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B77 Impact Test Data Processing

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ABSTRACT

With the system of equations presented, variables which describe the motion of B77 Impact Test Units can be obtained from high-speed motion picture data. Equations are developed that describe the kinematics of the body in terms of linear and angular positions, velocities and acceleration, flight path azimuth and elevation angles, and total angle of attack and windward meridian angle. The dynamics of the body are described in terms of its linear and angular momentum, its kinetic and potential energies, and the external forces and moments.

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B77 IMPACT TEST DATA PROCESSING

1. Introduction

In the B77 impact test program, both instrumented and noninstrumented units (ITUs) are impacted on targets at various velocities and attitudes. During each test, the units are photographed by several high-speed motion picture cameras. The sequences of photographs obtained are processed using an image matching technique to obtain the position and attitude of the body at discrete times. From this data, the following variables that describe the motion of the body during impact can be obtained: velocity and acceleration of various points in the body, angular velocity and acceleration, linear and angular momentum, kinetic and potential energies, and external forces and moments. Other variables that describe the trajectory of the body, such as the trajectory azimuth and flight path angles, the total angle of attack, and the windward meridian angle, can also be obtained.

This report presents a derivation of a system of equations used to obtain the variables that describe the motion of the body from the measured values of position and attitude. Section 2 of this report defines the coordinate systems used in the analysis. Equations which describe the kinematics and the dynamics of the body are developed in Sections 3 and 4 respectively. Section 5 describes the data processing procedure which can be used to obtain the variables of interest. A digital computer program that performs the required numerical computations is included in Appendix A. Sample data plots are shown in Appendix B.

2. Coordinate Systems

The Cartesian coordinate systems used in this analysis are defined by triads of mutually orthogonal unit vectors $(\vec{i}, \vec{j}, \vec{k})$ that define the positive directions of the coordinate system axes. Since all coordinate systems are right-handed, the position and orientation of any coordinate system are uniquely specified by the location of the origin and the directions of any two of the vectors of the triad $(\vec{i}, \vec{j}, \vec{k})$.

For this analysis, an Earth-fixed coordinate system is assumed as the inertial reference coordinate system. As shown in Figure 1, the origin of the inertial coordinate system ($\vec{i}_i$, $\vec{j}_i$, $\vec{k}_i$) is located at the midpoint of a line segment between bench marks B. M. 1 and B. M. 2; the $\vec{k}_i$ axis coincides with the local vertical and is directed upward; and the $\vec{j}_i$ axis coincides with the line between B. M. 1 and B. M. 2 and is directed from B. M. 1 toward B. M. 2.

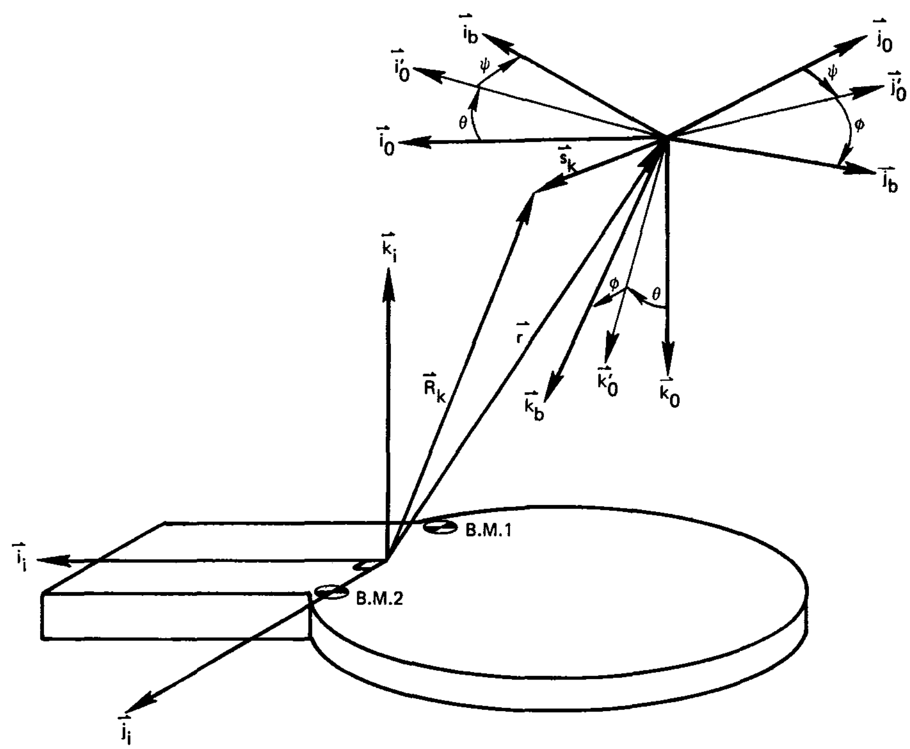


Figure 1. Inertial, Local Reference and Body-Fixed Coordinate Systems

The local reference coordinate system ($\vec{i}_0$, $\vec{j}_0$, $\vec{k}_0$) is obtained from the inertial coordinate system by a translation so that the origin coincides with a point fixed in the body, followed by a positive rotation about the $\vec{i}_0$ axis through an angle of 180°. The position of the point is described by the vector $\vec{r}$.

The body-fixed coordinate system ($\vec{i}_b$, $\vec{j}_b$, $\vec{k}_b$) is obtained from the local reference coordinate system by an ordered sequence of three positive rotations, as illustrated in Figure 1. The first rotation is through an angle θ about $\vec{j}_0$ to obtain (i_0', j_0, k_0'); the second rotation is through an angle ψ

about $\vec{k}_0'$ to obtain $(\vec{i}_b, \vec{j}_0', \vec{k}_0')$; the third rotation is through an angle ϕ about $\vec{i}_b$ to obtain $(\vec{i}_b, \vec{j}_b, \vec{k}_b)$.

3. Kinematics

The vector $\vec{r}$, which describes the position of the origin of the body-fixed coordinate system, can be expressed in terms of its components along the inertial coordinate system axes as

$$\vec{r} = \vec{i}_i x + \vec{j}_i y + \vec{k}_i z \tag{3.1}$$

The velocity of this point

$$\vec{v} = \frac{d\vec{r}}{dt} \tag{3.2}$$

can be expressed either in terms of its components along the inertial coordinate system axes as

$$\vec{v} = \vec{i}_i \dot{x} + \vec{j}_i \dot{y} + \vec{k}_i \dot{z} \tag{3.3}$$

where

$$\dot{x} = \frac{dx}{dt}$$

$$\dot{y} = \frac{dy}{dt}$$

$$\dot{z} = \frac{dz}{dt} \tag{3.4}$$

or in terms of its components along the body-fixed coordinate system axes as

$$\vec{v} = \vec{i}_b u + \vec{j}_b v + \vec{k}_b w \tag{3.5}$$

These two sets of components are related by

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = A(\theta,\psi,\phi)\, B(\pi) \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} \tag{3.6}$$

where $A(\theta, \psi, \phi)$ is a matrix defined by

$$A(\theta, \psi, \phi) = B(\phi)\, C(\psi)\, D(\theta) \tag{3.7}$$

$$B(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \tag{3.8}$$

$$C(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.9}$$

$$D(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \tag{3.10}$$

Let

$$\vec{\omega} = \vec{i}_b p + \vec{j}_b q + \vec{k}_b r \tag{3.11}$$

represent the angular velocity of the body expressed in terms of its components along the body-fixed coordinate system axes. The components p, q, and r are related to the time rates of change of the Euler angles

$$\begin{pmatrix} \dot{\theta} \\ \dot{\psi} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} \frac{d\theta}{dt} \\ \frac{d\psi}{dt} \\ \frac{d\phi}{dt} \end{pmatrix} \tag{3.12}$$

by

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = Q(\psi, \phi) \begin{pmatrix} \dot{\theta} \\ \dot{\psi} \\ \dot{\phi} \end{pmatrix} \tag{3.13}$$

where

$$Q(\psi, \phi) = \begin{bmatrix} \sin\psi & 0 & 1 \\ \cos\phi\cos\psi & \sin\phi & 0 \\ -\sin\phi\cos\psi & \cos\phi & 0 \end{bmatrix} \tag{3.14}$$

Let

$$\vec{R}_k = \vec{r} + \vec{s}_k \tag{3.15}$$

represent the position of the k^{th} arbitrarily chosen point which is fixed in the body. The velocity of this point is then

$$\vec{V}_k = \frac{d\vec{R}_k}{dt}$$

$$= \vec{v} + \frac{\delta\vec{s}_k}{\delta t} + \vec{\omega} \times \vec{s}_k \tag{3.16}$$

where $\frac{\delta\vec{s}_k}{\delta t}$ represents the velocity of the point as seen in the body-fixed coordinate system. Since, by hypothesis, the point is fixed in the body,

$$\frac{\delta\vec{s}_k}{\delta t} = 0 \tag{3.17}$$

and Equation (3.16) becomes

$$\vec{V}_k = \vec{v} + \vec{\omega} \times \vec{s}_k \tag{3.18}$$

This vector can be expressed in terms of its components along the body-fixed coordinate system axes as

$$\vec{V}_k = \vec{i}_b\, U_k + \vec{j}_b\, V_k + \vec{k}_b\, W_k \tag{3.19}$$

where

$$\begin{pmatrix} U_k \\ V_k \\ W_k \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \begin{pmatrix} qs_{kz} - rs_{ky} \\ rs_{kx} - ps_{kz} \\ ps_{ky} - qs_{kx} \end{pmatrix} \tag{3.20}$$

and s_{kx}, s_{ky}, and s_{kz} are the components of $\vec{s}_k$ in the body-fixed coordinate system, i.e.,

$$\vec{s}_k = \vec{i}_b\, s_{kx} + \vec{j}_b\, s_{ky} + \vec{k}_b\, s_{kz} \tag{3.21}$$

The acceleration of $\vec{R}_k$ is given by

$$\begin{aligned}
\vec{A}_k &= \frac{d\vec{V}_k}{dt} \\
&= \frac{d}{dt}(\vec{v} + \vec{\omega} \times \vec{s}_k) \\
&= \vec{a} + \vec{\omega} \times \left(\frac{\delta\vec{s}_k}{\delta t} + \vec{\omega} \times \vec{s}_k\right) + \left(\frac{\delta\vec{\omega}}{\delta t} + \vec{\omega} \times \vec{\omega}\right) \times \vec{s}_k \\
&= \vec{a} + \vec{\omega} \times (\vec{\omega} \times \vec{s}_k) + \vec{b} \times \vec{s}_k
\end{aligned} \tag{3.22}$$

where

$$\vec{a} = \frac{d\vec{v}}{dt} \tag{3.23}$$

is the acceleration of the origin of the body-fixed coordinate system, and where

$$\vec{b} = \frac{\delta\vec{\omega}}{\delta t} \tag{3.24}$$

is the angular acceleration of the body.

The linear acceleration can be expressed either in terms of its components along the inertial or body-fixed coordinate system axes as

$$\begin{aligned}
\vec{a} &= \vec{i}_i\ddot{x} + \vec{j}_i\ddot{y} + \vec{k}_i\ddot{z} \\
&= \vec{i}_b\dot{u} + \vec{j}_b\dot{v} + \vec{k}_b\dot{w}
\end{aligned} \tag{3.25}$$

where

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} \frac{d\dot{x}}{dt} \\ \frac{d\dot{y}}{dt} \\ \frac{d\dot{z}}{dt} \end{pmatrix} \tag{3.26}$$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = A(\theta, \psi, \phi)\, B(\pi) \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} \tag{3.27}$$

The angular acceleration can be expressed in terms of its components along the body-fixed coordinate system axes as

$$\vec{b} = \vec{i}_b \dot{p} + \vec{j}_b \dot{q} + \vec{k}_b \dot{r} \tag{3.28}$$

where

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} \frac{dp}{dt} \\ \frac{dq}{dt} \\ \frac{dr}{dt} \end{pmatrix} \tag{3.29}$$

With these results, $\vec{A}_k$ can be expressed in terms of its components along the body-fixed coordinate system axes as

$$\vec{A}_k = \vec{i}_b A_{kx} + \vec{j}_b A_{ky} + \vec{k}_b A_{kz} \tag{3.30}$$

where

$$\begin{pmatrix} A_{kx} \\ A_{ky} \\ A_{kz} \end{pmatrix} = \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} + \begin{pmatrix} p(qs_{ky} + rs_{kz}) - (q^2 + r^2)s_{kx} \\ q(rs_{kz} + ps_{kx}) - (r^2 + p^2)s_{ky} \\ r(ps_{kx} + qs_{ky}) - (p^2 + q^2)s_{kz} \end{pmatrix} + \begin{pmatrix} \dot{q}s_{kz} - \dot{r}s_{ky} \\ \dot{r}s_{kx} - \dot{p}s_{kz} \\ \dot{p}s_{ky} - \dot{q}s_{kx} \end{pmatrix} \tag{3.31}$$

With reference now to Figure 2, the azimuth angle ξ is the angle between the projection of the velocity vector $\vec{v}$ on the $(\vec{i}_i, \vec{j}_i)$ plane and the axis defined by $\vec{i}_i$. The flight path angle β is the angle between the velocity vector and its projection on the $(\vec{i}_i, \vec{j}_i)$ plane. These angles are given by

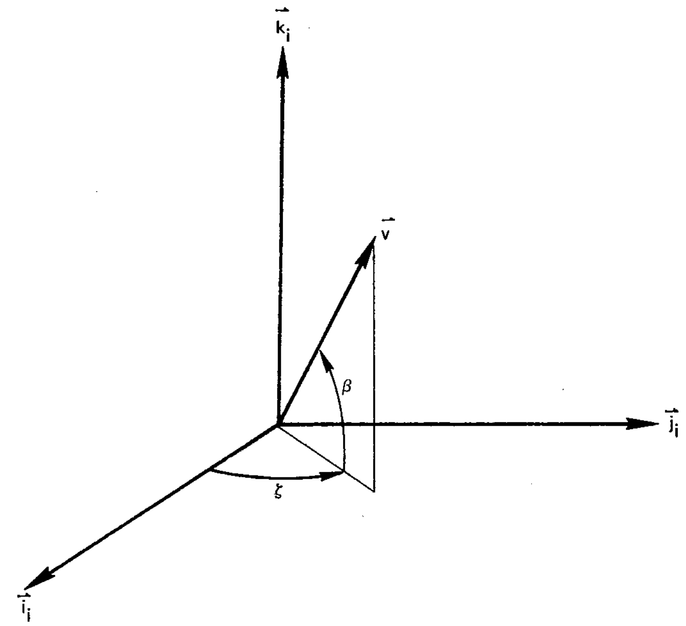


Figure 2. Trajectory Azimuth and Flight Path Angles

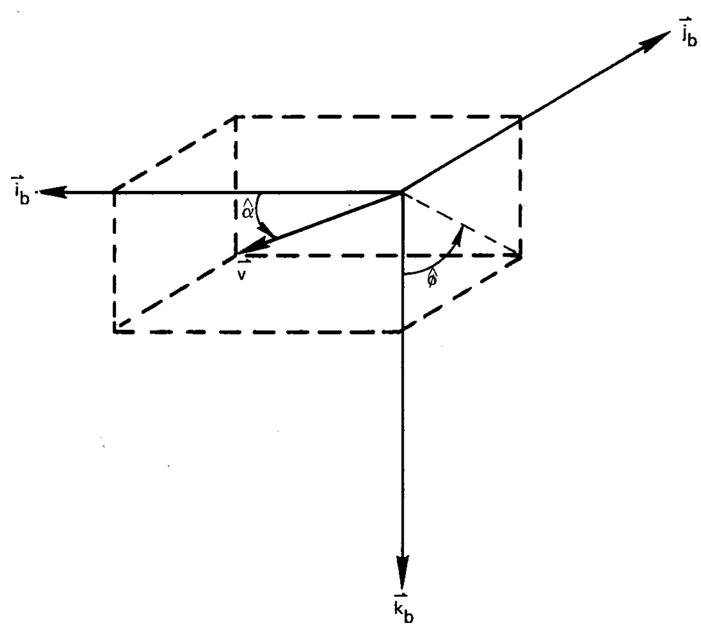


Figure 3. Total Angle of Attack and Windward Meridian Angle

$$\zeta = \text{atan}\left(\frac{\dot{y}}{\dot{x}}\right) \tag{3.32}$$

$$\beta = \text{atan}\ \frac{\dot{z}}{\sqrt{\dot{x}^2 + \dot{y}^2}} \tag{3.33}$$

As illustrated in Figure 3, the total angle of attack $\hat{\alpha}$ is the angle between the velocity vector $\vec{v}$ and $\vec{i}_b$. The windward meridian angle $\hat{\phi}$ is the angle between projection of $\vec{v}$ on the $(\vec{j}_b, \vec{k}_b)$ plane and $\vec{k}_b$. These angles are given by

$$\hat{\alpha} = \text{atan}\left(\frac{\sqrt{u^2 + w^2}}{u}\right) \tag{3.34}$$

$$\hat{\phi} = \text{atan}\left(\frac{v}{w}\right) \tag{3.35}$$

The magnitude of the velocity of the origin of the body-fixed coordinate system is given by

$$\begin{aligned} \nu &= \sqrt{u^2 + v^2 + w^2} \\ &= \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \end{aligned} \tag{3.36}$$

The magnitude of the velocity of the k^{th} point fixed in the body is given by

$$\nu_k = \sqrt{U_k^2 + V_k^2 + W_k^2} \tag{3.37}$$

4. Dynamics

With reference to Figure 4, let m_i represent the mass of the i^{th} infinitesimal mass particle of the body and let its position be described by the vector

$$\vec{r}_i = \vec{r} + \vec{\rho}_i \tag{4.1}$$

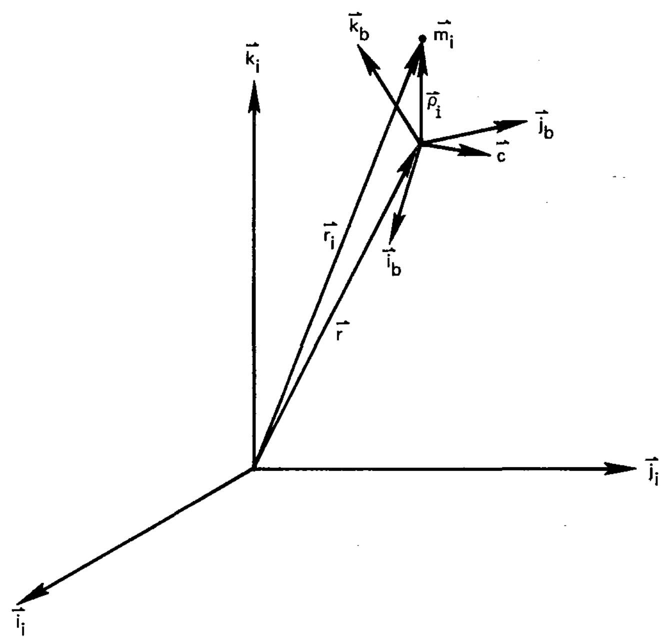


Figure 4. Center of Mass and ith Mass Particle

The kinetic energy associated with the body is the sum of the kinetic energies of the individual mass particles and is given by

$$T = \frac{1}{2} \sum_i m_i \frac{d\vec{r}_i}{dt} \cdot \frac{d\vec{r}_i}{dt} \tag{4.2}$$

The velocity of the i^{th} mass particle is given by

$$\frac{d\vec{r}_i}{dt} = \vec{v} + \frac{\delta\vec{\rho}_i}{\delta t} + \vec{\omega} \times \vec{\rho}_i \tag{4.3}$$

where $\frac{\delta \rho_i}{\delta t}$ represents the velocity of the i^{th} mass particle as seen in the body-fixed coordinate system. If the body is assumed to be rigid,

$$\frac{\delta\vec{\rho}_i}{\delta t} = 0 \tag{4.4}$$

and consequently

$$\frac{d\vec{r}_i}{dt} = \vec{v} + \vec{\omega} \times \vec{\rho}_i \tag{4.5}$$

With this result, Equation (4.2) can be written as

$$\begin{aligned}
T &= \frac{1}{2} \sum_i m_i (\vec{v} + \vec{\omega} \times \vec{\rho}_i) \cdot (\vec{v} + \vec{\omega} \times \vec{\rho}_i) \\
&= \frac{1}{2} \sum_i m_i \vec{v} \cdot \vec{v} + \sum_i m_i \vec{v} \cdot (\vec{\omega} \times \vec{\rho}_i) + \frac{1}{2} \sum_i m_i (\vec{\omega} \times \vec{\rho}_i) \cdot (\vec{\omega} \times \vec{\rho}_i) \\
&= \frac{1}{2} \overline{m}\vec{v} \cdot \vec{v} + \vec{v} \cdot (\vec{\omega} \times \overline{m}\vec{c}) + \frac{1}{2} \vec{\omega} \cdot \sum_i (\vec{\rho}_i \times m_i (\vec{\omega} \times \vec{\rho}_i)) \\
&= \frac{1}{2} \overline{m}\vec{v} \cdot \vec{v} + \vec{v} \cdot (\vec{\omega} \times \overline{m}\vec{c}) + \frac{1}{2} \vec{\omega} \cdot \vec{\vec{I}} \vec{\omega}
\end{aligned} \tag{4.6}$$

where $\vec{\vec{I}}$ represents the inertia tensor defined by

$$\vec{\vec{I}} \vec{\omega} = \sum_i (\vec{\rho}_i \times m_i (\vec{\omega} \times \vec{\rho}_i)) \tag{4.7}$$

The total mass of the body is represented by

$$\bar{m} = \sum_i m_i \tag{4.8}$$

and the center of mass of the body in the body-fixed coordinate system is represented by

$$\vec{c} = \frac{\sum_i m_i \vec{\rho}_i}{\bar{m}} \tag{4.9}$$

Equation (4.6) can be written in the form

$$T = \frac{1}{2}\vec{v} \cdot (\bar{m}\vec{v} + \vec{\omega} \times \bar{m}\vec{c}) + \frac{1}{2}\vec{\omega} \cdot \vec{\vec{I}}\vec{\omega} + \frac{1}{2}\vec{v} \cdot (\vec{\omega} \times \bar{m}\vec{c}) \tag{4.10}$$

Since

$$\vec{v} \cdot (\vec{\omega} \times \bar{m}\vec{c}) = \vec{\omega} \cdot (\bar{m}\vec{c} \times \vec{v}) \tag{4.11}$$

it follows that

$$\begin{aligned} T &= \frac{1}{2}\vec{v} \cdot (\bar{m}\vec{v} + \vec{\omega} \times \bar{m}\vec{c}) + \frac{1}{2}\vec{\omega} \cdot (\vec{\vec{I}}\vec{\omega} + \bar{m}\vec{c} \times \vec{v}) \\ &= \frac{1}{2}\vec{v} \cdot \vec{G} + \frac{1}{2}\vec{\omega} \cdot \vec{H} \end{aligned} \tag{4.12}$$

where

$$\vec{G} = \bar{m}\,(\vec{v} + \vec{\omega} \times \vec{c}) \tag{4.13}$$

represents the linear momentum of the body and

$$\vec{H} = \vec{\vec{I}}\vec{\omega} + \bar{m}\vec{c} \times \vec{v} \tag{4.14}$$

represents the angular momentum about the origin of the body-fixed coordinate system.

With reference now to Equation (4.7), the components of the vector $\vec{\vec{I}}\vec{\omega}$ can be conveniently expressed in terms of its components along the body-fixed coordinate system axes in the following manner. Let

$$\vec{\rho}_i = \vec{i}_b x_i + \vec{j}_b y_i + \vec{k}_b z_i \tag{4.15}$$

be the representation of $\vec{\rho}_i$ in the body-fixed coordinate system. Then

$$\begin{aligned}\vec{\vec{I}}\vec{\omega} &= \sum_i \vec{\rho}_i \times m_i \, (\vec{\omega} \times \vec{\rho}_i) \\ &= \vec{i}_b (I_{xx} p + I_{xy} q + I_{xz} r) \\ &+ \vec{j}_b (I_{yx} p + I_{yy} q + I_{yz} r) \\ &+ \vec{k}_b (I_{zx} p + I_{zy} q + I_{zz} r)\end{aligned} \tag{4.16}$$

where

$$I_{xx} = \sum_i m_i (y_i^2 + z_i^2) \tag{4.17}$$

$$I_{yy} = \sum_i m_i (x_i^2 + z_i^2) \tag{4.18}$$

$$I_{zz} = \sum_i m_i (x_i^2 + y_i^2) \tag{4.19}$$

$$I_{xy} = I_{yx} = -\sum_i m_i x_i y_i \tag{4.20}$$

$$I_{yz} = I_{zy} = -\sum_i m_i y_i z_i \tag{4.21}$$

$$I_{zx} = I_{xz} = -\sum_i m_i z_i x_i \tag{4.22}$$

are the moments and products of inertia in the body-fixed coordinate system.

With these results, the linear momentum can be expressed in terms of its components along the body-fixed coordinate system axes as

$$\vec{G} = \vec{i}_b g_x + \vec{j}_b g_y + \vec{k}_b g_z \tag{4.23}$$

where

$$\begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} = \begin{bmatrix} 0 & \bar{m}c_z & -\bar{m}c_y \\ -\bar{m}c_z & 0 & \bar{m}c_x \\ \bar{m}c_y & -\bar{m}c_x & 0 \end{bmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} + \begin{bmatrix} \bar{m} & 0 & 0 \\ 0 & \bar{m} & 0 \\ 0 & 0 & \bar{m} \end{bmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{4.24}$$

The angular momentum can also be expressed in terms of its components along the body-fixed coordinate system axes as

$$\vec{H} = \vec{i}_b h_x + \vec{j}_b h_y + \vec{k}_b h_z \tag{4.25}$$

where

$$\begin{pmatrix} h_x \\ h_y \\ h_z \end{pmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} + \begin{bmatrix} 0 & -\bar{m}c_z & \bar{m}c_y \\ \bar{m}c_z & 0 & -\bar{m}c_x \\ -\bar{m}c_y & \bar{m}c_x & 0 \end{bmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{4.26}$$

Equation (4.12) can now be written as

$$T = \frac{1}{2} \begin{pmatrix} p \\ q \\ r \\ u \\ v \\ w \end{pmatrix}' \cdot \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} & 0 & -\bar{m}c_z & \bar{m}c_y \\ I_{yx} & I_{yy} & I_{yz} & \bar{m}c_z & 0 & -\bar{m}c_x \\ I_{zx} & I_{zy} & I_{zz} & -\bar{m}c_y & \bar{m}c_x & 0 \\ 0 & \bar{m}c_z & -\bar{m}c_y & \bar{m} & 0 & 0 \\ -\bar{m}c_z & 0 & \bar{m}c_x & 0 & \bar{m} & 0 \\ \bar{m}c_y & -\bar{m}c_x & 0 & 0 & 0 & \bar{m} \end{bmatrix} \begin{pmatrix} p \\ q \\ r \\ u \\ v \\ w \end{pmatrix} \tag{4.27}$$

The potential energy of the body is the sum of the potential energies of the individual mass properties. The potential energy due to gravity is

$$\begin{aligned} V &= \sum_n m_n \vec{r}_n \cdot \vec{k}_i g \\ &= \vec{k}_i g \cdot \sum_n m_n (\vec{r} + \vec{\rho}_n) \\ &= \bar{m} g\, \vec{k}_i \cdot (\vec{r} + \vec{c}) \end{aligned} \tag{4.28}$$

where g represents the acceleration of gravity, which is assumed to be constant.

The vector $\vec{c}$ can be expressed in terms of its components along the inertial coordinate system axes as

$$\vec{c} = \vec{i}_i c_1 + \vec{j}_i c_2 + \vec{k}_i c_3 \tag{4.29}$$

where

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = [A(\theta, \psi, \phi)\, B(\pi)]^{-1} \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix} \tag{4.30}$$

Since $A(\theta, \psi, \phi)$ and $B(\phi)$ are orthonormal matrices, Equation (4.30) can be written as

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = B'(\pi)\, A'(\theta, \psi, \phi) \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix} \tag{4.31}$$

where A' and B' denote the transposes of A and B respectively. Equation (4.28) can now be written as

$$\begin{aligned} V &= \bar{m} g\, \vec{k}_i \cdot (\vec{r} + \vec{c}) \\ &= \bar{m} g\, \vec{k}_i \cdot [\vec{i}_i (x + c_1) + \vec{j}_i (y + c_2) + \vec{k}_i (z + c_3)] \\ &= \bar{m} g\, (z + c_3) \end{aligned} \tag{4.32}$$

The total energy associated with the body is the sum of the potential and kinetic energies.

$$E = T + V \tag{4.33}$$

The external force and moment acting on the body are given by

$$\vec{F} = \frac{d\vec{G}}{dt}$$

$$= \frac{d}{dt}\,[\overline{m}(\vec{v} + \vec{\omega} \times \vec{c})]$$

$$= \overline{m}\left[\vec{a} + \vec{\omega} \times \left(\frac{\delta\vec{c}}{\delta t} + \vec{\omega} \times \vec{c}\right) + \left(\frac{\delta\vec{\omega}}{\delta t} + \vec{\omega} \times \vec{\omega}\right) \times \vec{c}\right]$$

$$= \overline{m}\left[\vec{a} + \vec{\omega} \times (\vec{\omega} \times \vec{c}) + \frac{\delta\vec{\omega}}{\delta t} \times \vec{c}\right] \tag{4.34}$$

and

$$\vec{M} = \frac{d\vec{H}}{dt} - (\vec{\omega} \times \vec{c}) \times m\vec{v}$$

$$= \frac{d}{dt}\,[\vec{\vec{I}}\,\vec{\omega} + \overline{m}\,\vec{c} \times \vec{v}]$$

$$= \vec{\vec{I}}\left(\frac{\delta\vec{\omega}}{\delta t} + \vec{\omega} \times \vec{\omega}\right) + \overline{m}\left[\vec{c} \times \vec{a} + \left(\frac{\delta\vec{c}}{\delta t} + \vec{\omega} \times \vec{c}\right) \times \vec{v}\right]$$

$$= \vec{\vec{I}}\,\frac{\delta\vec{\omega}}{\delta t} + \overline{m}\left[\vec{c} \times \vec{a}\right] \tag{4.35}$$

The force and moment can be expressed either in terms of their components along the body-fixed or the inertial coordinate system axes as

$$\vec{F} = \vec{i}_b X + \vec{j}_b Y + \vec{k}_b Z$$

$$= \vec{i}_i f_x + \vec{j}_i f_y + \vec{k}_i f_z \tag{4.36}$$

and

$$\vec{M} = \vec{i}_b L + \vec{j}_b M + \vec{k}_b N$$

$$= \vec{i}_i \ell + \vec{j}_i m + \vec{k}_i n \tag{4.37}$$

where

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \overline{m}\left[\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} + \begin{pmatrix} p(qc_y + rc_z) - (q^2 + r^2)c_x \\ q(rc_z + pc_x) - (r^2 + p^2)c_y \\ r(pc_x + qc_y) - (p^2 + q^2)c_z \end{pmatrix} + \begin{pmatrix} \dot{q}c_z - \dot{r}c_y \\ \dot{r}c_x - \dot{p}c_z \\ \dot{p}c_y - \dot{q}c_x \end{pmatrix}\right] \tag{4.38}$$

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = B'(\pi)\, A'(\theta, \psi, \phi) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \tag{4.39}$$

and

$$\begin{pmatrix} L \\ M \\ N \end{pmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} + \bar{m} \left[\begin{pmatrix} \dot{w}_{cy} - \dot{v}c_z \\ \dot{u}c_z - \dot{w}c_x \\ \dot{v}c_x - \dot{u}c_y \end{pmatrix} + \begin{pmatrix} w(rc_x - pc_z) - v(pc_y - qc_x) \\ u(pc_y - qc_x) - w(qc_z - rc_y) \\ v(qc_z - rc_y) - u(rc_x - pc_z) \end{pmatrix} \right] \tag{4.40}$$

$$\begin{pmatrix} \ell \\ m \\ n \end{pmatrix} = B'(\pi)\, A'(\theta, \psi, \phi) \begin{pmatrix} L \\ M \\ N \end{pmatrix} \tag{4.41}$$

With reference now to Equations (4.23) through (4.26), the linear and angular momentum vectors can be expressed in terms of their components in the inertial coordinate system as

$$\vec{G} = \vec{i}_i G_x + \vec{j}_i G_y + \vec{k}_i G_z \tag{4.42}$$

and

$$\vec{H} = \vec{i}_i H_x + \vec{j}_i H_y + \vec{k}_i H_z \tag{4.43}$$

respectively where

$$\begin{pmatrix} G_x \\ G_y \\ G_z \end{pmatrix} = B'(\pi)\, A'(\theta, \psi, \phi) \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} \tag{4.44}$$

and

$$\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = B'(\pi)\, A'(\theta, \psi, \phi) \begin{pmatrix} h_x \\ h_y \\ h_z \end{pmatrix} \tag{4.45}$$

5. Data Processing Procedures

The procedures that use the previously derived equations to process the impact test data are described in this section. The constants and parameters needed to process the data are listed in Table I. All constants, parameters, and variables must be in a consistent system of units. The English system of units with mass in slugs, length in feet, and time in seconds is specified to be consistent with existing capabilities. The parameters which describe the inertial properties of the body, the mass and the moments and products of inertia, are assumed to be piece-wise constant functions of time.

TABLE I

CONSTANTS AND PARAMETERS

Symbol	Units	Remarks
s_{kx}	ft	$k = 1, 2, \ldots, K$; Location of k^{th} point in the body at which velocity and acceleration are to be computed.
s_{ky}	ft	
s_{kz}	ft	
c_x	ft	Location of the center of mass (piece-wise constant functions of time)
c_y	ft	
c_z	ft	
$\bar{m}$	slugs	Mass of the body (piece-wise constant function of time)
I_{xx}	slug-ft^2	Moments and products of inertia (piece-wise constant functions of time)
I_{yy}	slug-ft^2	
I_{yz}	slug-ft^2	
$I_{xy} = I_{yx}$	slug-ft^2	
$I_{yz} = I_{zy}$	slug-ft^2	
$I_{zx} = I_{xz}$	slug-ft^2	
g	ft/sec^2	Acceleration of gravity

The input variables are listed in Table II and the output variables in Table III. The input data consists of the three variables x, y, and z that describe the position of one point fixed in the body (the origin of the body-fixed coordinate system) and the three variables θ, ψ, and ϕ that describe the angular position of the body. The values of these variables are obtained at discrete instants of time from photographs taken by high-speed motion picture cameras.

TABLE II

INPUT VARIABLES

Symbol	Units	Remarks
x	feet	Location of the origin of the body-fixed coordinate system.
y	feet	
z	feet	
ψ	radians*	Euler angles which describe angular position of the body
θ	radians*	
ϕ	radians*	

*The computer program described in Appendix A uses angles measured in degrees rather than radians.

TABLE III

OUTPUT VARIABLES

Symbol	Units	Remarks
$\dot{x}$ $\dot{y}$ $\dot{z}$	ft/sec ft/sec ft/sec	Inertial components of body velocity
$\dot{\psi}$ $\dot{\theta}$ $\dot{\phi}$	radians/sec* radians/sec* radians/sec*	Time rate of change of Euler angles
u v w	radians/sec* radians/sec* radians/sec*	Body components of body velocity
p q r	radians/sec* radians/sec* radians/sec*	Body components of angular velocity
U_k V_k W_k	ft/sec ft/sec ft/sec	Body components of velocity of k^{th} point fixed in the body
$\ddot{x}$ $\ddot{y}$ $\ddot{z}$	ft/sec^2 ft/sec^2 ft/sec^2	Inertial components of acceleration
$\dot{p}$ $\dot{q}$ $\dot{r}$	rad/sec^2* rad/sec^2* rad/sec^2*	Body components of angular acceleration
$\dot{u}$ $\dot{v}$ $\dot{w}$	ft/sec^2 ft/sec^2 ft/sec^2	Body components of acceleration

*The computer program described in Appendix A uses angles measured in degrees rather than radians.

TABLE III (continued)

Symbol	Units	Remarks
A_{kx} A_{ky} A_{kz}	ft/sec^2 ft/sec^2 ft/sec^2	Body components of acceleration of k^{th} point fixed in the body
ζ	radians*	Azimuth angle
β	radians*	Flight path angle
$\hat{\alpha}$	radians*	Total angle of attack
$\hat{\phi}$	radians*	Windward meridian angle
ν	ft/sec	Magnitude of velocity $\vec{v}$
ν_k	ft/sec	Magnitude of velocity $\vec{V}_k$
g_x g_y g_z	slug-ft/sec slug-ft/sec slug-ft/sec	Body components of linear momentum
h_x h_y h_z	slug-ft/sec slug-ft/sec slug-ft/sec	Body components of angular momentum
T	ft-lbs	Kinetic energy of body
V	ft-lbs	Potential energy of body
E	ft-lbs	Total energy of body
X Y Z	lbs lbs lbs	Body components of external force
f_x f_y f_z	lbs lbs lbs	Inertial components of external force

*The computer program described in Appendix A uses angles measured in degrees rather than radians.

TABLE III (continued)

Symbol	Units	Remarks
L	ft-lbs	Body components of external moment about the origin of the body-fixed coordinate system
M	ft-lbs	
N	ft-lbs	
ℓ	ft-lbs	Inertial components of external moment about the origin of the body-fixed coordinate system
m	ft-lbs	
n	ft-lbs	
G_x	slug-ft/sec	Inertial components of linear momentum
G_y	slug-ft/sec	
G_z	slug-ft/sec	
H_x	slug-ft/sec	Inertial components of angular momentum
H_y	slug-ft/sec	
H_z	slug-ft/sec	

The variables $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{\theta}$, $\dot{\psi}$, $\dot{\phi}$ defined by Equations (3.4) and (3.12) can be obtained by numerical differentiation techniques. These techniques require input data over relatively large intervals of time to compute the derivatives at any one instant of time. Consequently, these six variables must be computed in a preliminary pass of the data.

Once the six derivatives mentioned above have been obtained, the elements of the matrix $A(\theta, \psi, \phi)$ as defined by Equations (3.7) through (3.10) can be computed, and the variables u, v, and w can be computed from Equation (3.6). Similarly, the elements of the matrix $E(\psi, \phi)$ as defined by Equation (3.14) can be computed and the variables p, q, and r obtained from Equation (3.13). The values of the variables U_k, V_k, and W_k $(k = 1, 2, \ldots, K)$ can be computed from Equation (3.20).

A second pass at the data is required to obtain the derivatives $\ddot{x}$, $\ddot{y}$, $\ddot{z}$, $\dot{p}$, $\dot{q}$, and $\dot{r}$ as defined by Equations (3.26 and (3.29). The values of the variables $\dot{u}$, $\dot{v}$, and $\dot{w}$ can then be obtained from Equation (3.27), and the values of the variables A_{kx}, A_{ky}, and A_{kz} $(k = 1, 2, \ldots, K)$ can be obtained from Equation (3.31). The variables ζ, β, $\hat{\alpha}$, $\hat{\phi}$, ν, and ν_k $(k = 1, 2, \ldots, K)$ can be obtained from Equations (3.32) through (3.37) respectively.

Next, the components of linear momentum (g_x, g_y, and g_z) and the components of angular momentum (h_x, h_y, and h_z) can be obtained from Equations (4.24) and (4.26) respectively. The kinetic energy T can be obtained from Equation (4.27). The value of c_3 can be obtained from Equation (4.31), and the potential energy V can then be computed from Equation (4.32). The total energy is obtained from Equation (4.32). The components of external force in the body-fixed coordinate system X, Y, and Z and in the inertial system f_x, f_y, and f_z can be obtained from Equations (4.38) and (4.39). Similarly, the components of the external moment in the two coordinate systems L, M, and N and ℓ, m, and n can be obtained from Equations (4.40) and (4.41).

APPENDIX A--USER'S INSTRUCTIONS FOR PROGRAM IMPACT

Introduction

The purpose of program IMPACT is to compute angular velocities and accelerations, linear velocities and accelerations of various points on a body, linear and angular momentum, kinetic and potential energies, and the external forces and moments from data obtained from high-speed photographic film of an impacting body. The equations used are described in the main text of this document. The program IMPACT exists on a permanent file named MACHLIB which is a part of Division 8411's Secondary Processing Library for the CDC 6600 computer. The program uses the word-addressable Master Disc File which is standard to all programs in the Secondary Processing Library.

File Structure

Master Disc File

The Master Disc File is a word-addressable file as described in the CDC 6000 Computer Systems User's Guide for Cyber Record Manager.

Word-Addressable File - General--The word-addressable file is one of several file formats available in the Record Manager System used by FORTRAN extended. The word-addressable file can be considered as one continuous string of data starting at word 1 at the beginning of the file and ending at word N, the last word in the file (see Figure A-1). Every word is uniquely identified by a number from 1 to N. The word-addressable files can be accessed randomly or sequentially and can only exist on mass storage. The user must do all of the bookkeeping to read or write data.

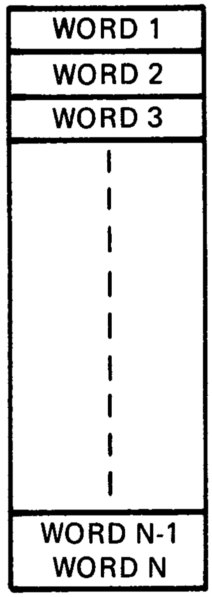


Figure A-1. Word-Addressable File

The word-addressable files used by the Data Reduction programs are specified record type U. This means that the records are not defined, and the user must specify a record length on both read and write operations. The Record Manager System cannot determine where the U type record begins or ends on a file. There are no ENDFILE indicators written on the files and they would be ignored if written.

The logical file names are defined in the program and are not listed in the program statement. With a few exceptions, the input file is named MDF and the output file MDF1. If any external file is attached, the user must be sure that the file has been renamed to conform with the logical file name internal to the program. This is done by using a FILE card and a LDSET card.

FILE (ℓfn old, LFN = ℓfn new)

LDSET (FILES = ℓfn old)

Word-Addressable File - Data Reduction--The following is a brief description of the word-addressable file (WAF) used by the Data Reduction Secondary Processing Libraries. The structure maintains the original formal of the Master Disc File (MDF) with modifications necessary to make the file word-addressable.

The first 900 words are reserved for the word address table that is maintained by the user for read and write operations. The first data entry is at address 901 and is always the IDT record for the first data channel. An IDT record is entered for each channel of data in a frame or (data node) for the subfile. The IDT records must be written out in the same sequence as the sequence of the data in the data frame.

Following the IDT records are the data. The first word of the data record is a key word (DTA) followed by a second word which normally indicates the number of words in the record. (For the word-addressable file, the second word is not used.) The data follows these two words in one continuous string to the end.

A terminator record (referred to as an END record) is written at the end of the data which consists of three words, END, 1, 0. This constitutes a subfile, and the number of subfiles that can be written is dependent upon the space available in the word address table.

If the WAF is written with one channel of data per subfile, 149 channels of data may be written. This means that there is room for only 149 unique identifiers in the word address table. However, if the WAF is written with 16 channels per frame, a maximum of 384 channels of data can be written out; these are located in a total of only 24 subfiles. Figure A-2 illustrates the file structure.

The IDT Record

The IDT record is a 128-word record which identifies a single channel of information in the MDF. The record also has the necessary parameters and labeling information to generate a plot of that data without reading in additional parameter cards. The description of the IDT record follows.

WORD	CHAR	FORMAT	ITEM	ORIGIN	COMMENTS
1	1-3	BCD	RECORD LABEL	PROG	ALWAYS IDT
2	1-10	FXP	NUMBER OF WORDS	PROG	ALWAYS 126
3	1-8	BCD	PARAMETER IDENTIFICATION	IDT	BLANKS REMOVED
4	1-10	FXP	THT FILE NUMBER	PROG	
5	1-10	FXP	THT REEL NUMBER	LABEL	
6	1-10	FXP	KEYWORD	PROG	INITIALLY ZERO
7	1-10	FLP	INTERPOLATE DELTA TIME	PROG	
8	1-10	FXP	NUMBER OF TIME GAPS	PROG	
9-18	1-100	FLP	SCALE TIME OF TIME GAPS	PROG	
19-28	1-100	FLP	MAGNITUDE OF TIME GAPS	PROG	
29	1-10	BCD	DATA BASE REEL NO.	LBL	
30	1-10	BCD	DATA BASE ISSUE NO.	LBL	
31	1-10	FXP	TYPE DATA FLAG	--	PICKED UP FROM COMMON
32	1-10	FXP	NO. POINTS IN TRANSFORM / FOURID OR OUTPUT POINTS FROM FILTER		
33	1-5	BCD	TEST NUMBER	LBL	
34	1	BCD	TEST NUMBER SUFFIX	LBL	
35	1-3	BCD	TEST RUN NUMBER	LBL	
36	1-8	BCD	TEST DATE	LBL	

37-38	1-16	BCD	CHARGE NUMBER	LBL	
39-40	1-20	BCD	TPE NAME	LBL	
41	1-4	BCD	TPE ORGANIZATION	LBL	
42-43	1-20	BCD	DATA ANALYST	LBL	
44-45	1-16	BCD	TEST ITEM	LBL	
46	1-5	BCD	RECORDING SOURCE		PICKED UP FROM IND. PROG.
47-49	1-40	FXP	UNUSED SPACE	PROG	ALWAYS ZERO
50	1-8	BCD	PARAMETER IDENTIFICATION	PLT	UNCHANGED FROM THT
51	1-10	BCD	PARAMETER UNITS	PLT	
52	1-10	BCD	PLOT DATA UNITS	PLT	
53	1-10	FLP	PLOT CONVERSION FACTOR	PLT	
54	1-4	BCD	PLOT CLASSIFICATION	PLT	
55	1-4	BCD	PLOT SIZE CODE	PLT	
56	1-10	FLP	PLOT GRID SIZE	PLT	
57	1-10	FXP	PLOT AXIS CODE	PLT	
58	1-10	FXP	PLOT SYMBOL CODE	PLT	
59-62	1-40	BCD	PLOT Y ANNOTATION	PLT	
63-66	1-40	BCD	PLOT TIME ANNOTATION	PLT	
67	1-10	BCD	PLOT TIME UNITS	PLT	
68	1-10	FLP	PLOT TIME CONVERSION	PLT	
69	1-10	FLP	FILTER DELAY CORRECTION	PLT	
70	1-10	FLP	PLOT REFERENCE TIME	PLT	
71-78	1-80	BCD	PLOT TITLE	PLT	
79	1-10	FLP	PLOT START TIME	PLT	
80	1-10	FLP	PLOT STOP TIME	PLT	
81	1-10	FLP	PLOT DATA MINIMUM	PLT	
82	1-10	FLP	PLOT DATA MAXIMUM	PLT	
83	1-10	FXP	TAPE TRACK NO.	PLT	
84	1-10	FLP	VCO CENTER FREQ. (KHZ)	PLT	
85	1-10	FLP	VCO DEVIATION (KHZ)	PLT	
86-87	1-20	FXP	UNUSED SPACE	PROG	ALWAYS ZERO
88	1-10	FLP	STATIC CORRECTION	NOR	
89	1-10	FLP	STATIC VALUE	NOR	
90	1-10	FLP	STD. DEV. STATIC CORR.	NOR	
91	1-10	FXP	NO. SAMPLES STATIC CORR.	NOR	
92	1-10	FLP	HIGH BAND EDGE	NOR	
93	1-10	FLP	CENTER BAND	NOR	
94	1-10	FLP	LOW BAND EDGE	NOR	
95	1-10	FLP	UPPER CAL. LEVEL	NOR	
96	1-10	FLP	LOWER CAL. LEVEL	NOR	
97	1-10	FLP	CAL. STD. DEV.	NOR	
98	1-10	FLP	STD. DEV. UPPER CAL.	NOR	
99	1-10	FLP	STD. DEV. LOWER CAL.	NOR	
100	1-10	FXP	NO. SAMPLES UPPER CAL.	NOR	
101	1-10	FXP	NO. SAMPLES LOWER CAL.	NOR	
102	1-10	FXP	NO. DATA SAMPLES / SEGMENT (PSD)		
103	1-10	FLP	DUPL. OF WD 95 - NOR INFO ZERO'D AT TIMES OR		
104	1-10	FLP	DUPL. OF WD 96 - USED FOR COMMENTS (SEE BELOW)		
105-106	1-40	FXP	UNUSED SPACE	PROG	ALWAYS ZERO
107-112	1-60	FXP	RUN START TIME	RUN	
113	1-10	FLP	DATA MINIMUM VALUE	MMX	
114	1-10	FLP	DATA MAXIMUM VALUE	MMX	
115	1-10	FXP	NO. OF ERRORS	MMX	
116	1-10	FXP	NUMBER OF OPERATIONS	MMX	
117-128	1-120	BCD	OPERATIONS	MMX	

SOME SECONDARY PROCESSING PROGRAMS PLACE A COMMENT IN THE NOR AREA.

WORD 88 - 'COMMENT ' OR 'WNDTNL ' (LEFT JUSTIFIED)
WORD 89
THRU 100- MESSAGE AREA
WORD 101- BLANK AS ARE ANY UNUSED WORDS ABOVE

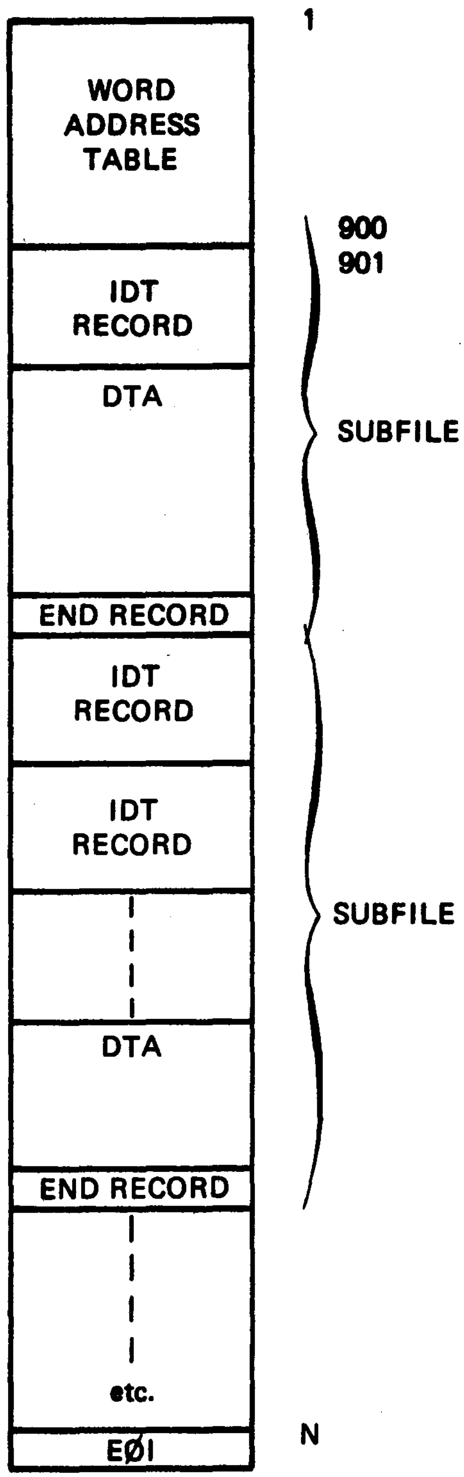


Figure A-2. File Structure

Test Data Input

The input file to program IMPACT must be a word-addressable Master Disc File as described on page 35. This file contains the data recovered from the high-speed photographic film. The following information is required for each test.

Symbol	Channel Identifier	Units	Remarks
x	X	Feet	Location of the origin of the body fixed co-ordinate system
y	Y	Feet	
z	Z	Feet	
ψ	Yaw	Degrees	Euler angles which describe angular position of body
θ	Pitch	Degrees	
ϕ	Roll	Degrees	

Parameter Card Input

Parameter Cards

1. Control information may be entered in one of three card formats:

   a. The fixed-field card-identifier format consists of a 1-to-5-character card identifier beginning in Column 1, followed by one or more fixed-data fields as defined by the program.

   b. The free-field card-identifier format corresponds to a fixed-field card and has the following syntax:

      ID=data field, data field, ..., data field

      where the card identifier ID may begin in any column. Blank fields are appended or surplus fields deleted from the right, if necessary, so that a free-field card defines the same parameters as the fixed-field card with the same identifier.

   c. The parameter-identifier format has the following syntax:

ID_1=data field, ID_2=data field, ..., ID_n=data field

where ID_j is a 1-to-10-character parameter identifier defined by the program.

2. Three types of data fields are used.

   a. Integer - A number written in I format may appear anywhere in the field.

   b. Real - A number written in I, F, or E format may appear anywhere in the field.

   c. Name - A 1-to-10-character alphanumeric parameter may appear anywhere in the field.

   Internal blank characters are ignored in all three types of fields.

Parameter Card Description

The ten parameter cards defined for program IMPACT have the following card identifiers: CHAN, TIME, SK, SK1, SK2, MP, MP1, MPR, MPR1, and PASS. The information for all of the cards except the PASS card may be entered in any of the three formats described on pages 40 and 41. The PASS card information can be entered only in the fixed-field format. Each card is described in the fixed-field, card-identifier format.

1. CHAN card - Identifies up to seven channels of data to be selected for input.

Column	Input Parameter Identifier	Type	Description
1-4	---	--	CHAN
5-10	---	--	Blank
11-20	ID1	Name	Channel Name
21-30	ID2	"	" "
31-40	ID3	"	" "
41-50	ID4	"	" "
51-60	ID5	"	" "
61-70	ID6	"	" "
71-80	ID7	"	" "

2. TIME card - Defines the time span of interest and the time increment.

Column	Parameter Identifier	Type	Description
1-4	--	--	TIME
5-10	--	--	Blank
11-20	START	Real	Time in seconds, referenced to plot reference time. Default is time of first sample.
21-30	STOP	Real	Time in seconds, referenced to plot reference time. Default is time of last sample.
31-40	DELT	Real	Time increment for interpolation. A value of 0 indicates data will not be interpolated.
41-50	NPT	Integer	Maximum number of points to be taken. (Not implemented)
51-80	--	--	Blank

3. SK card - Defines the locations of point 1 and point 2 in the body at which velocities and accelerations are to be computed.

Symbol	Column	Identifier	Type	Description
	1-2	--	--	SK
	3-10	--	--	Blank
S_{1x}	11-20	SKX1	Real	X-coordinate of first point in body
S_{1y}	21-30	SKY1	Real	Y-coordinate of first point in body
S_{1z}	31-40	SKZ1	Real	Z-coordinate of first point in body
S_{2x}	41-50	SKX2	Real	X-coordinate of second point in body

Symbol	Column	Identifier	Type	Description
S_{2y}	51-60	SKY2	Real	Y-coordinate of second point in body
S_{2z}	61-70	SKZ2	Real	Z-coordinate of second point in body
	71-80	--	--	Blank

4. SKI card - Defines the locations of point 3 and point 4 in the body at which velocities and acceleration are to be computed.

Symbol	Column	Parameter Identifier	Type	Description
	1-3	--	--	SK1
	4-10	--	--	Blank
S_{3x}	11-20	SKX3	Real	X-coordinate of third point in body
S_{3y}	21-30	SKY3	Real	Y-coordinate of third point in body
S_{3z}	31-40	SKZ3	Real	Z-coordinate of third point in body
S_{4x}	41-50	SKX4	Real	X-coordinate of fourth point in body
S_{4y}	51-60	SKY4	Real	Y-coordinate of fourth point in body
S_{4z}	61-70	SKZ4	Real	Z-coordinate of fourth point in body
	71-80	--	--	Blank

5. SL2 card - Defines the locations of point 5 and point 6 in the body at which velocities and accelerations are to be computed.

Symbol	Column	Parameter Identifier	Type	Description
	1-3	--	--	SK2
	4-10	--	--	Blank
S_{5x}	11-20	SKX5	Real	X-coordinate of fifth point in body
S_{5y}	21-30	SKY5	Real	Y-coordinate of fifth point in body
S_{5z}	31-40	SKZ5	Real	Z-coordinate of fifth point in the body
S_{6x}	41-50	SKX6	Real	X-coordinate of sixth point in body
S_{6y}	51-60	SKY6	Real	Y-coordinate of sixth point in body
S_{6z}	61-70	SKZ6	Real	Z-coordinate of sixth point in body
	71-80	--	--	Blank

6. MP card - Defines the mass properties of the body at beginning of data.

Symbol	Column	Parameter Identifier	Type	Description
	1-2	--	--	MP
	3-10	--	--	Blank
	11-20	T1	Real	Time for application of mass property data of body
$\bar{m}$	21-30	BMASS1	Real	Mass of body in slugs
c_x	31-40	CMX1	Real	X-coordinate of center of mass in feet
c_y	41-50	CMY1	Real	Y-coordinate of center of mass in feet
c_z	51-60	CMZ1	Real	Z-coordinate of center of mass in feet
I_{xx}	61-70	XIX1	Real	X-moment of inertia
I_{yy}	71-80	YIY1	Real	Y-moment of inertia

7. MP1 card - Continuation of the MP card to define the remaining mass properties of the body.

Symbol	Column	Parameter Identifier	Type	Description
	1-3	--	--	MP1
	4-10	--	--	Blank
I_{zz}	11-20	ZIZ1	Real	Z-moment of inertia
$I_{xy}=I_{yx}$	21-30	XIY1	Real	XY-product of inertia
$I_{yz}=I_{zy}$	31-40	YIZ1	Real	YZ-product of inertia
$I_{zx}=I_{xz}$	41-50	ZIX1	Real	ZX-product of inertia
	51-80	--	--	Blank

8. MPR card - Defines the second set of mass properties of the body and the time these values are to be used.

Symbol	Column	Parameter Identifier	Type	Description
	1-3	--	--	MPR
	4-10	--	--	Blank
	11-20	T2	Real	Time for application of this set of mass property data
$\bar{m}$	21-30	BMASS2	Real	Mass of the body in slugs
c_x	31-40	CMX2	Real	X-coordinate in feet of center of mass
c_y	41-50	CMY2	Real	Y-coordinate in feet of center of mass
c_z	51-60	CMZ2	Real	Z-coordinate in feet
I_{xx}	61-70	XIX2	Real	X-moment of inertia
I_{yy}	71-80	YIY2	Real	Y-moment of inertia

9. MPR1 card - Continuation of the MPR card to define the remaining mass properties of the body.

Symbol	Column	Parameter Identifier	Type	Description
	1-4	--	--	MPR1
	5-10	--	--	Blank
I_{zz}	11-20	ZIZ2	Real	Z-moment of inertia
I_{xy}	21-30	XIY2	Real	X-Y product of inertia
I_{yz}	31-40	YIZ2	Real	Y-Z product of inertia
I_{zx}	41-50	ZIX2	Real	Z-X product of inertia
	51-80	--	--	Blank

10. PASS card - Indicates the end of a set of parameter cards.

Column	Parameter Identifier	Type	Description
1-4	--	--	PASS
5	--	--	Blank
6-10	--	Integer	Fatal error limit. Default = 10
11-15	--	Integer	Nonfatal error limit. Default = 20

Parameter Card Organization

The parameter cards are organized into a set using any or all of the cards described on pages 42 through 46 (sections 2 through 10) and terminated by a PASS card. The cards are order-independent. The parameters read in for a pass remain until they are redefined by a succeeding set of parameter cards. Parameters associated with continuation cards (MP1 and MPR1) are blanked when the primary card MP and MPR are read.

Diagnostic Messages

All diagnostic information is listed on a file named TAPE61. Program DUMP61 lists and summarizes the diagnostic information.

Errors are classified into two types, "fatal" and "nonfatal." A fatal error is one in which one or more channels of information cannot be computed. A nonfatal error is one which can be corrected by substitution of a default option. The data is processed, but the result may not be what was originally requested.

The first and second fields on the PASS card are the fatal and nonfatal error limits, respectively. If either error limit is reached, the program will "exit."

Output Data File

The data computed by program IMPACT is written out in seven or more subfiles in Division 8411's standard word addressable Master Disc File format as described on pages 36-39. The IDT records have been modified to accompany the output data so that the MDF can be easily read and the data plotted by the program MDFPLT. The file can also be used as input to any of the secondary data processing programs on the permanent library file, DRWALIB.

The content of the file is as follows:

Subfile No. 1

Symbol	Channel Identifier	Units	Remarks
x	X(SM)	Feet	Smoothed x coordinate of the body
y	Y(SM)	Feet	Smoothed y coordinate of the body
z	Z(SM)	Feet	Smoothed z coordinate of the body
ψ	PSI(SM)	Degrees	Smoothed yaw angle of the body
θ	THE(SM)	Degrees	Smoothed pitch angle of the body
ϕ	PHI(SM)	Degrees	Smoothed roll angle of the body

Subfile No. 2

Symbol	Channel Identifier	Units	Remarks
$\dot{x}$	XD	FT/SEC	ψ inertial component of velocity
$\ddot{x}$	XDD	FT/S/S	X inertial component of acceleration
$\dot{y}$	YD	FT/SEC	Y inertial component of velocity
$\ddot{y}$	YDD	FT/S/S	Y inertial component of acceleration
$\dot{z}$	ZD	FT/SEC	Z inertial component of velocity
$\ddot{z}$	ZDD	FT/S/S	Z inertial component of acceleration
$\dot{\theta}$	THED	DEG/SEC	Time rate of change of pitch
$\dot{\psi}$	PSID	DEG/SEC	Time rate of change of yaw
$\dot{\phi}$	PHID	DEG/SEC	Time rate of change of roll
p	PP	DEG/SEC	
q	QQ	DEG/SEC	Body components of angular velocity
r	RR	DEG/SEC	
$\dot{p}$	PPD	DEG/S/S	
$\dot{q}$	QQD	DEG/S/S	Body components of angular acceleration
$\dot{r}$	RRD	DEG/S/S	

Subfile No. 3

Symbol	Channel Identifier	Units	Remarks
u	UU	FT/SEC	
v	VV	FT/SEC	Body components of velocity
w	WW	FT/SEC	
ν	NU	FT/SEC	Magnitude of velocity
$\dot{u}$	UUD	FT/S/S	
$\dot{v}$	VVD	FT/S/S	Body components of acceleration
$\dot{w}$	WWD	FT/S/S	
ζ	ZETA	DEGREES	Azimuth angle
β	BETA	DEGREES	Flight path angle
$\hat{\alpha}$	ALPH	DEGREES	Total angle of attack
$\hat{\phi}$	PHIH	DEGREES	Windward meridian angle

Subfile No. 4 or 4 and 5 (Depending on the number of points selected.)

Symbol	Channel Identifier	Units	Remarks
U_1	U1	FT/SEC	Body components of velocity at first selected point of the body
V_1	V1	FT/SEC	
W_1	W1	FT/SEC	
ν_1	NU1	FT/SEC	Magnitude of velocity at 1^{st} point
.	.	.	
.	.	.	
.	.	.	
U_k		FT/SEC	Body components of velocity at k^{th} selected point of the body
V_k		FT/SEC	
W_k		FT/SEC	
ν_k		FT/SEC	Magnitude of velocity at k^{th} point

Subfile No. 5 or 6 and 7

Symbol	Channel Identifier	Units	Remarks
A_{1x}	A1X	FT/S/S	Body components of acceleration at first selected point of the body
A_{1y}	A1Y	FT/S/S	
A_{1z}	A1Z	FT/S/S	
.	.	.	
.	.	.	
.	.	.	
A_{kx}	AkX	FT/S/S	Body components of acceleration at k^{th} selected point of the body
A_{ky}	AkY	FT/S/S	
A_{kz}	AkZ	FT/S/S	

NOTE: The program has the capability of computing velocities and accelerations for up to six selected points in the body. The number of subfiles required for data output will depend on the number of points selected. If only two locations are selected, the data is written out in two subfiles. One subfile contains the velocities; the other contains the accelerations. If six body locations are selected, a total of four subfiles are written: two containing velocity data and two containing acceleration data.

Subfile No. 6 or 8

Symbol	Channel Identifier	Units	Remarks
g_x	GX	SLUG-FT/S	Body components of linear momentum
g_y	GY	SLUG-FT/S	
g_z	GZ	SLUG-FT/S	
h_x	HX	SLUG-FT/S	Body components of angular momentum
h_y	HY	SLUG-FT/S	
h_z	HZ	SLUG-FT/S	
T	KE	FT-LBS	Kinetic energy of the body
V	PE	FT-LBS	Potential energy of the body
E	TE	FT-LBS	Total energy of the body
X	BX	LBS	Body components of external force
Y	BY	LBS	
Z	BZ	LBS	
f_x	FX	LBS	Inertial components of external force
f_y	FY	LBS	
f_z	FZ	LBS	
L	LL	FT-LBS	X body component of external moment

Subfile No. 7 or 9

Symbol	Channel Identifier	Units	Remarks
M	MM	FT-LBS	y body component of external moments
N	NN	FT-LBS	z body component of external moments
ℓ	SL	FT-LBS	Inertial components of external moment
m	SM	FT-LBS	
n	SN	FT-LBS	
G_x	GGX	SLUG-FT/S	Inertial components of linear momentum
G_y	GGY	SLUG-FT/S	
G_z	GGZ	SLUG-FT/S	

Subfile No. 7 or 9 (continued)

Symbol	Channel Identifier	Units	Remarks
H_x	HHX	SLUG-FT/S	Inertial components of angular momentum
H_y	HHY	SLUG-FT/S	
H_z	HHZ	SLUG-FT/S	

Program Execution

Program IMPACT is on a permanent library named MACHLIB. The program also uses subroutines that are located on DRWALIB. To load and execute program IMPACT, both libraries must be attached as illustrated.

```
ATTACH(MLIB, MACHLIB, ID=EDHDR)
ATTACH(WALIB, DRWALIB, ID=EDHDR)
LIBRARY(MLIB, WALIB)
LIBLOAD(MLIB, IMPACT)
EXECUTE.
```

The program is looking for an input file with a logical name MDF; therefore when attaching the data file it must have the logical file name MDF.

```
ATTACH(MDF, IMP129A, ID=B77DTA)
```

The output file has a logical file name MDF1. Any subsequent use of this file may require that the file be given a new logical file name to agree with the file name internal to the program. If a person wants to plot the data from the output file, it would have to be given the new name MDF by using the following SCOPE cards.

```
FILE(MDF, LFN=MDFL)
LIBLØAD(WALIB, MDFPLT)
LDSET(FILES=MDF)
LDSET(LIB=FTNLIBY)
EXECUTE.
PLT75.
```

The CDC 6600 SCOPE cards are followed by an EØR(7, 8, 9) card. This card is followed by the input parameter cards which are in turn followed by another EØR card.

JØB card

SCOPE cards

$^{7}8_{9}$ EØR card

parameter cards

$^{7}8_{9}$ EØR card

$^{6}{}^{7}8_{9}$ End of job card.

To accomplish the smoothing and compute the first and second derivatives, the subroutine SMØØ from the Sandia Mathematical Program Library[1] is used. The subroutine computes the parameters of a smoothing spline fit to the data. This subroutine uses an array of error estimates which provide for a looser or tighter fit of the data. The results of the numerical differentiation are not the most satisfactory; however, they are about as good as we can obtain with the techniques now available. It is necessary to make several runs with adjusted error estimates to produce reasonable derivatives.

[1] Huddleston, R. E. and Jefferson, T. H., User's Guide to Sandia Mathematical Program Library at Livermore, Sandia Laboratories, Livermore, SAND76-8209, March 1976.

APPENDIX B--SAMPLE DATA PLOTS

Appendix B contains some sample plots of the more meaningful data, computed by the program IMPACT, such as the body components of linear and angular velocity, the potential and kinetic energy, body components of external and inertial force, and the inertial components of linear and angular momentum. Not shown is the diagnostic information computed by the program for each of the sample plots.

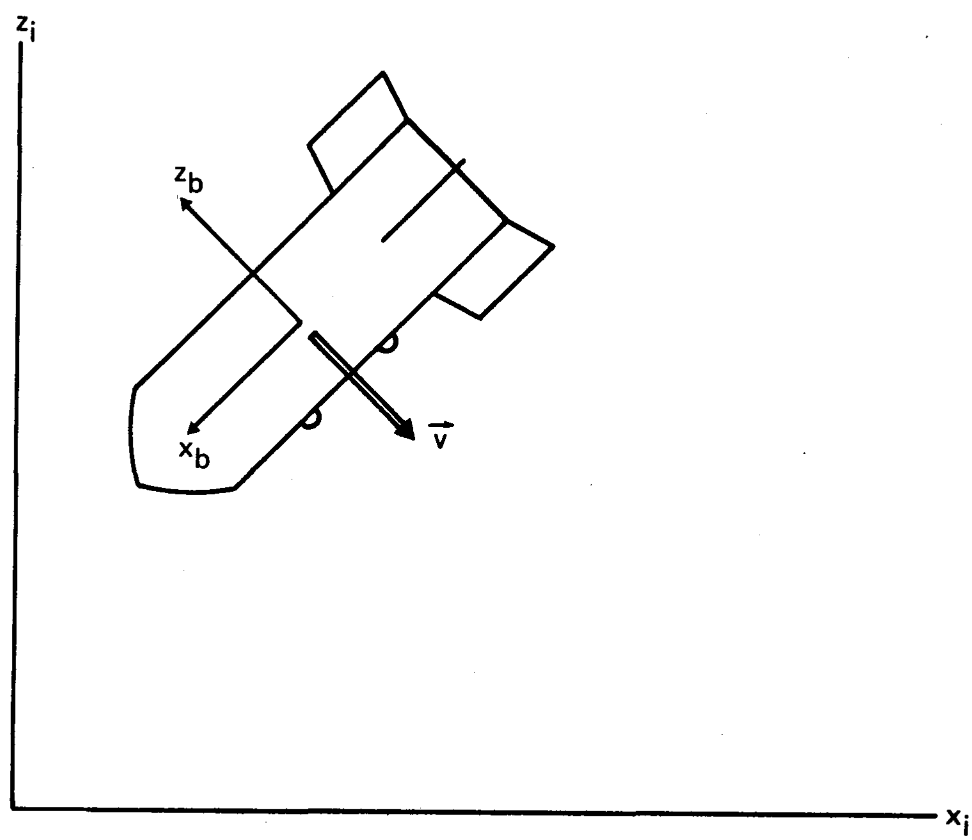
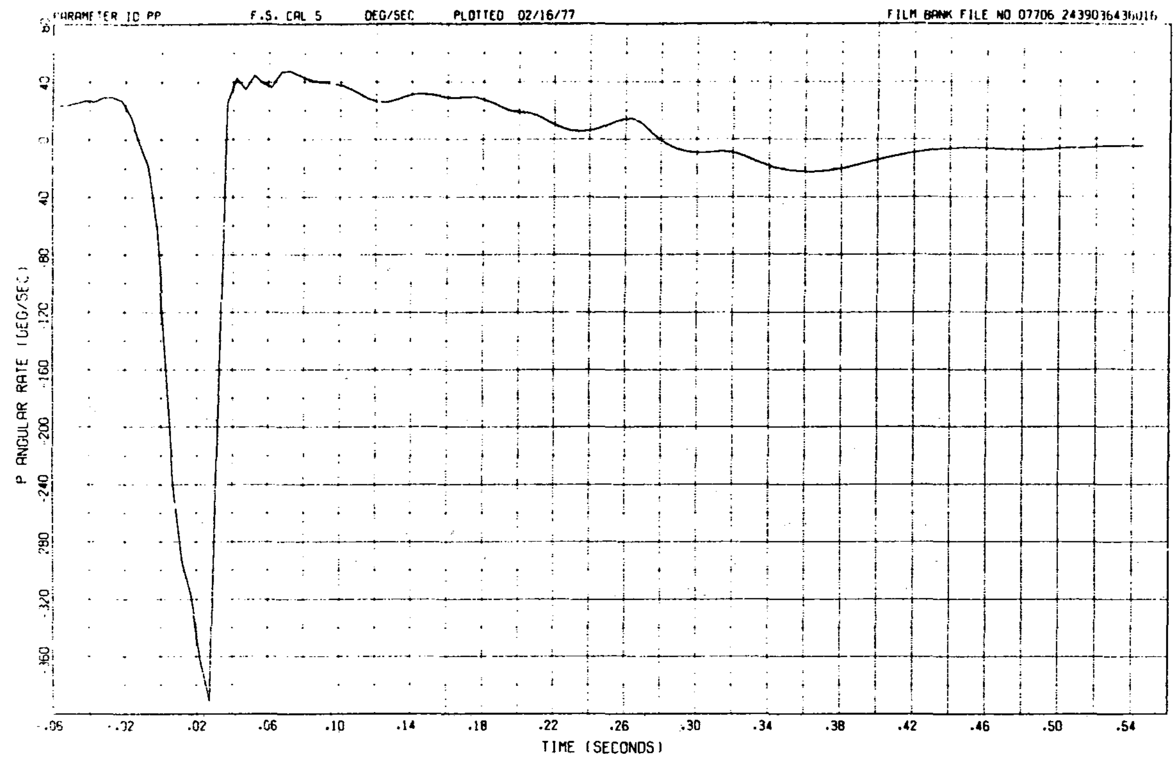
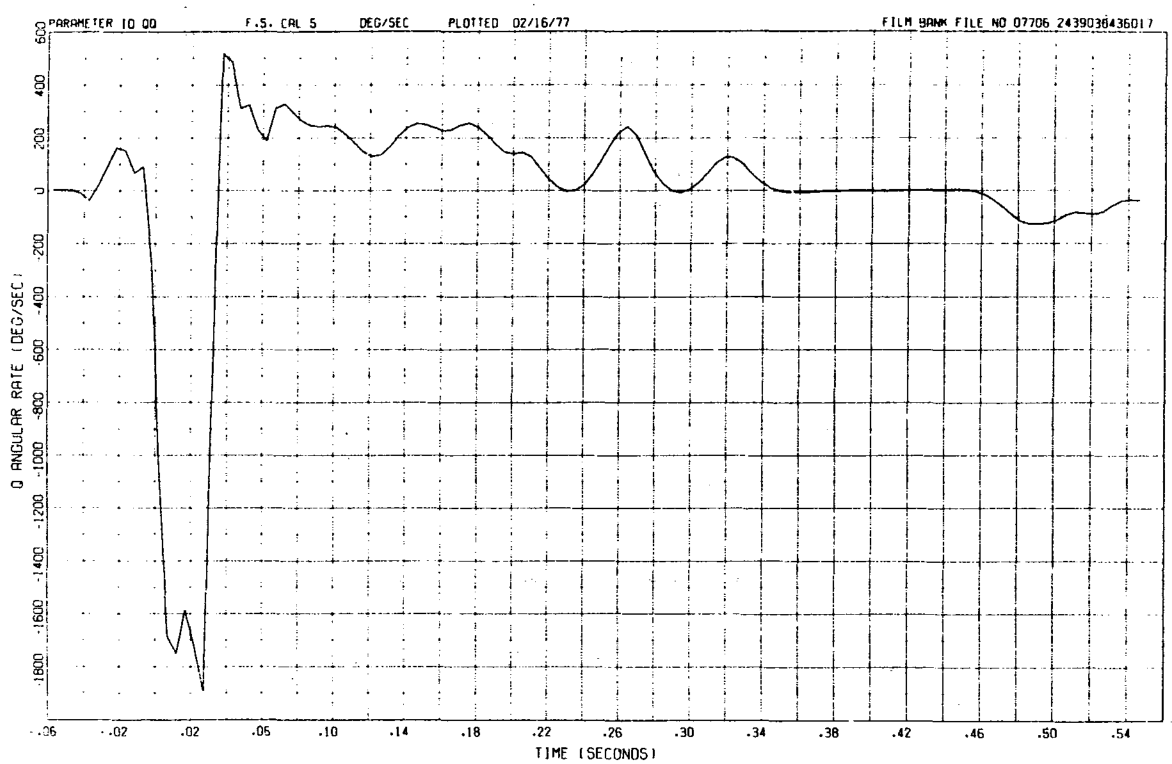
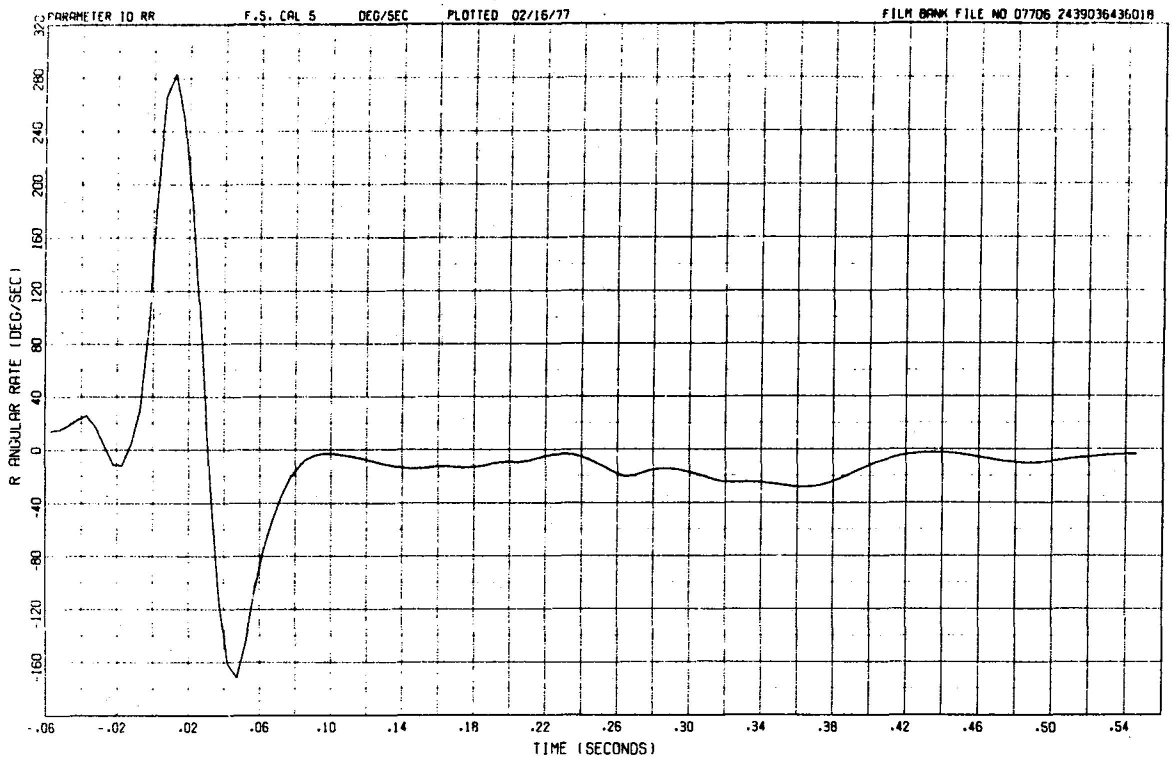
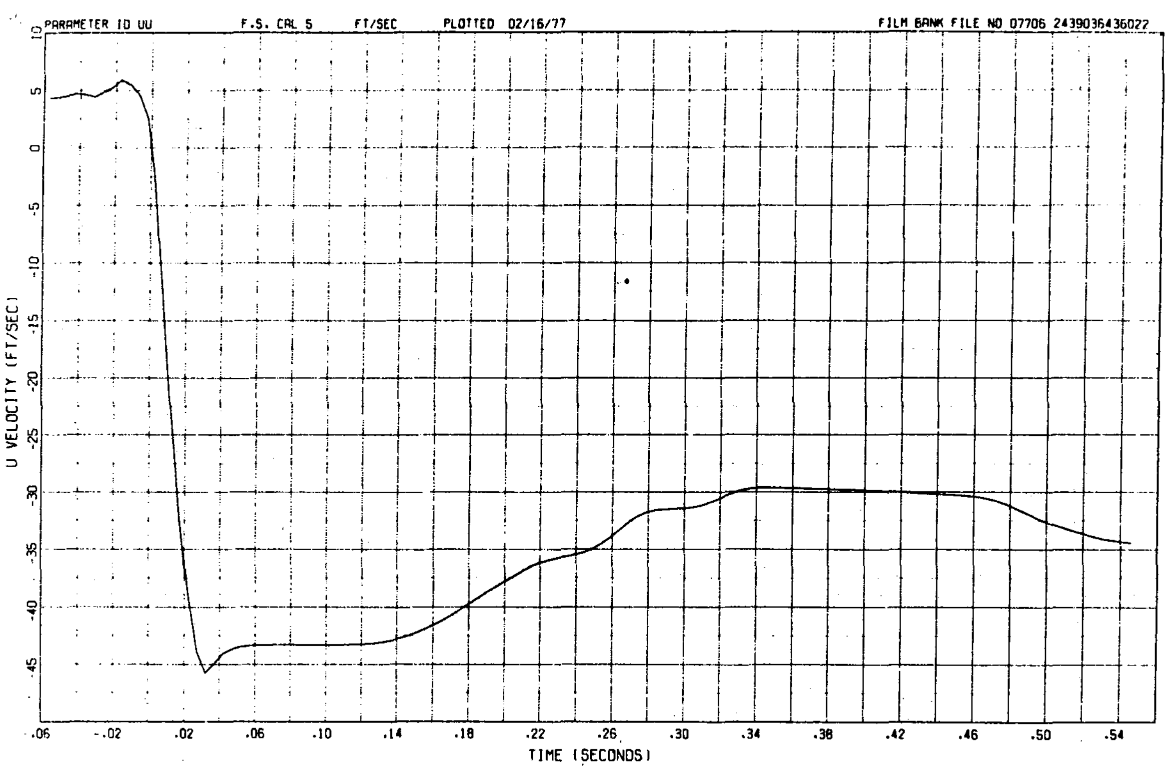


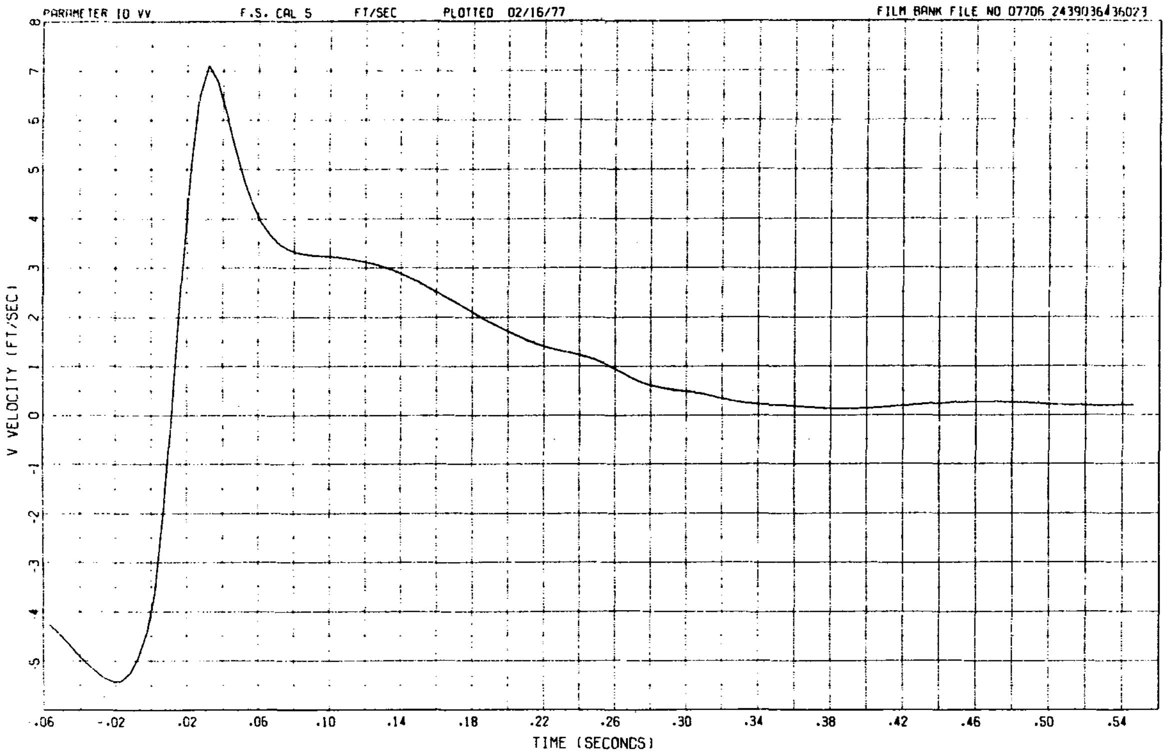
Figure B-1. Initial Conditions for ITU-130A

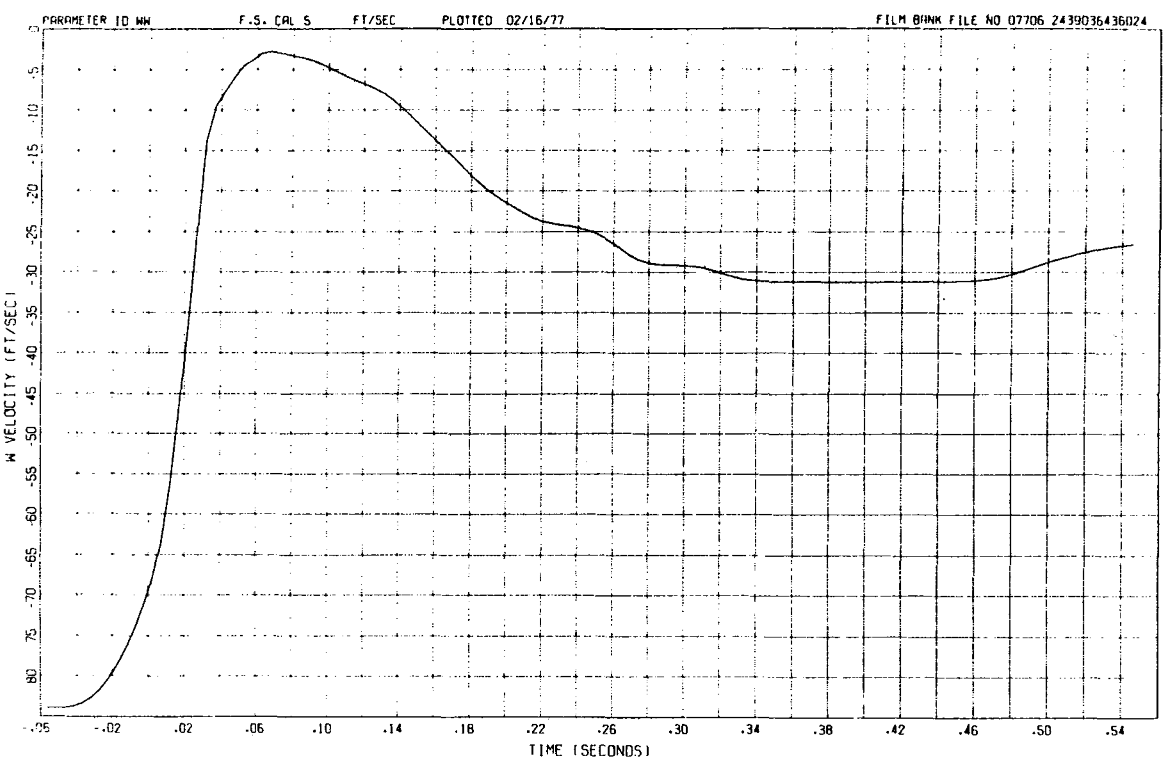

PARAMETER ID PP
F.S. CAL 5
DEG/SEC
PLOTTED 02/16/77
FILM BANK FILE NO 07706 243903643
P ANGULAR RATE (DEG/SEC)
TIME (SECONDS)

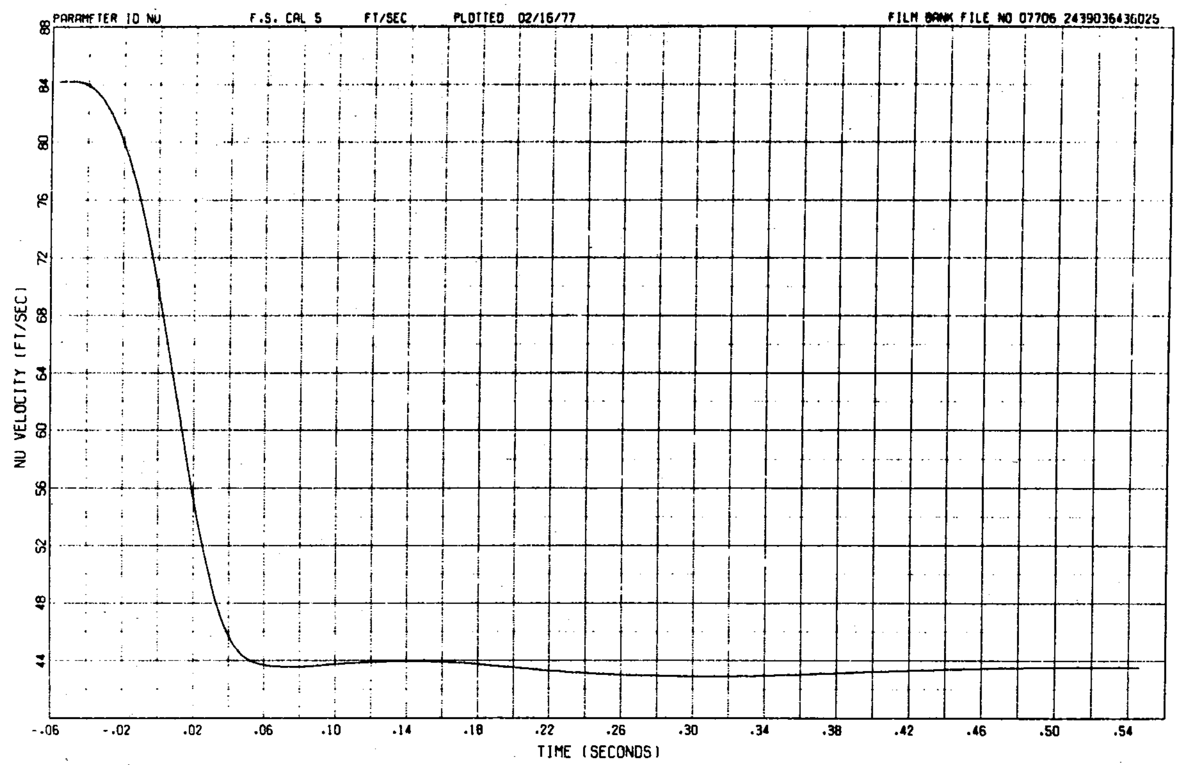

PARAMETER ID QQ
F.S. CAL 5
DEG/SEC
PLOTTED 02/16/77
FILM BANK FILE NO 07706 243903643601
Q ANGULAR RATE (DEG/SEC)
TIME (SECONDS)

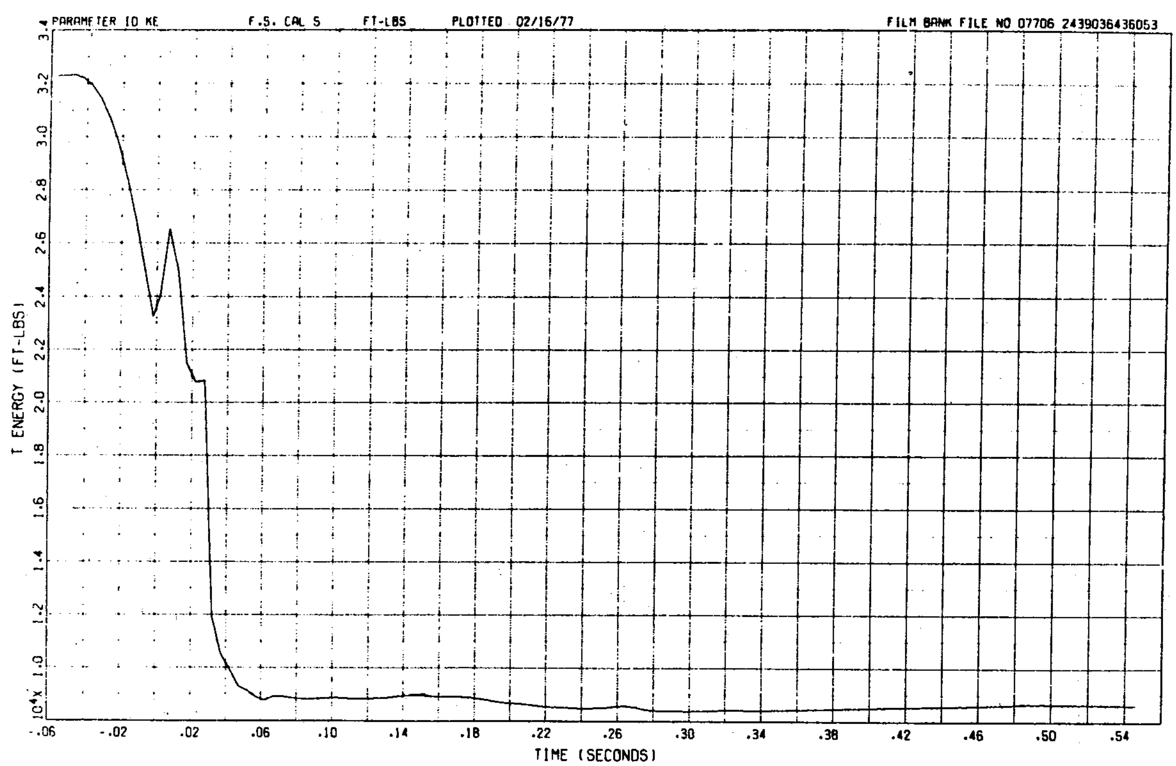

PARAMETER ID RR
F.S. CAL 5
DEG/SEC
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436018
R ANGULAR RATE (DEG/SEC)
320
280
240
200
160
120
80
40
0
-40
-80
-120
-160
-.06
-.02
.02
.06
.10
.14
.18
.22
.26
.30
.34
.38
.42
.46
.50
.54
TIME (SECONDS)

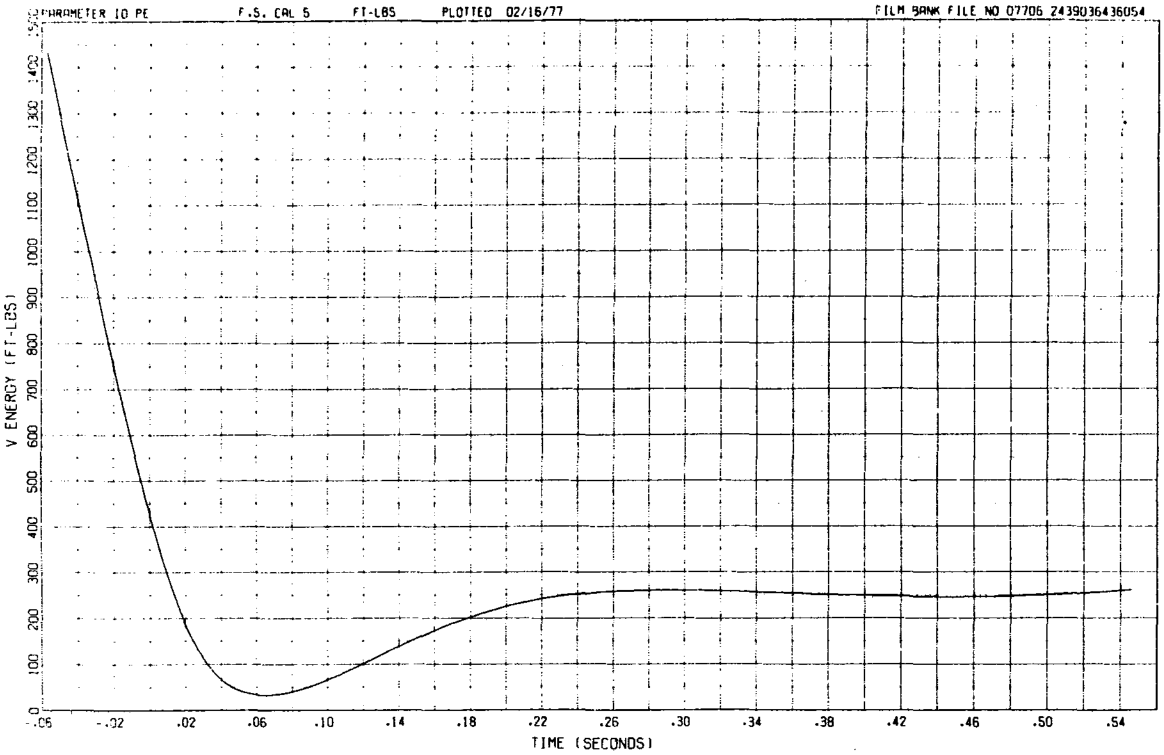

PARAMETER ID UU
F.S. CAL 5
FT/SEC
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436022
U VELOCITY (FT/SEC)
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-.06
-.02
.02
.06
.10
.14
.18
.22
.26
.30
.34
.38
.42
.46
.50
.54
TIME (SECONDS)

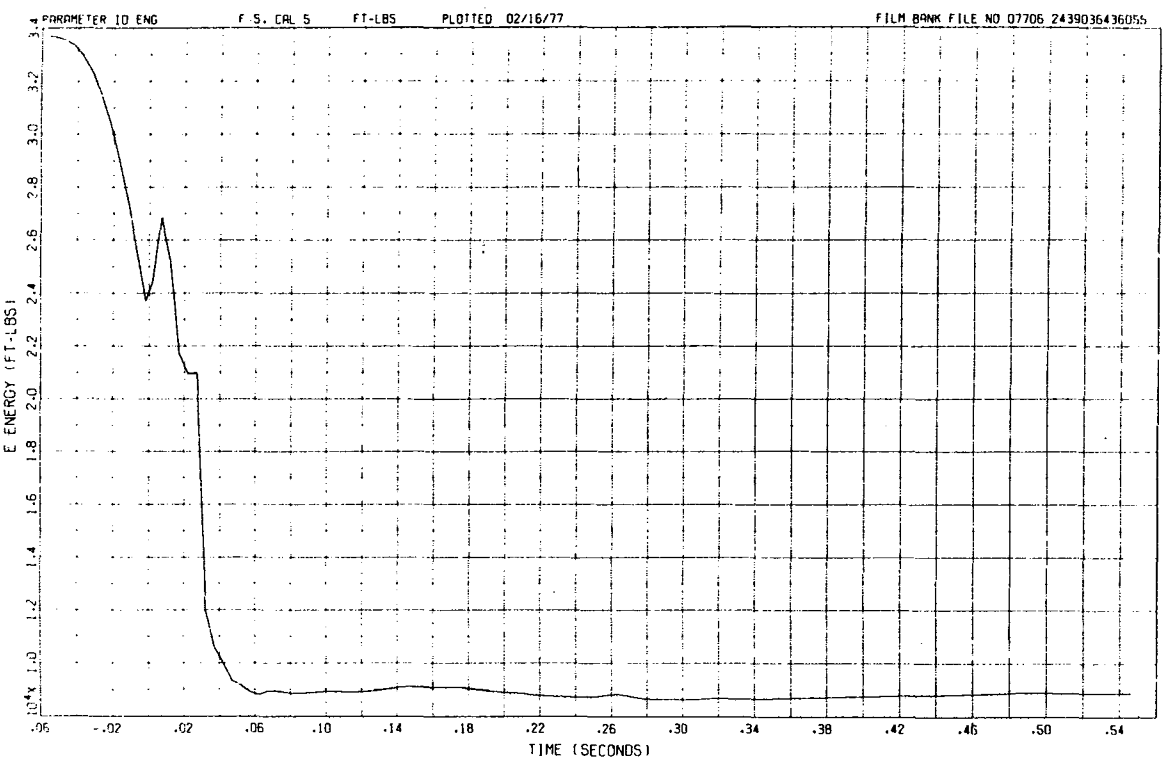

PARAMETER ID VV
F.S. CAL 5
FT/SEC
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436023
V VELOCITY (FT/SEC)
TIME (SECONDS)

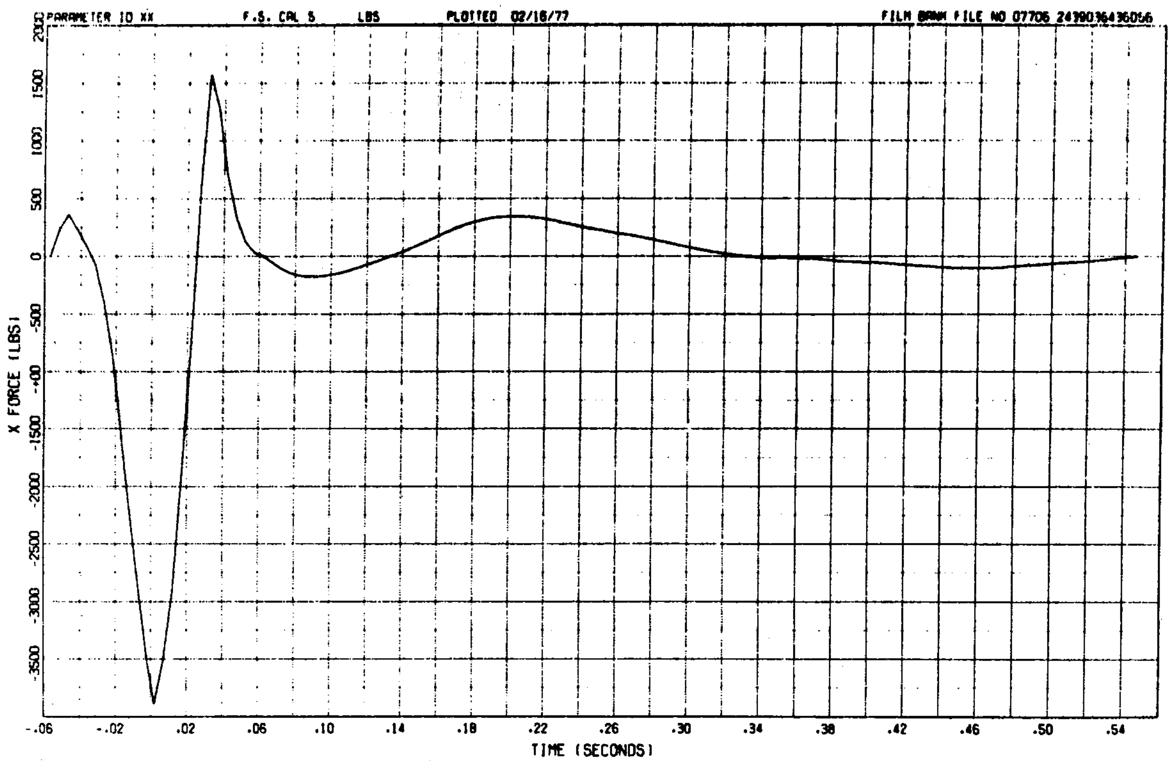

PARAMETER ID WW
F.S. CAL 5
FT/SEC
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436024
W VELOCITY (FT/SEC)
TIME (SECONDS)

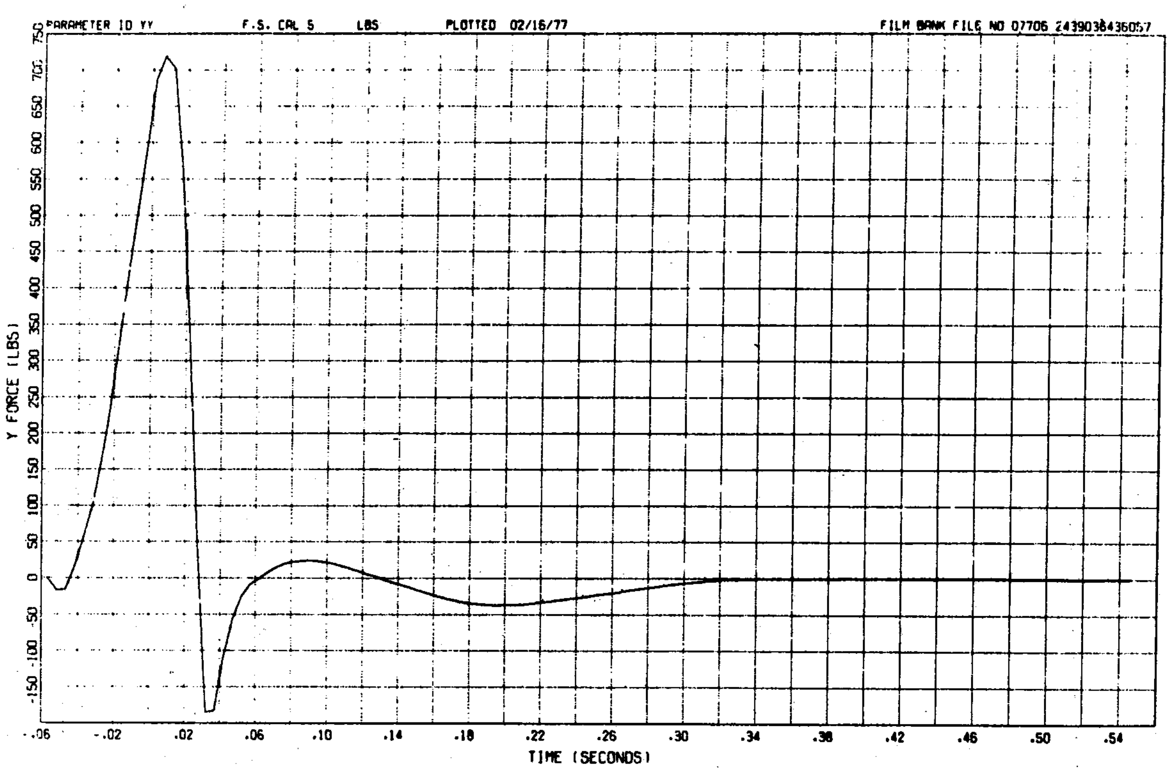

PARAMETER ID NU
F.S. CAL 5
FT/SEC
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436025
NU VELOCITY (FT/SEC)
88
84
80
76
72
68
64
60
56
52
48
44
-.06
-.02
.02
.06
.10
.14
.18
.22
.26
.30
.34
.38
.42
.46
.50
.54
TIME (SECONDS)

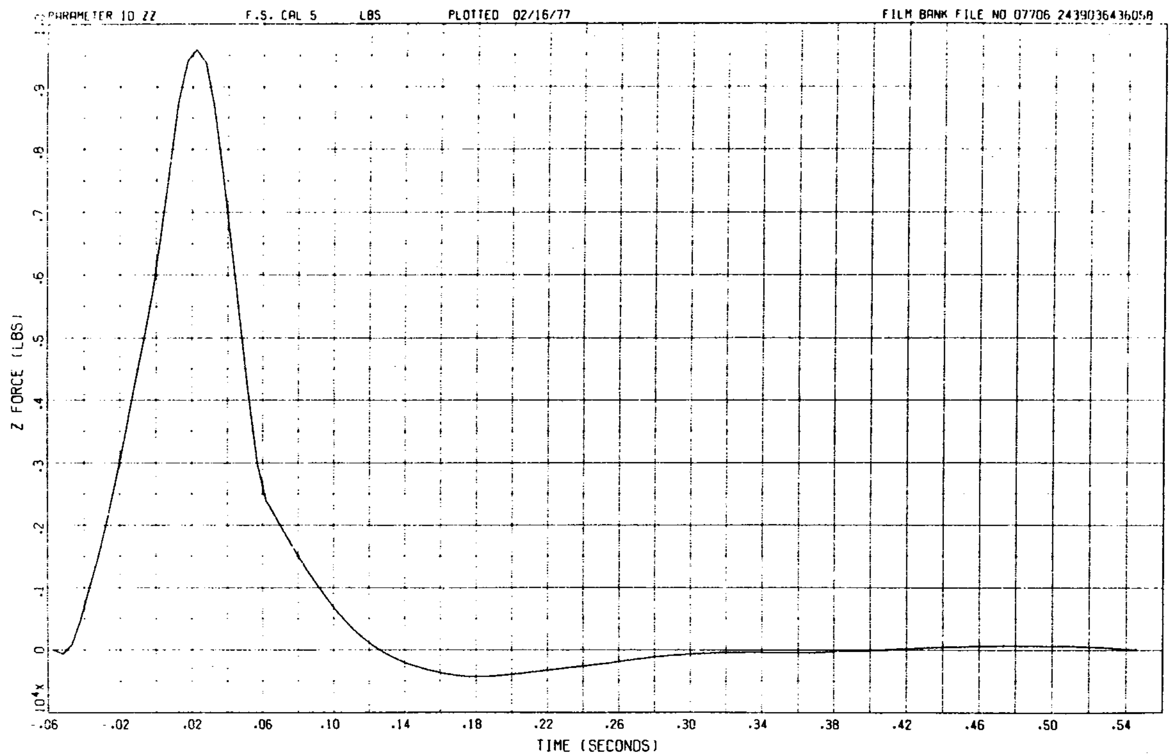

PARAMETER ID KE
F.S. CAL 5
FT-LBS
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436053
T ENERGY (FT-LBS)
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
10⁴X
-.06
-.02
.02
.06
.10
.14
.18
.22
.26
.30
.34
.38
.42
.46
.50
.54
TIME (SECONDS)

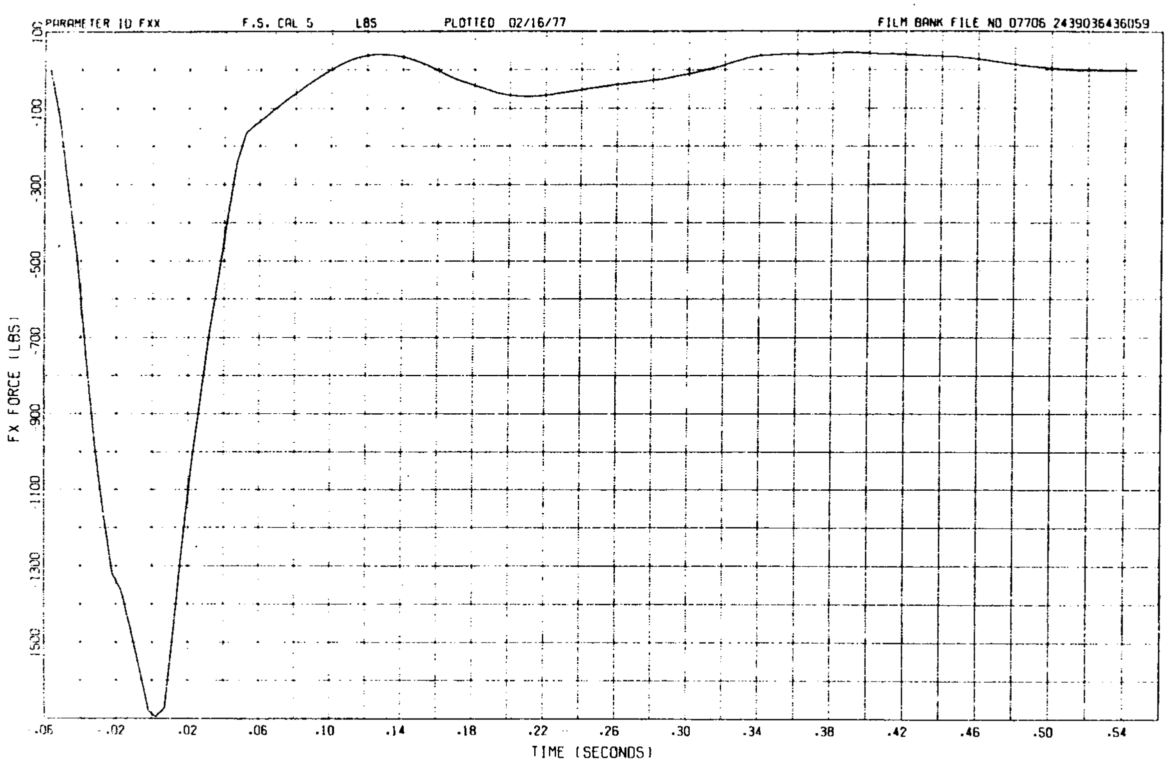

PARAMETER ID PE
F.S. CAL 5
FT-LBS
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436054
V ENERGY (FT-LBS)
TIME (SECONDS)

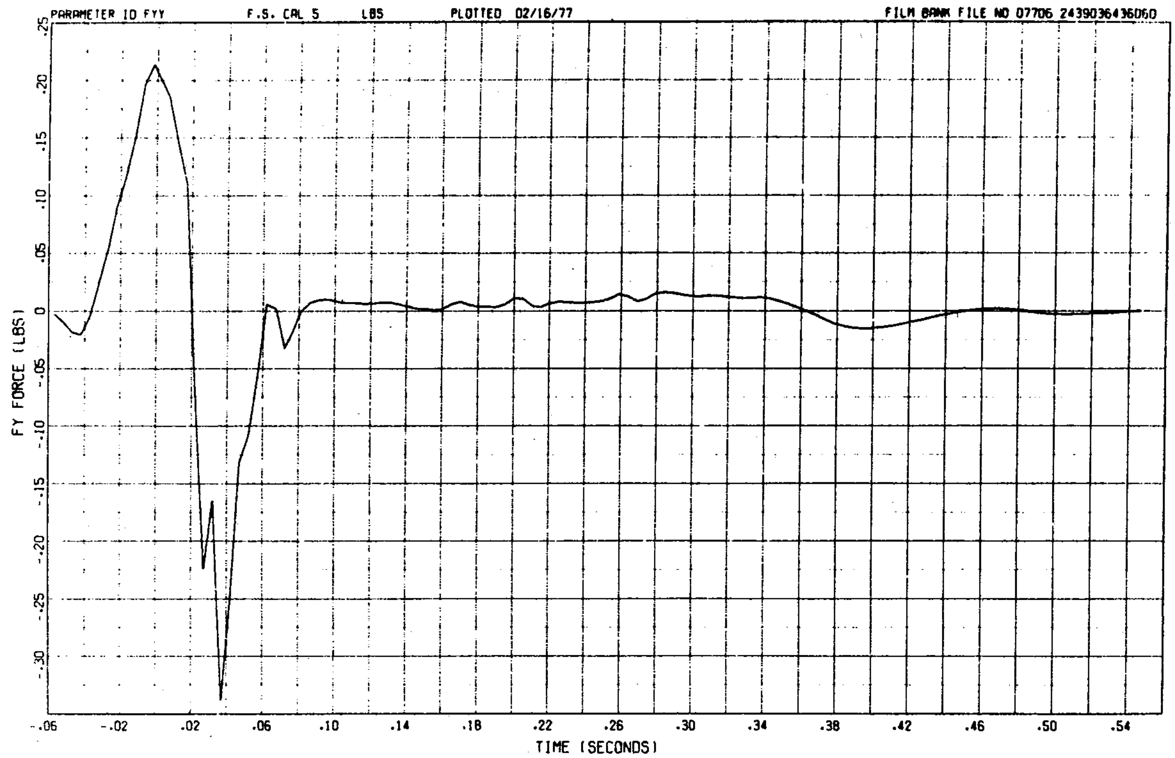

PARAMETER ID ENG
F.S. CAL 5
FT-LBS
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436055
E ENERGY (FT-LBS)
TIME (SECONDS)

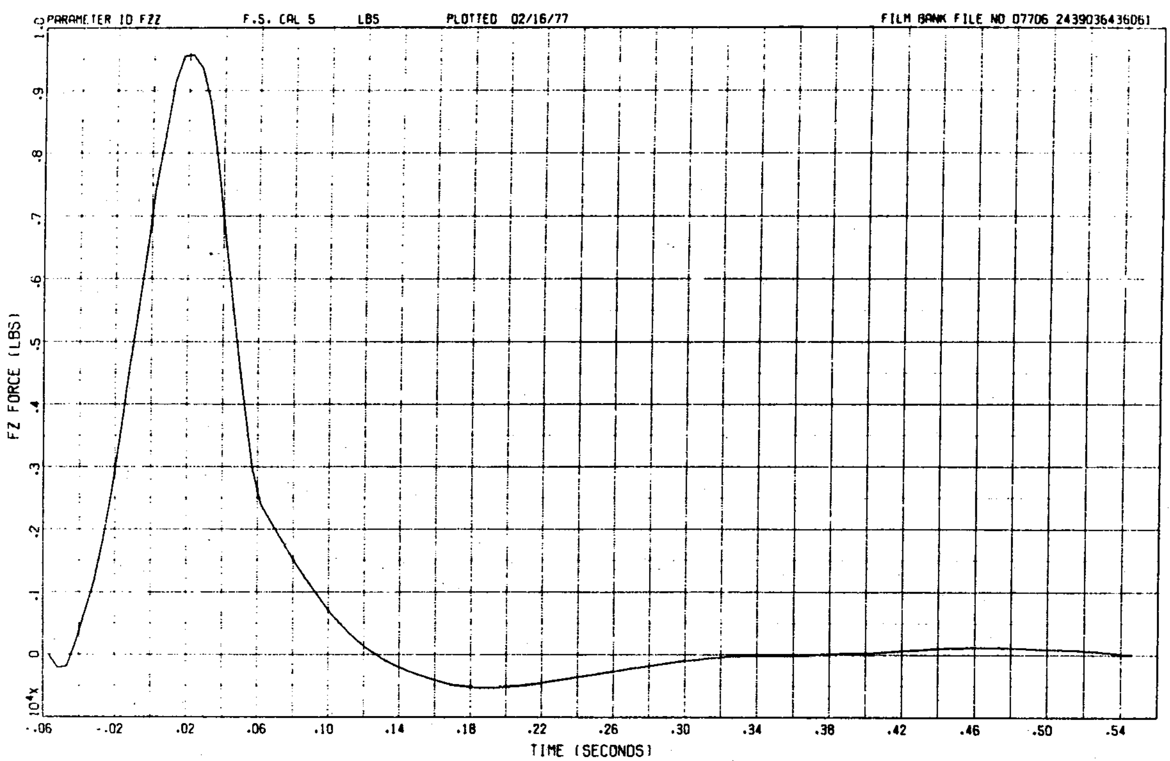

PARAMETER ID XX
F.S. CAL 5
LBS
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436056
X FORCE (LBS)
2000
1500
1000
500
0
-500
-1000
-1500
-2000
-2500
-3000
-3500
-.06
-.02
.02
.06
.10
.14
.18
.22
.26
.30
.34
.38
.42
.46
.50
.54
TIME (SECONDS)

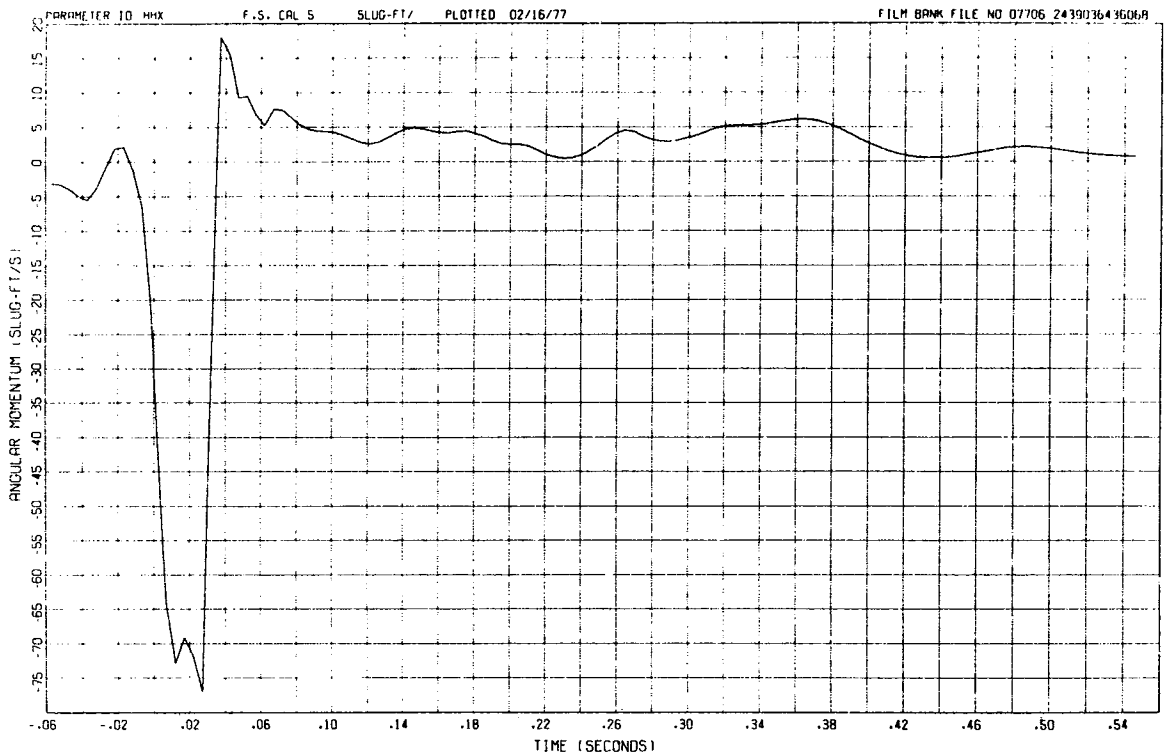

PARAMETER ID YY
F.S. CAL 5
LBS
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436057
Y FORCE (LBS)
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
-50
-100
-150
-.06
-.02
.02
.06
.10
.14
.18
.22
.26
.30
.34
.38
.42
.46
.50
.54
TIME (SECONDS)

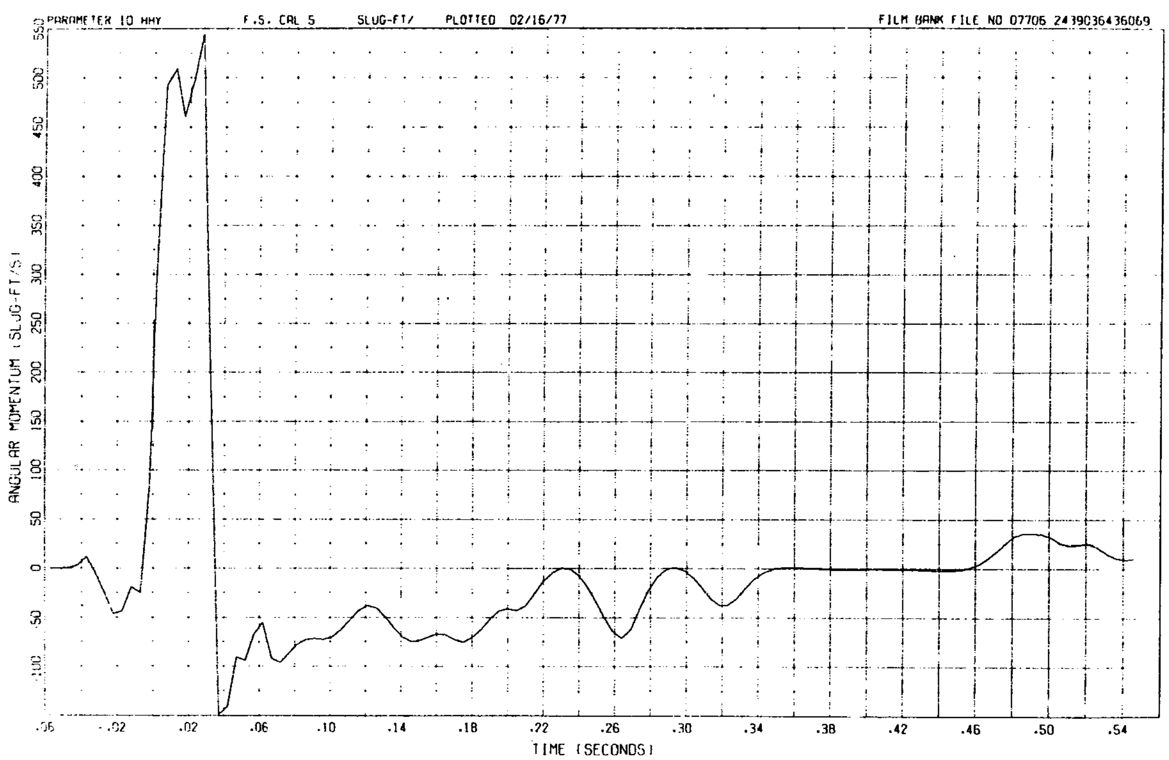

PARAMETER ID ZZ F.S. CAL 5 LBS PLOTTED 02/16/77 FILM BANK FILE NO 07706 2439036436058
Z FORCE (LBS)
10^4 x
TIME (SECONDS)

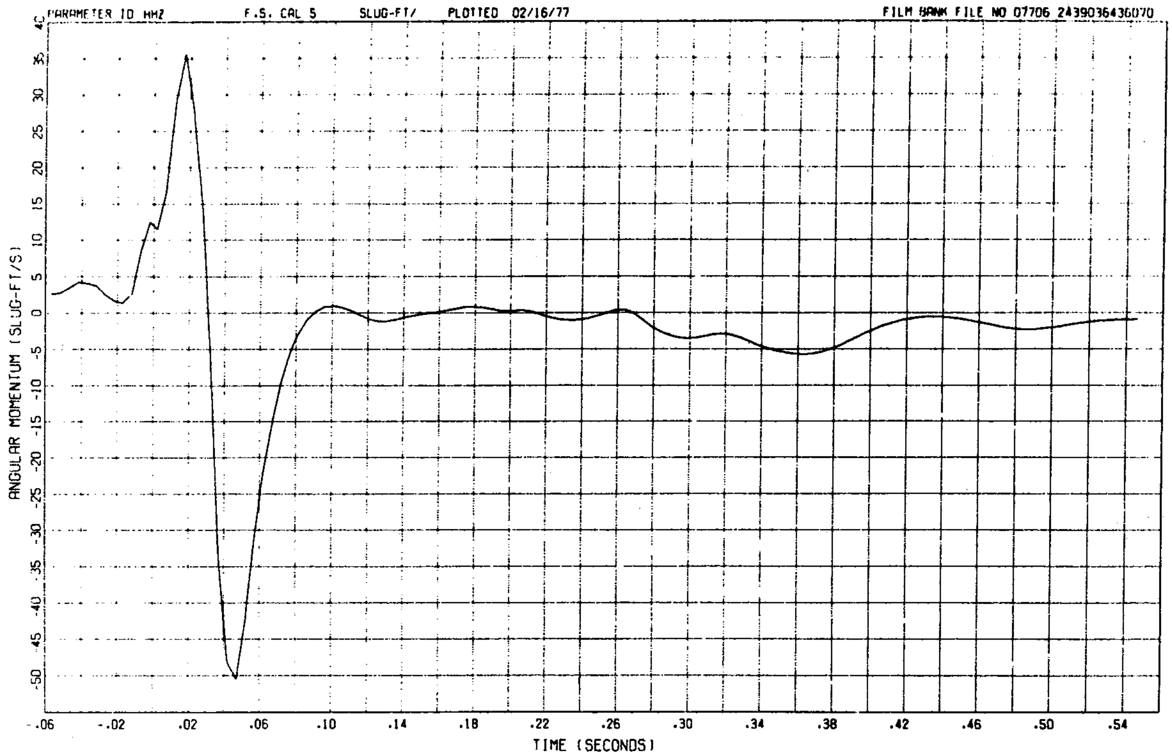

PARAMETER ID FXX F.S. CAL 5 LBS PLOTTED 02/16/77 FILM BANK FILE NO 07706 2439036436059
FX FORCE (LBS)
TIME (SECONDS)

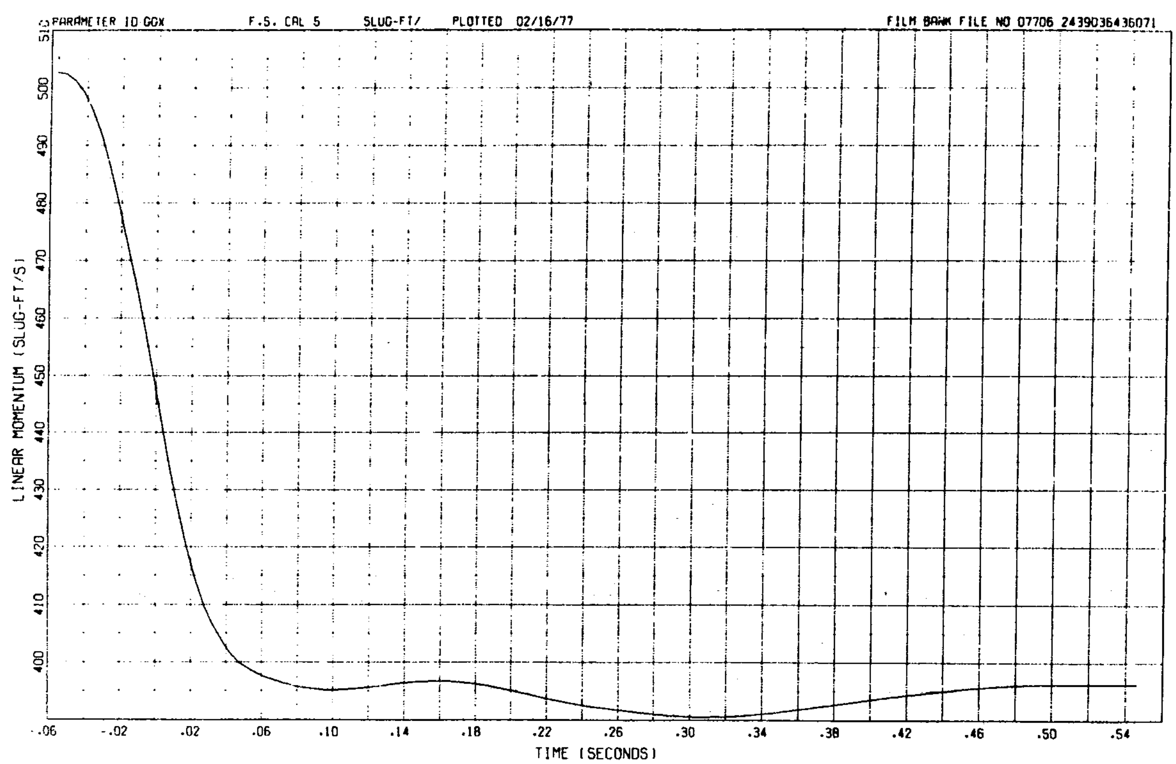

PARAMETER ID FYY F.S. CAL 5 LBS PLOTTED 02/16/77 FILM BANK FILE NO 07706 2439036436060
FY FORCE (LBS)
TIME (SECONDS)

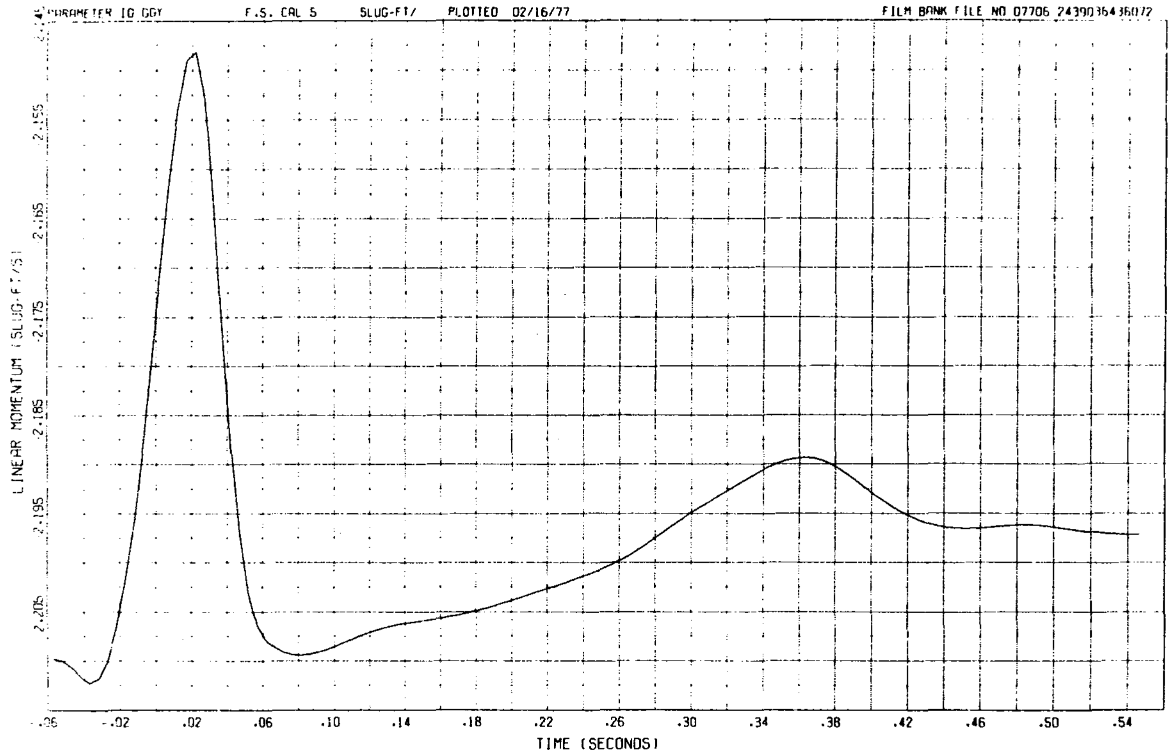

PARAMETER ID FZZ F.S. CAL 5 LBS PLOTTED 02/16/77 FILM BANK FILE NO 07706 2439036436061
FZ FORCE (LBS)
10^4 x
TIME (SECONDS)

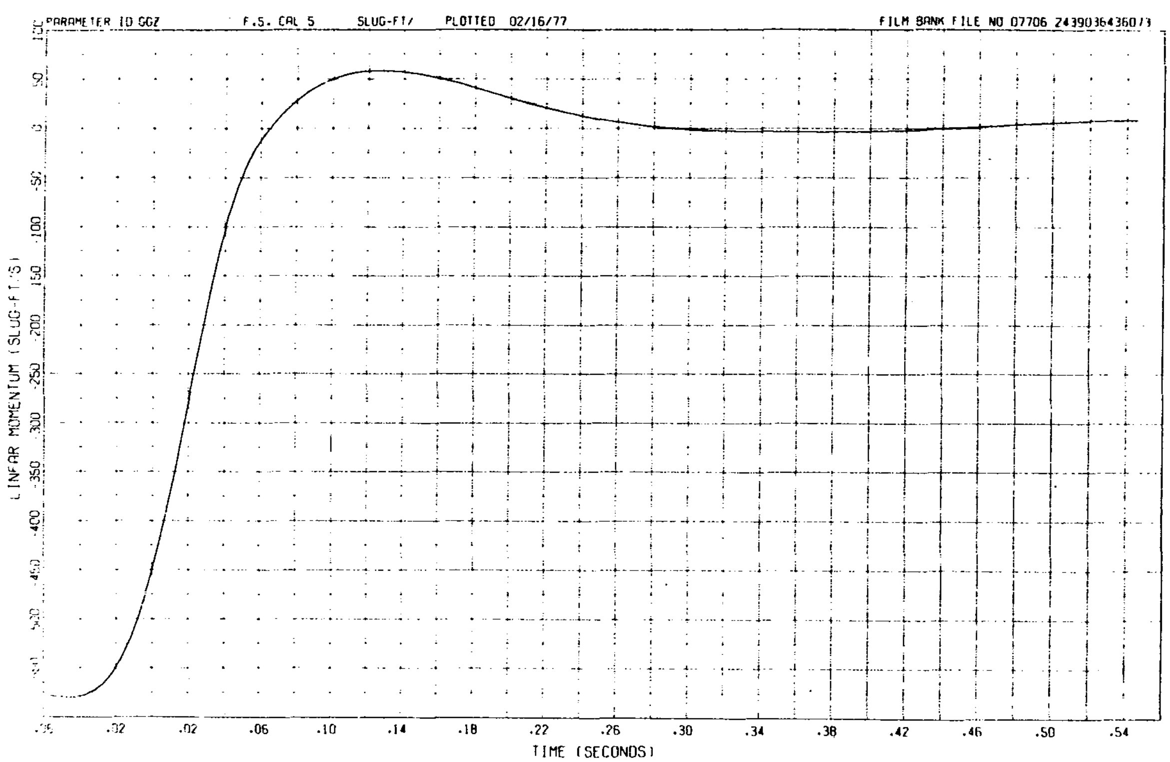

PARAMETER 10 HHX
F.S. CAL 5
SLUG-FT/
PLOTTED 02/16/77
FILM BANK FILE NO 07706 243903643606B
ANGULAR MOMENTUM (SLUG-FT/S)
TIME (SECONDS)


PARAMETER 10 HHY
F.S. CAL 5
SLUG-FT/
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436069
ANGULAR MOMENTUM (SLUG-FT/S)
TIME (SECONDS)


PARAMETER ID HHZ
F.S. CAL 5
SLUG-FT/
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436070
ANGULAR MOMENTUM (SLUG-FT/S)
TIME (SECONDS)


PARAMETER ID GGX
F.S. CAL 5
SLUG-FT/
PLOTTED 02/16/77
FILM BANK FILE NO 07706 2439036436071
LINEAR MOMENTUM (SLUG-FT/S)
TIME (SECONDS)


PARAMETER ID GGY
F.S. CAL 5
SLUG-FT/
PLOTTED 02/16/77
FILM BANK FILE NO 07706 243903643 6072
LINEAR MOMENTUM (SLUG-FT/S)
TIME (SECONDS)


PARAMETER ID GGZ
F.S. CAL 5
SLUG-FT/
PLOTTED 02/16/77
FILM BANK FILE NO 07706 24390364360 73
LINEAR MOMENTUM (SLUG-FT/S)
TIME (SECONDS)

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