the sizing nomograms. This procedure results in a single $POA_0/S$ combination. The designer must select another array configuration and repeat this procedure to define other $POA_0/S$ combinations. If the selected array configuration results in a $POA_0$ value that does not lie within the four $POA_0$ values obtained from the sizing nomograms, that array configuration is discarded.

**Component specifications**

For the motor/pump set, we will select a 1-hp submersible motor/centrifugal pump set. The operating current-voltage (I-V) curve of the motor is shown in Figure 7. The capacity ($Q$) of the pump is given by

$$Q = [2 \ g \ (H-h)]^{0.5} \ A \quad (7)$$

where $H$ is the total head developed by the pump, $h$ is static head, $A$ is the discharge pipe area, and $g$ is acceleration due to gravity (9.8 m/sec$^2$). The total head can be approximated from

$$H = H_0 \ (\text{RPM}/\text{RPM}_0)^2 \quad (8)$$
Figure 7. I-V curves for array and motor at 20°C.

where $H_0$ is the total head at RPM$_0$. The RPM is given by

$$\text{RPM} = \frac{[V - (I \text{ R}_\text{ap})]}{K_{\text{sp}}} \quad (9)$$

where $R_{\text{ap}}$ and $K_{\text{sp}}$ are characteristics of the motor. The following values were used for the motor/pump parameters.

- $A = 0.000025 \text{ m}^2$
- $H_0 = 59 \text{ m at 3432 RPM}$
- $R_{\text{ap}} = 0.387 \text{ ohms}$
- $K_{\text{sp}} = 0.0158 \text{ W/RPM}$

We will select a 40-watt module with the following characteristics.

- area $0.36 \text{ m}^2$
- maximum power voltage (Vmp) 16.50
- maximum power current (Imp) 2.45
- open circuit voltage (Voc) 20.60
- short circuit current (Isc) 2.74
- temperature coefficients
  - voltage ($C_v$) $0.0783 \text{ V/°C}$
  - current ($C_i$) $0.0022 \text{ A/°C}$

**Single-day simulation**

The system's operating voltage is defined by the intersection of the array and motor I-V curves. The array consists of parallel sets (Npar) of a string of modules in series (Nser). The array's voltage
is the module's voltage multiplied by Nser, and the array's current is the module's current multiplied by Npar. Figure 7 shows array I-V curves (at 20°C) for three parallel sets of two modules in series at various POA insolations. These I-V curves were derived using the module parameters and the I-V model found in "Residential Photovoltaic System Design Handbook" (7). For a given array configuration, the operating voltage and consequently the \( \eta_{\text{in}} \) path efficiency (product of the array and motor efficiencies) will vary with POA insulation and temperature.

To determine the \( \eta_{\text{in}} \) path efficiency for the design month, the designer must perform a single-day simulation that is representative of an average day. Using techniques found in "Fundamentals of Solar Radiation" (8), we can construct hourly POA insolation values for an average day from the average horizontal insolation, the latitude, and the tilt angle. Table 6 gives the hourly values of POA insolation, operating voltage, the array output, and pump output (hydraulic energy) at various array temperatures for the array configuration represented in Figure 7. The \( \eta_{\text{in}} \) path efficiency is the total

| Table 6. Single-day simulation for three parallel sets of two modules in series (2.16 m²). |
|---------------------------------|--------|--------|--------|--------|
| 10°C hour | POA\(_W/m²\) | Voltage | Array Output | Pump Output |
| 6-7, 18-19 | 41 | 9.8 | 3.3 | 0.0 |
| 7-8, 17-18 | 164 | 17.9 | 23.6 | 4.4 |
| 8-9, 16-17 | 337 | 25.0 | 67.4 | 13.9 |
| 9-10, 15-16 | 485 | 30.0 | 118.7 | 18.2 |
| 10-11, 14-15 | 601 | 32.8 | 148.9 | 20.6 |
| 11-12, 13-14 | 672 | 34.3 | 168.8 | 21.8 |
| totals | 4598 | | | 157.9 |

\[ \eta_{\text{in}} = \frac{157.9}{4598 \times 2.16} = 0.016 \]

| 20°C hour | POA\(_W/m²\) | Voltage | Array Output | Pump Output |
| 6-7, 18-19 | 41 | 8.2 | 2.8 | 0.0 |
| 7-8, 17-18 | 164 | 17.4 | 23.2 | 4.6 |
| 8-9, 16-17 | 337 | 25.1 | 68.0 | 13.9 |
| 9-10, 15-16 | 485 | 30.0 | 112.2 | 18.1 |
| 10-11, 14-15 | 601 | 32.5 | 144.3 | 20.3 |
| 11-12, 13-14 | 672 | 33.8 | 161.7 | 21.4 |
| totals | 4598 | | | 156.7 |

\[ \eta_{\text{in}} = \frac{156.7}{4598 \times 2.16} = 0.016 \]
Table 6. Continued

<table>
<thead>
<tr>
<th>30°C hour</th>
<th>POA (W/m²)</th>
<th>Voltage</th>
<th>Array Output (W)</th>
<th>Pump Output (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-7, 18-19</td>
<td>41</td>
<td>6.6</td>
<td>2.3</td>
<td>0.0</td>
</tr>
<tr>
<td>7-8, 17-18</td>
<td>164</td>
<td>17.4</td>
<td>23.3</td>
<td>4.7</td>
</tr>
<tr>
<td>8-9, 16-17</td>
<td>337</td>
<td>25.1</td>
<td>67.4</td>
<td>13.9</td>
</tr>
<tr>
<td>9-10, 15-16</td>
<td>485</td>
<td>29.6</td>
<td>109.6</td>
<td>17.9</td>
</tr>
<tr>
<td>10-11, 14-15</td>
<td>601</td>
<td>32.0</td>
<td>138.1</td>
<td>19.9</td>
</tr>
<tr>
<td>11-12, 13-14</td>
<td>672</td>
<td>33.2</td>
<td>153.1</td>
<td>20.9</td>
</tr>
<tr>
<td>totals</td>
<td>4598</td>
<td></td>
<td>154.8</td>
<td></td>
</tr>
</tbody>
</table>

\[ \eta_{in} = \frac{154.8}{(4598 \times 2.16)} = 0.016 \]

hydraulic energy divided by the total POA insolation times the array area. As the array temperature increases, the total array output decreases from 525.7 Wh/day at 10°C to 493.8 Wh/day at 30°C. The increase in array temperature, on the other hand, decreases the operating voltage, which increases the pump's efficiency. The net result is that the pump's output is nearly constant at about 157 Wh/day. Consequently, \( \eta_{in} \) is essentially constant at 0.016. For this system, the array temperature has no effect on \( \eta_{in} \) and can be ignored.

Select array configuration/storage capacity combinations

Single-day simulations were performed for all possible array configurations from a total of 1 to 20 modules. Table 7 lists the array configuration/storage capacity combinations that give POA values within the four POAₐ values obtained from the sizing nomograms. If there are no overriding constraints on either the array size or storage capacity, the designer would choose the least costly array configuration/storage capacity combination.

Table 7. Possible array configuration/storage capacity combinations for water pumping system.

<table>
<thead>
<tr>
<th># modules (Nser, Npar)</th>
<th>( A_a ) (m²)</th>
<th>( \eta_{in} )</th>
<th>POAₐ (kWh/m²-day)</th>
<th>S</th>
<th>Capacity (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 (2,3)</td>
<td>2.16</td>
<td>0.016</td>
<td>3.94</td>
<td>2.16</td>
<td>21.6</td>
</tr>
<tr>
<td>8 (2,4)</td>
<td>2.88</td>
<td>0.014</td>
<td>3.38</td>
<td>1.78</td>
<td>17.8</td>
</tr>
<tr>
<td>9 (3,3)</td>
<td>3.24</td>
<td>0.011</td>
<td>3.82</td>
<td>2.08</td>
<td>20.8</td>
</tr>
<tr>
<td>10 (2,5)</td>
<td>3.60</td>
<td>0.012</td>
<td>3.15</td>
<td>1.62</td>
<td>16.2</td>
</tr>
<tr>
<td>12 (3,4)</td>
<td>4.32</td>
<td>0.010</td>
<td>3.15</td>
<td>1.62</td>
<td>16.2</td>
</tr>
<tr>
<td>12 (4,3)</td>
<td>4.32</td>
<td>0.008</td>
<td>3.94</td>
<td>2.16</td>
<td>21.6</td>
</tr>
<tr>
<td>15 (5,3)</td>
<td>5.40</td>
<td>0.007</td>
<td>3.60</td>
<td>1.93</td>
<td>19.3</td>
</tr>
</tbody>
</table>

17
SUMMARY

We have presented a quantitative sizing procedure for stand-alone PV/storage systems. The technique involves the following four steps.

Step One. Define Site- and Application-Specific Parameters.

Site-specific parameters
- average horizontal insolation (December or June)
- latitude.

Application-specific parameters
- average daily demand
- loss-of-load probability (LOLP).

Step Two. Determine Array Sizes in terms of Design Insolations.

- four sets of design insolations.
- each sets contains a series of five design insolations, one for each tilt angle option.
- select tilt angle.

Step Three. Determine Storage Capacities in terms of Days of Storage.

- four values of days of storage, one for each set of design insolations.

Step Four. Translate Design Insolations and Days of Storage into Array Areas and Storage Capacities.

- define system configuration and component specifications.

- determine path efficiencies as defined by equations (1) and (2).

- use equations (3) and (4) to select array configurations/storage capacity combinations.
REFERENCES


APPENDIX A1.

SIZING NOMOGRAMS FOR SIZE SET 1
Figure A1a. Array-sizing nomogram.
Size set 1 - tilt angle = latitude minus 20°

Figure A1b. Array-sizing nomogram.
Size set 1 - tilt angle = latitude minus 10°
Figure A1c. Array-sizing nomogram.
Size set 1 - tilt angle = latitude

Figure A1d. Array-sizing nomogram.
Size set 1 - tilt angle = latitude plus 10°
Figure A1e. Array-sizing nomogram.
Size set 1 - tilt angle = latitude plus 20°

Figure A1f. Storage-sizing nomogram.
Size set 1
APPENDIX A2.

SIZING NOMOGRAMS FOR SIZE SET 2
Figure A2a. Array-sizing nomogram. Size set 2 - tilt angle = latitude minus 20°

Figure A2b. Array-sizing nomogram. Size set 2 - tilt angle = latitude minus 10°
Figure A2c. Array-sizing nomogram.
Size set 2 - tilt angle = latitude

Figure A2d. Array-sizing nomogram.
Size set 2 - tilt angle = latitude plus 10°
Figure A2e. Array-sizing nomogram.
Size set 2 – tilt angle = latitude plus 20°

Figure A2f. Storage-sizing nomogram.
Size set 2
APPENDIX A3.

SIZING NOMOGRAMS FOR SIZE SET 3
Figure A3a. Array-sizing nomogram.
Size set 3 - tilt angle = latitude minus 20°

Figure A3b. Array-sizing nomogram.
Size set 3 - tilt angle = latitude minus 10°
Figure A3c. Array-sizing nomogram.
Size set 3 - tilt angle = latitude

Figure A3d. Array-sizing nomogram.
Size set 3 - tilt angle = latitude plus 10°
Figure A3e. Array-sizing nomogram.
Size set 3 - tilt angle = latitude plus 20°

Figure A3f. Storage-sizing nomogram.
Size set 3
APPENDIX A4.

SIZING NOMOGRAMS FOR SIZE SET 4
Figure A4a. Array-sizing nomogram.
Size set 4 - tilt angle = latitude minus 20°

Figure A4b. Array-sizing nomogram.
Size set 4 - tilt angle = latitude minus 10°
Figure A4c. Array-sizing nomogram.
Size set 4 - tilt angle = latitude

Figure A4d. Array-sizing nomogram.
Size set 4 - tilt angle = latitude plus 10°
Figure A4e. Array-sizing nomogram.
Size set 4 - tilt angle = latitude plus 20°

Figure A4f. Storage-sizing nomogram.
Size set 4
APPENDIX B. Development of Sizing Nomograms

Basis of Loss-of-Load Simulation Model

The usefulness of the sizing nomograms depends on two factors: the accuracy and applicability of the simulation model. If the model is accurate, but only valid for one system, the nomograms are of little use. The same result holds if the model is applicable to any system, but is grossly inaccurate. Normally, as a performance model becomes more generic, it becomes less accurate. There is, however, an acceptable compromise.

Simulation model research at Sandia has shown that the method used to calculate the POA insolation dominates the accuracy of the performance predictions (1). The effects of other factors, such as how the array efficiency varies with temperature and insolation, tend to be second order. Therefore, as long as the POA insolation calculations are accurate, one can use constant efficiencies for the components without sacrificing accuracy. This does not imply that the values of the component efficiencies are unimportant. What it does mean is that if appropriate values are used for the constant efficiencies, the difference between LOLPs predicted using hourly calculated efficiencies and the constant efficiencies will be negligible. This approach is acceptable for most stand-alone systems. In this analysis, the appropriate values for the efficiencies are the average daily efficiencies for the design month.

Derivation of Loss-of-Load Simulation Model

The use of constant component efficiencies allows one to develop an LOLP simulation model in which all system-specific parameters are incorporated into two user-supplied parameters. The system is modeled by grouping the components into three subsystems, an array subsystem, a storage subsystem, and a demand subsystem, in such a way that the energy or product flows in and out of the storage subsystem can be represented by

\[ S_i = S_{i-1} + \left( \frac{\text{POA}_i}{\text{POA}_0} \right) - \text{LF}_{hr} \]

(1B)

where \( S_i \) is the normalized stored energy available for hour \( i \), \( \text{POA}_i \) is the plane of array insolation for hour \( i \), \( \text{POA}_0 \) is the design insolation, \( \text{LF}_{hr} \) is the fraction of the daily energy demand that occurs during hour \( i \), and \( S_{i-1} \) is the normalized stored energy available from the previous hour. Equation (1B) is derived below.

The amount of energy generated by the array subsystem (\( E_{a,i} \)) is given by

\[ E_{a,i} = \text{POA}_i A \eta_a \]

(2B)

where \( A \) is the array area, and \( \eta_a \) is the efficiency of the array subsystem. The energy flow in and out of the storage subsystem is
given by

\[ C_i = C_{i-1}^+ + \frac{E_{ai}}{f} \frac{\eta_{s, in}}{\eta_{1}} - \left[ LF_{hr} \frac{E_{i}}{\eta_{1}} - E_{ai} (1-f) \right] / \eta_{s, out} \]  

(3B)

where \( C_i \) and \( C_{i-1} \) are the available stored energies at the current and previous time steps, \( E_{i} \) is the daily energy demand, \( \eta_{1} \) is the average daily load subsystem efficiency, \( \eta_{s, in} \) and \( \eta_{s, out} \) are storage and retrieval efficiencies of the storage subsystem, and \( f \) is the fraction of \( E_{ai} \) that is added to storage.

The factor \( f \) was set equal to 1 which forces all energies to flow through the storage subsystem. As we shall see later, this choice of \( f \) allows the derivation of equation (1B). Although the value of 1 for \( f \) is does representative of the actual energy flows, it does not introduce error into the LOLP. Equation (3B) then reduces to

\[ C_i = C_{i-1}^+ + \frac{E_{ai}}{f} \frac{\eta_{s, in}}{\eta_{1}} - LF_{hr} \frac{E_{i}}{\eta_{1}} / \eta_{s, out} \]  

(4B)

At this point, it is convenient to introduce the concept of the design insolation, \( POA_{0} \), from which the array size is determined. It is defined as

\[ \frac{E_{i}}{\eta_{out}} = POA_{0} A \eta_{in}. \]  

(5B)

The parameter \( \eta_{in} \) is referred to as the path efficiency from insolation to storage and is defined as

\[ \eta_{in} = \eta_{a} \frac{\eta_{s, in}}{\eta_{1}}. \]  

(6B)

The parameter \( \eta_{out} \) is referred to as the path efficiency from storage to demand and is defined as

\[ \eta_{out} = \frac{\eta_{s, out}}{\eta_{1}}. \]  

(7B)

By solving equation (5B) for \( A \eta_{a} \) and substituting into equation (2B), we obtain

\[ \frac{E_{ai}}{\eta_{s, in}} = \frac{POA_{i}}{POA_{0}} \frac{E_{i}}{\eta_{out}} \]  

(8B)

Substituting equations (7B) and (8B) into equation (4B) and dividing through by \( E_{i}/\eta_{out} \), we obtain

\[ S_i = S_{i-1}^+ + \frac{POA_{i}}{POA_{0}} - LF_{hr} \]  

where \( S_i \) and \( S_{i-1} \) are defined as

\[ S_i = C_i / \left( E_{i}/\eta_{out} \right) \]  

(10B)

and are subject to the following bounds

\[ 0 \leq S_i \leq S_0 \]  

(11B)

where \( S_0 \) is the normalized storage capacity.

The loss-of-load for hour \( i \) (LOL\(_i\)) represents the amount of
load not satisfied and is defined as

$$LOL_i = \begin{cases} 0 & \text{if } E_i \geq LF_{hr} \\ LF_{hr} - E_i & \text{if } E_i < LF_{hr} \end{cases}$$ (12B)

where $E_i$ is the energy available for the load and is defined as

$$E_i = S_i + POA_i/POA_0.$$ (13B)

The long-term loss-of-load probability (LOLP) is the average of the $LOL_i$ values and is estimated as shown in Figure B1.

Figure B1. Calculation of loss-of-load probability (LOLP)
Testing of the Loss-of-Load Simulation Model

The loss-of-load (LOL) simulation model was verified by comparing its LOLP values against LOLP values estimated using the proven PVFORM simulation model (2). PVFORM consists of subroutines that estimate array efficiency, battery charge and discharge efficiencies, and inverter efficiency. The array efficiency varies with cell temperature which depends on wind speed, ambient temperature, and insolation. The battery charge and discharge efficiencies depend on battery state-of-charge and the inverter efficiency depends on the load-factor. All these efficiencies can vary significantly. This comparison is therefore of valid test of using constant component efficiencies. The parameters required to run PVFORM were set as shown in Table B1.

**TABLE B1. PVFORM Parameters.**

<table>
<thead>
<tr>
<th>Array</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>INOCT (installed operating cell temperature)</td>
<td>48°C</td>
</tr>
<tr>
<td>Reference efficiency at INOCT</td>
<td>10%</td>
</tr>
<tr>
<td>Maximum power temperature coefficient (default)</td>
<td>-0.4%/°C</td>
</tr>
<tr>
<td>Size</td>
<td>See Table B2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Battery</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum charge efficiency</td>
<td>80%</td>
</tr>
<tr>
<td>Maximum discharge efficiency</td>
<td>90%</td>
</tr>
<tr>
<td>Minimum state-of-charge</td>
<td>0.2</td>
</tr>
<tr>
<td>Capacity</td>
<td>see Table B2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total daily demand (constant)</td>
<td>120 kWh ac</td>
</tr>
<tr>
<td>Peak demand (default)</td>
<td>9 kW</td>
</tr>
<tr>
<td>Minimum demand (default)</td>
<td>3 kW</td>
</tr>
<tr>
<td>Hourly profile (default)</td>
<td>residential</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inverter</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating</td>
<td>10 kW ac</td>
</tr>
<tr>
<td>Full-load efficiency</td>
<td>92%</td>
</tr>
</tbody>
</table>

Comparisons were made with data from four sites, Albuquerque, NM, Charleston, TN, Medford, OR, and Seattle, WA. The LOL simulation model was run with the constant array efficiencies given in Table B2, battery charge and discharge efficiencies of 80 and 90 %, a minimum state-of-charge of 0.2, and an inverter efficiency of 91.2% with the residential hourly load profile. The plane-of-array calculations in PVFORM and the LOL simulation model are identical. The resulting LOLP values are shown in Table B2.

LOLPs were estimated at small and large battery capacities at each site to give comparisons at high and low values of LOLP. In all cases, the error induced by using constant efficiencies and a value of 1 for f is on the order of ± 50% and is within the uncertainty associated with the LOLP values given by this sizing technique.
test shows that the LOL simulation model is accurate when correct values for the average daily path efficiencies are used. Guidelines for determining these path efficiencies are given in the main body of this report.

Table B2. Comparison of LOL simulation model and PVFORM

<table>
<thead>
<tr>
<th>Site</th>
<th>Array_Size* (m²)</th>
<th>Battery Cap (kWh)</th>
<th>LOL simulation</th>
<th>PVFORM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albuquerque</td>
<td>365.0 (10.7)</td>
<td>150</td>
<td>0.038</td>
<td>0.027</td>
</tr>
<tr>
<td>Charleston</td>
<td>585.0 (10.1)</td>
<td>150</td>
<td>0.077</td>
<td>0.065</td>
</tr>
<tr>
<td>Medford</td>
<td>1266.0 (9.9)</td>
<td>150</td>
<td>0.045</td>
<td>0.039</td>
</tr>
<tr>
<td>Seattle</td>
<td>1825.0 (8.2)</td>
<td>158</td>
<td>0.090</td>
<td>0.080</td>
</tr>
</tbody>
</table>

* Number in parentheses is the average daily array efficiency in the design month as determined using PVFORM.

Generation of Sizing Nomograms

**Basis of sizing nomograms**

Given a value for POA₀ and the SOLMET data from one site, one can generate a curve of LOLP as a function of S₀ using the LOL simulation model described above. These analyses were performed over a range of S₀ values from 1 to 15. The lower value represents the minimum storage required for a stand-alone system, and the upper value represents the amount of storage at which LOLP approaches zero. We found that it was possible to define a set of POA₀'s, one for each site, so that the deviation between the LOLP curves from all twenty sites became very small and the POA₀ values could be correlated with the average December POA insolation (3). This result formed the basis for generating sizing nomograms in which the array size is given by readily available insolation data, and the storage capacity is given by the desired LOLP.

This preliminary work was done assuming that the array was tilted at latitude. Furthermore, insolation data are most commonly available as horizontal insolation and not POA insolation. Our first objective was to generate sizing nomograms that were applicable for a range of tilt angles from latitude minus 20° to latitude plus 20° and required horizontal insolation data as input instead of POA insolation data. A second objective was to generate nomograms that allow the designer to choose from several combinations of array size/storage capacity that give the same LOLP. These combinations would range from large array and small storage capacity to small array size and large storage capacity allowing the designer to select
the most cost-effective combination depending on the cost of the array and storage. The next section describes the approach used to meet these objectives.

**Method used to generate nomograms**

The overall approach was to define an appropriate correlation between the POA\(_0\) and average December POA insolations and then structure a parameter estimation routine to search for correlation parameters that minimize the deviation between the LOLP curves. The correlation was defined as

\[
POA_0 = a_0 + a_1 POA + a_2 POA^2 - a_3 POA^3, \quad (14B)
\]

where \(a_0 \ldots a_3\) are the correlation parameters. To eliminate the POA insolation as an input to the routine, it was related to the average December horizontal insolation using the method of Liu and Jordan (4).

The routine was structured as shown in Figure B2. The sequence of steps shown in Figure B2 are as described below.

**Step 1.** Select values for the tilt and \(S_0\). The resulting correlations will only be valid for the selected tilt angle.

**Step 2.** Use the LOL simulation model and SOLMET data to generate values of LOLP for a range of POA\(_0\) values and a preset value of \(S_0\). The values of LOLP define discrete functions \(f_n\)

\[
LOLP_n = f_n(POA_0, S_0), \quad (15B)
\]

where the subscript \(n\) denotes the site. The functions \(f_n\) are made up of 350 LOLP values corresponding to POA\(_0\) values from .2 to 7 kWh at an interval of .2 kWh.

**Step 3.** Calculate an average December POA insolation value for each site from the average horizontal insolation, tilt angle of the array, latitude of the site, and the ground albedo. The average horizontal insolations were derived directly from the SOLMET data. The ground albedo values had no discernible effect on the correlations. A value of .3 was used for all sites.

**Step 4.** Input an initial estimate of \(a_0 \ldots a_3\).

**Step 5.** Calculate a POA\(_0\) value for each site using equation (14B) and the current values for \(a_0 \ldots a_3\).

**Step 6.** Determine an LOLP value for each site that corresponds to the POA\(_0\) value calculated in Step 5 using the functions \(f_n\) defined in Step 2. The LOLP values are determined by interpolating between the points in \(f_n\).

**Step 7.** Calculate the least squares difference between the LOLP\(_n\) values and a preset value LOLP\(_0\). In this type of analysis, one normally calculates an average LOLP value and uses that value to calculate the difference. This technique
Figure B2. Structure of parameter estimations routine.

cannot be used in this analysis because the LOLP values go to zero as POA decreases (i.e., the array size increases). If the difference was calculated from the average, the parameter estimation routine would always give the trivial solution in which \( (POA_0)_n = 0 \), \( LOLP_n = 0 \), and the difference \( = 0 \).

The approach used in this analysis also provides a convenient means to generate correlations for various array/storage combinations. By fixing \( LOLP_0 \) at some value and increasing \( S_0 \), we force the correlation to generate higher \( POA_0 \) values (smaller array sizes). The net result
is the same LOLP\textsubscript{n} values at larger storage capacities (larger $S_0$s) and smaller array sizes (larger POA\textsubscript{0}s). Correlations were generated with $S_0$ set equal to 2.5, 4.0, 5.5, and 7.0.

Note that the difference is only calculated one $S_0$ value rather than over the entire range of $S_0$ values. This greatly simplified coding and executing the parameter estimation routine and did not reduce the accuracy of the correlations.

NOTE - Step 8 was performed using a function minimization routine provided by R. J. Hanson, Sandia National Laboratories. A description of the technique can be found elsewhere (5).

Step 8. Determine if the differences are minimized. If they are, go to Step 9. If not, estimate new values of $a_0$ ... $a_3$ and return to Step 4.

Step 9. The current values for $a_0$ ... $a_3$ define the correlation for the current tilt angle. This correlation is used to generate the array-sizing nomograms. The storage-sizing nomograms are generated by calculating LOLP values for $S$ values from 1 to 14.5 using the LOL simulation model and the POA\textsubscript{0} values defined by the correlation.

Four sets of correlations were defined by twenty runs of the parameter estimation routine. The four sets represent four array/storage combinations generated by varying the value of $S_0$. Each set contains five correlations corresponding to tilt angles of latitude plus -20, -10, 0, 10, and 20°. The parameters $a_0$ ... $a_3$ for all twenty correlations are given in Table B3. The tilt angle had no effect on the storage-sizing nomograms. Each set of nomograms therefore has a single storage-sizing nomogram that represents all tilt angles.
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</table>
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